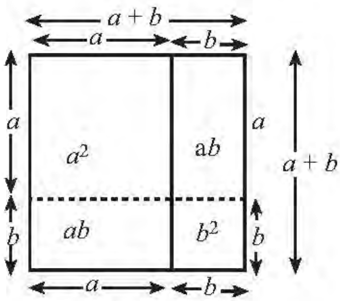
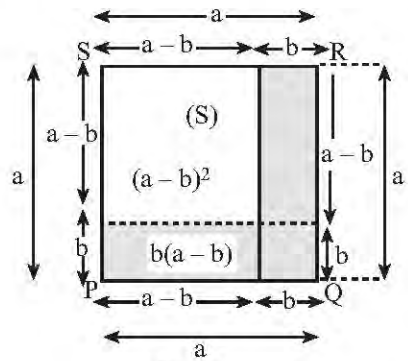
Lesson 9 Factorization

9.0 Review

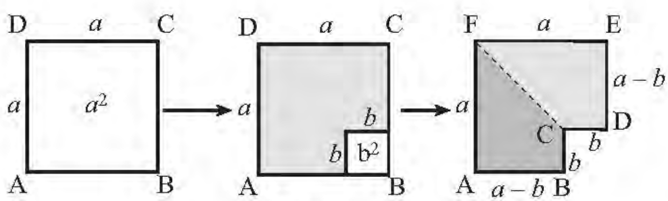
Make different groups in your class. Observe the following pictures carefully and find the area of each figure. Present it in your class.



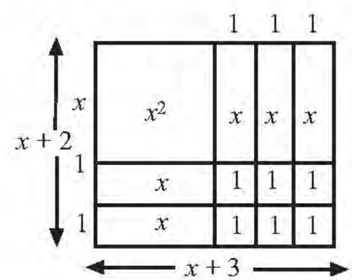
चित्र (a)



चित्र (b)



चित्र (c)



चित्र (d)

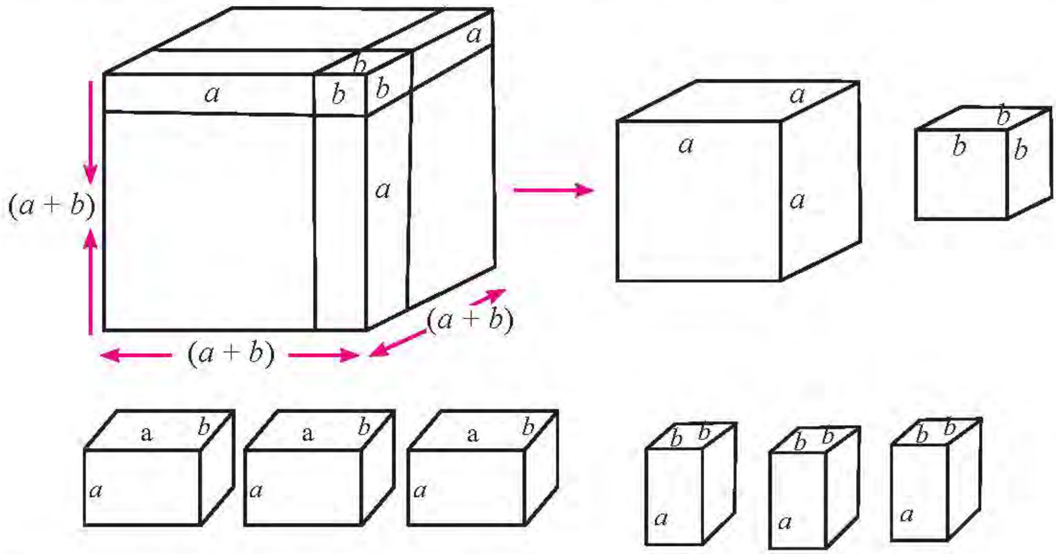
9.1 Factorization of the expression in the form of $(a + b)^3$, $(a^3 + b^3)$, $(a - b)^3$ and $(a^3 - b^3)$

9.1.1 Geometrical concept of $(a + b)^3$

Activity 1

Make different groups of students in your class. Take a soft cubical object in each group eg. a piece of soap. Mark in the ratio of $a{:}b$ on the object's length, breadth, and height as shown in figure first. Now, cut the object from marked places. Observe the number of pieces and shape of each piece in your group. Present the conclusions in your class.

Find the volume of each piece. Compare the volume of the cube and the sum of the volume of all pieces. Discuss this relationship in your class.



For example, the conclusion of group A is as follows:

The volume of the cube = Volume of some of all pieces.

$$\text{Thus, } (a+b)^3 = a^3 + b^3 + a^2b + a^2b + a^2b + ab^2 + ab^2 + ab^2$$
$$= a^3 + 3a^2b + 3ab^2 + b^3$$

$$= a^3 + 3ab\,(a+b) + b^3$$

(i) $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

(ii) $(a+b)^3 = a^3 + 3ab\,(a+b) + b^3$ (Make 3ab common from second and third term)

(iii) $(a+b)^3 = (a+b) \times (a+b) \times (a+b)$

9.1.2 Geometrical concept of $(a-b)^3$

Activity 2

Make different groups of students in your class. Take a soft cubical object each group eg. a piece of soap. Suppose the length of each edge is a unit. Draw a line on (a-b) unit of each edge and cut it into eight different pieces as shown in the following figure. Find the volume of all pieces separately. Compare the volume of the cube and the sum of the volume of all pieces. Discuss this relationship in your class. For example, the conclusion of group B is as follows:

Now, the volume of the cube = Volume of some of all pieces

$$a^3 = (a-b)^3 + b^3 + (a-b)^2 \cdot b + (a-b)^2 \cdot b + (a-b)^2 b + (a-b)b^2 + (a-b)b^2 + (a-b)b^2$$

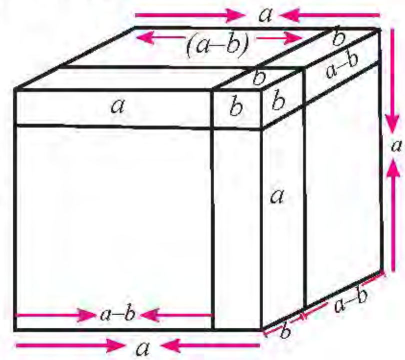
$$= (a-b)^3 + b^3 + 3(a-b)^2 b + 3(a-b)b^2$$

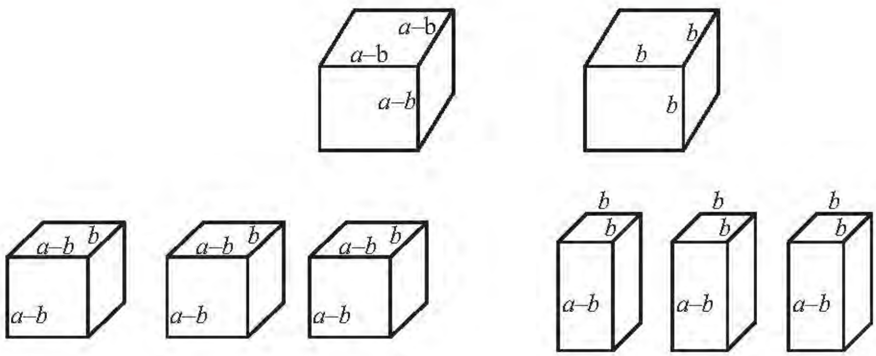
$$= (a-b)^3 + b^3 + 3b(a^2 - 2ab + b^2) + 3ab^2 - 3b^3$$

$$= (a-b)^3 + b^3 + 3a^2 b - 6ab^2 + 3b^3 + 3ab^2 - 3b^3$$

$$= (a-b)^3 + 3a^2 b - 3ab^2 + b^3$$

or, $(a-b)^3 = a^3 - 3a^2 b + 3ab^2 - b^3$





$= a^3 - 3ab(a-b) - b^3$ ($\because$ Take 3ab common from second and third terms)

(a) $(a-b)^3 = a^3 - 3ab^2 + 3ab^2 - b^3$

(b) $(a-b)^3 = a^3 - 3ab(a-b) - b^3$

(c) $(a-b)^3 = (a-b) \times (a-b) \times (a-b)$

9.1.3 Simplified form of (a^3+b^3)

Activity 3

Make different groups of students in your class. Using the geometrical concept of $(a + b)^3$ and $(a - b)^3$, discuss the simplified form of $(a^3 + b^3)$ in your group.

Here, the conclusion of group C is:

We know that,

$$(a + b)^3 = a^3 + b^3 + 3ab\,(a + b)$$

or, $(a + b)^3 = 3ab\,(a + b) + a^3 + b^3$

or, $(a + b)\,(a + b)^2 - 3ab\,(a + b) = a^3 + b^3$

or, $(a + b)\,\{(a + b)^2 - 3ab\} = a^3 + b^3$ [$\because$ $(a + b)$ is common on both]

or, $(a + b)\,(a^2 + 2ab + b^2 - 3ab) = a^3 + b^3$ [$\because$ using formula of $(a + b)^2$]

or, $(a + b)\,(a^2 - ab + b^2) = a^3 + b^3$

(i) $a^3 + b^3 = (a + b)^3 - 3ab\,(a + b)$

(ii) $a^3 + b^3 = (a + b)\,(a^2 - ab + b^2)$

9.1.4 Simplified form of $(a^3 - b^3)$

We know that,

$$(a - b)^3 = a^3 - b^3 - 3ab\,(a - b)$$

or, $(a - b)^3 + 3ab\,(a - b) = a^3 - b^3$

or, $(a - b)\,(a - b)^2 + 3ab\,(a - b) = a^3 - b^3$

or, $(a - b)\,\{(a - b)^2 + 3ab\} = a^3 - b^3$ [$\because$ Taking common $(a - b)$]

or, $(a - b)\,(a^2 - 2ab + b^2 + 3ab) = a^3 - b^3$

(i) $a^3 - b^3 = (a - b)^3 + 3ab\,(a - b)$

(ii) $a^3 - b^3 = (a - b)\,(a^2 + ab + b^2)$

Example 1

Find the cube of $(x + 3)$ using formula:

Solution,

Here,

Cube of $(x + 3)$

$= (x + 3)^3$

$= (x)^3 + 3 \cdot x^2 \cdot 3 + 3 \cdot x \cdot 3^2 + 3^3$ $\quad\quad$ $[(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3]$

$= x^3 + 9x^2 + 27x + 27$

Example 2

Find the cube of (x – 2):

Solution,

Here, cube of $(x – 2)$

$= (x – 2)^3$

$= (x)^3 – 3 \cdot x^2 \cdot 2 + 3 \cdot x \cdot 2^2 – (2)^3$ $\quad\quad$ $[(a – b)^3 = a^3 –3a^2b + 3ab^2 – b^3]$

$= x^3 – 6x^2 + 12x – 8$

Example 3

Factorize:

(a) $(3x + 5y)^3$ $\quad\quad$ (b) $(2x – 7y)^3$

Solution,

Here,

(a) $(3x + 5y)^3 = (3x + 5y)(3x + 5y)(3x + 5y)$

(b) $(2x – 7y)^3 = (2x – 7y)(2x – 7y)(2x – 7y)$

Example 4

Factorize:

(a) $8x^3 + y^3$ $\quad\quad$ b) $\dfrac{p^3}{q^3} - \dfrac{q^3}{p^3}$

Solution,

Here,

(a) $8x^3 + y^3$

$= (2x)^3 + (y)^3$

$= (2x + y)\{(2x)^2 – 2x \cdot y + (y)^2\}$ $\quad\quad$ $[\because a^3 + b^3 = (a + b)(a^2 – ab + b^2)]$

$= (2x + y)(4x^2 – 2xy + y^2)$

(b) $\frac{p^3}{q^3} - \frac{q^3}{p^3} = \left(\frac{p}{q}\right)^3 - \left(\frac{q}{p}\right)^3$

$$= \left(\frac{p}{q} - \frac{q}{p}\right)\left(\frac{p^2}{q^2} + 1 + \frac{q^2}{p^2}\right) \; [\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)]$$

$$= \left(\frac{p}{q} - \frac{q}{p}\right)\left\{\left(\frac{p}{q}\right)^2 + \left(\frac{q}{p}\right)^2 + 1\right\}$$

$$= \left(\frac{p}{q} - \frac{q}{p}\right)\left\{\left(\frac{p}{q} + \frac{q}{p}\right)^2 - 2\frac{p}{q}.\frac{q}{p} + 1\right\}$$

$$= \left(\frac{p}{q} - \frac{q}{p}\right)\left\{\left(\frac{p}{q} + \frac{q}{p}\right)^2 - 1\right\}$$

$$= \left(\frac{p}{q} - \frac{q}{p}\right)\left\{\left(\frac{p}{q} + \frac{q}{p}\right)^2 - (1)^2\right\}$$

$$= \left(\frac{p}{q} - \frac{q}{p}\right)\left(\frac{p}{q} + 1 + \frac{q}{p}\right)\left(\frac{p}{q} - 1 + \frac{q}{p}\right)$$

Exercise 9.1

1. Find the cube of following expressions using formula:

(a) $(x + 1)$ (b) $(x - 3)$

(c) $(x + 4)$ (d) $(2x - 1)$

(e) $(3y + 2b)$ (f) $\left(\frac{x}{2} - 1\right)$

2. Write the following expressions in the expanded form:

(a) $(2x + 3y)^3$ (b) $(5a - 8b)^3$

3. Write the given expressions in the form of $(a + b)^3$:

(a) $27a^3 + 108a^2b + 144ab^2 + 64b^3$

(b) $8x^3 + 36x^2y + 54xy^2 + 27y^3$

4. Write the following expressions in the form of $(a - b)^3$:

(a) $64m^3 - 48m^2n + 12\,mn^2 - n^3$

(b) $125p^3 - 150p^2q + 60pq^2 - 8q^3$

5. **Factorize the following:**

(a) $8x^4 + 27x$ (b) $x^3 + 8y^3$ (c) $1 + 125x^3$

(d) $8x^3 + \frac{1}{x^3}$ (e) $(a + b)^3 + 1$ (f) $x^6 + y^6$

6. **Factorize the following expressions:**

(a) $250m^4 - 2m$ (b) $x^3y - 64y^4$ (c) $64p^6q^3 - 125$

(d) $(a - b)^3 - 8(a + b)^3$ (e) $\frac{x^3}{y^3} - \frac{y^3}{x^3}$ (f) $p^3 - \frac{1}{p^3}$

7. How many cubic feet wood is remain if we cut a pieces of cubical wood having length 7 ft from a cubical log of x feet length? ($x > 7$ ft)

Answer

1. (a) $x^3 + 3x^2 + 3x + 1$ (b) $x^3 - 9x^2 + 27x - 27$ (c) $x^3 + 12x^2 + 48x + 64$

   (d) $8x^3 - 12x^2 + 6x - 1$ (e) $27y^3 + 54y^2b + 36yb^2 + 8b^3$

   (f) $\frac{1}{8}x^3 - \frac{3}{4}x^2 + \frac{3}{2}x - 1$

2. (a) $(2x + 3y)(2x + 3y)(2x + 3y)$ (b) $(5a - 8b)(5a - 8b)(5a - 8b)$

   (c) $(10p - 6q)(10p - 6q)(10p - 6q)$ (d) $(12m - 5n)(12m - 5n)(12m - 5n)$

3. (e) $(3a + 4b)^3$ (f) $(2x + 3y)^3$

4. (a) $(4m - n)^3$ (b) $(5p - 2q)^3$

5. (a) $x(2x + 3)(4x^2 - 6x + 9)$ (b) $(x + 2y)(x^2 - 2xy + 4y^2)$

   (c) $(1 + 5x)(1 - 5x + 25x^2)$ (d) $\left(2x + \frac{1}{x}\right)\left(4x^2 - 2 + \frac{1}{x^2}\right)$

   (e) $(a + b + 1)(a^2 + 2ab + b^2 - a - b + 1)$

   (f) $(x^2 + y^2)(x^4 - x^2y^2 + y^4)$

6. (a) $2m(5m - 1)(25m^2 + 5m + 1)$

   (b) $y(x - 4y)(x^2 + 4xy + 16y^2)$

   (c) $(4p^2q - 5)(16p^4q^2 + 20p^2q + 125)$

   (d) $-(a + 3b)(7a^2 + 6ab + 3b^2)$

   (e) $\left(\frac{x}{y} - \frac{y}{x}\right)\left(\frac{y^2}{x^2} + 1 + \frac{y^2}{x^2}\right)$ (f) $\left(p - \frac{1}{p}\right)\left(p^2 + 1 + \frac{1}{p^2}\right)$

7. $(x^3 - 343)$ ft^3

9.2 Factorization of the expression in the form of $a^4 + a^2b^2 + b^4$

Activity 1

Discuss how we can factorize the expression, $a^4 + a^2b^2 + b^4$ with your friend.

Here,

$= (a^2)^2 + 2a^2b^2 - a^2b^2 + (b^2)^2$

$= (a^2 + b^2)^2 - (ab)^2$ $\quad [\because (a + b)^2 = a^2 + 2ab + b^2]$

$= (a^2 + b^2 + ab)(a^2 + b^2 - ab)$ $\quad [\because a^2 - b^2 = (a + b)(a - b)]$

$= (a^2 + ab + b^2)(a^2 - ab + b^2)$

$$a^4 + a^2b^2 + b^4 = (a^2 + ab + b^2)(a^2 - ab + b^2)$$

Activity 2

Can we factorize the expression $a^4 + a^2b^2 + b^4$ other than the method used to factorize in above Activity 1? Discuss in your group.

The method of factorization presented by a group is:

$a^4 + a^2b^2 + b^4$

$= (a^2)^2 + (b^2)^2 + a^2b^2$

$= (a^2 + b^2)^2 - 2a^2b^2 + a^2b^2$ $\quad [\because a^2 + b^2 = (a + b)^2 - 2ab]$

$= (a^2 + b^2)^2 - a^2b^2$

$= (a^2 + b^2)^2 - (ab)^2$

$= (a^2 + b^2 + ab)(a^2 + b^2 - ab)$ $\quad [\because a^2 - b^2 = (a + b)(a - b)]$

$= (a^2 + ab + b^2)(a^2 - ab + b^2)$

In above example, the terms having power 4 are colleced together and use the formula $(a + b)^2 - 2ab$ to factorize given expression.

Example 1

Factorize: $y^4 + y^2 + 1$

Solution,

Here, $y^4 + y^2 + 1$

$= (y^2)^2 + (1)^2 + y^2$

$= (y^2 + 1)^2 - 2y^2 \cdot 1 + y^2$ $\quad [\because a^2 + b^2 = (a + b)^2 - 2ab]$

$= (y^2 + 1)^2 - (y)^2$

$= (y^2 + 1 + y)(y^2 + 1 - y)$ $\quad [\because a^2 - b^2 = (a + b)(a - b)]$

$= (y^2 + y + 1)(y^2 - y + 1)$

Example 2

Factorize: $y^4 + 64$

Solution,

Here, $y^4 + 64$

$= (y^2)^2 + (8)^2$

$= (y^2 + 8)^2 - 2 \cdot y^2 \cdot 8$

$= (y^2 + 8)^2 - 16y^2$

$= (y^2 + 8)^2 - (4y)^2$

$= (y^2 + 8 + 4y)(y^2 + 8 - 4y)$

$= (y^2 + 4y + 8)(y^2 - 4y + 8)$

Example 3

Factorize: $49a^4 - 154a^2b^2 + 9b^4$

Solution,

Herer, $49a^4 - 154a^2b^2 + 9b^4$

$= (7a^2)^2 + (3b^2)^2 - 154a^2b^2$

$= (7a^2 + 3b^2)^2 - 2 \cdot 7a^2 \cdot 3b^2 - 154a^2b^2$

$= (7a^2 + 3b^2)^2 - 42a^2b^2 - 154a^2b^2$

$= (7a^2 + 3b^2)^2 - 196a^2b^2$

$= (7a^2 + 3b^2)^2 - (14ab)^2$

$= (7a^2 + 3b^2 + 14ab)(7a^2 + 3b^2 - 14ab)$

$= (7a^2 + 14ab + 3b^2)(7a^2 - 14ab + 3b^2)$

Example 4

Factorize: $p^4 - 3p^2 + 1$

Solution,

Here, $p^4 - 3p^2 + 1$

$$= (p^2)^2 + (1)^2 - 3p^2$$

$$= (p^2 - 1)^2 + 2 \cdot p^2 \cdot 1 - 3p^2 \qquad [\because a^2 + b^2 = (a - b)^2 + 2ab]$$

$$= (p^2 - 1)^2 - p^2$$

$$= (p^2 - 1)^2 - (p)^2$$

$$= (p^2 - 1 + p)\,(p^2 - 1 - p)$$

$$= (p^2 + p - 1)\,(p^2 - p - 1)$$

Example 5

Factorize: $\frac{x^4}{y^4} + \frac{x^2}{y^2} + 1$

Solution,

Here,

$$\frac{x^4}{y^4} + \frac{x^2}{y^2} + 1$$

$$= \left(\frac{x^2}{y^2}\right)^2 + (1)^2 + \frac{x^2}{y^2}$$

$$= \left(\frac{x^2}{y^2} + 1\right)^2 - 2 \cdot \frac{x^2}{y^2} \cdot 1 + \frac{x^2}{y^2}$$

$$= \left(\frac{x^2}{y^2} + 1\right)^2 - \frac{x^2}{y^2}$$

$$= \left(\frac{x^2}{y^2} + 1\right)^2 - \left(\frac{x}{y}\right)^2$$

$$= \left(\frac{x^2}{y^2} + 1 + \frac{x}{y}\right)\left(\frac{x^2}{y^2} + 1 - \frac{x}{y}\right)$$

$$= \left(\frac{x^2}{y^2} + \frac{x}{y} + 1\right)\left(\frac{x^2}{y^2} - \frac{x}{y} + 1\right)$$

Example 6

Factorize: $x^2 - 10x + 24 + 6y - 9y^2$

Solution,

Here,

$$\begin{aligned}
& x^2 - 10x + 24 + 6y - 9y^2 \\
=\ & (x)^2 - 2 \cdot x \cdot 5 + (5)^2 - (5)^2 + 24 + 6y - 9y^2 \\
=\ & (x-5)^2 - 25 + 24 + 6y - 9y^2 \\
=\ & (x-5)^2 - 1 + 6y - 9y^2 \\
=\ & (x-5)^2 - (1 - 6y + 9y^2) \\
=\ & (x-5)^2 - \{(1)^2 - 2 \cdot 1 \cdot 3y + (3y)^2\} \\
=\ & (x-5)^2 - (1-3y)^2 \\
=\ & (x - 5 + 1 - 3y)(x - 5 - 1 + 3y) \\
=\ & (x - 3y - 4)(x + 3y - 6)
\end{aligned}$$

Exercise 9.2

1. Factorize:

(a) $x^4 + x^2y^2 + y^4$ (b) $16x^4 + 7x^2 + 1$

(c) $16x^4 + 36x^2y^2 + 81y^4$ (d) $4m^4 + 35m^2n^2 + 121n^4$

(e) $48a^4 + 108a^2b^2 + 243b^4$ (f) $32p^4 + 72p^2q^2 + 162q^4$

2. Factorize:

(a) $x^4 + 4$ (b) $4x^4 + 81y^4$

(c) $64e^4 + f^4$ (d) $m^4 + 4n^4$

(e) $81x^4 + 64y^4$ (f) $y^4 + 324x^4$

3. Factorize following algebric expressions:

(a) $x^4 - 5x^2y^2 + 4y^4$ (b) $x^4 - 22x^2y^2 + 9y^4$

(c) $b^4 - 3b^2 + 1$ (d) $25x^4 - 34x^2y^2 + 9y^4$

(e) $49a^4 - 154a^2b^2 + 9b^4$ (f) $25a^5b - 9a^3b^3 + 16ab^5$

4. Factorize following expressions:

(a) $\frac{m^4}{n^4}+1+\frac{n^4}{m^4}$ (b) $y^4+\frac{1}{y^4}+1$

(c) $\frac{a^4}{b^4}-\frac{5a^2}{b^2}+4$ (d) $\frac{p^4}{q^4}+1+\frac{p^2}{q^2}$

(e) $\frac{a^4}{b^4}+1-\frac{7a^2}{b^2}$ (f) $x^4+\frac{1}{x^4}-7$

5. Factorize:

(a) $p^2-10p+24+6q-9q^2$ (b) $p^4-8p^2-33-14q-q^2$

(c) $a^2-12a-28+16b-b^2$ (d) $x^4+9-7x^2+2xy-y^2$

(e) $25x^2-49y^2+30x+70y-16$ (f) $49x^2+16y^2-64z^2+56xy+16z-1$

6. Factorize the expression $x^{10}-10x^6+9x^2$ and find its factors.

Answers

1. (a) $-x^2+xy+y^2)(x^2-xy+y^2)$ (b) $(4x^2+x+1)(4x^2-x+1)$
   (c) $-4x^2+6xy+9y^2)(4x^2-6xy+9y^2)$
   (d) $-2m^2+3mn+11n^2)(2m^2-3mn+11n^2)$
   (e) $3(4a^2+6ab+9b^2)(4a^2-6ab+9b^2)$
   (f) $2(4p^2+6pq+9q^2)(4p^2-6pq+9q^2)$
2. (a) $(x^2+2x+2)(x^2-2x+2)$ (b) $(2x^2+6xy+9y^2)(2x^2-6xy+9y^2)$
   (c) $(8e^2+4ef+f^2)(8e^2-4ef+f^2)$ (d) $(m^2+2mn+2n^2)(m^2-2mn+2n^2)$
   (e) $(9x^2+12xy+8y^2)(9x^2-12xy+8y^2)$
   (f) $(y^2+6xy+18x^2)(y^2-6xy+18x^2)$
3. (a) $(x^2+3xy+2y^2)(x^2-3xy+2y^2)$
   (b) $(x^2+4xy-3y^2)(x^2-4xy-3y^2)$ (c) $(b^2+b-1)-b^2-b-1)$
   (d) $(5x^2+2xy-3y^2)(5x^2-2xy-3y^2)$
   OR $(5x^2+8xy+3y^2)(5x^2-8xy+3y^2)$
   (e) $(7a^2+14ab+3b^2)(7a^2-14ab+3b^2)$
   (f) $ab(5a^2+7ab+4b^2)(5a^2-7ab+4b^2)$

4. (a) $\left(\frac{m^2}{n^2}+1+\frac{n^2}{m^2}\right).\left(\frac{m^2}{n^2}-1+\frac{n^2}{m^2}\right)$ (b) $\left(y^2+1+\frac{1}{y^2}\right).\left(y^2-1+\frac{1}{y^2}\right)$

(c) $\left(\frac{a^2}{b^2}+\frac{3a}{b}+2\right).\left(\frac{a^2}{b^2}-\frac{3a}{b}+2\right)$ (d) $\left(\frac{p^2}{q^2}+\frac{p}{q}+1\right).\left(\frac{p^2}{q^2}-\frac{p}{q}+1\right)$

(e) $\left(\frac{a^2}{b^2}+\frac{3a}{b}+1\right).\left(\frac{a^2}{b^2}-\frac{3a}{b}+1\right)$ (f) $\left(x^2+3+\frac{1}{x^2}\right).\left(x^2-3+\frac{1}{x^2}\right)$

5. (a) $(p+3q-6)(p-3q-4)$ (b) $[(\mathrm{p}^2+q+3)\text{-}p^2-q-11)]$

(c) $(a+b-14)(a-b+2)$ (d) $(x^2+\mathrm{x}-y-3)-(x^2-\mathrm{x}+y-3)$

(e) $\text{-}5x-7y+8)\text{ - }5x+7y-2)$

(f) $(7x^2+4y+8z-1)(7x^2+4y-8z+1)$

6. $x^2(x^2+3)(x^2+1)(x^2-3)(x+1)(x-1)$

Lesson 10 Highest Common Factor and Lowest Common Multiple

10.0 Review

Discuss with your nearest friends in your class and find the HCF and LCM of given algebraic expressions. Also, present it to your class:

$$2x^2 - 8y^2 \text{ and } 2x^4 + 16xy^3$$

Here, to find the HCF and LCM of given expressions we have to factorize the given expressions as;

First Expression $= 2x^2 - 8y^2$

$= 2(x^2 - 4y^2)$

$= 2\{x^2 - (2y)^2\}$

$= 2(x - 2y)(x + 2y)$

Second Expression $= 2x(x^3 + 8y^3)$

$= 2x\{(x)^3 + (2y)^3\}$

$= 2x(x + 2y)(x^2 - 2xy + 4y^2)$

Now, HCF = Common Factor $= 2 \times (x + 2y) = 2(x + 2y)$

LCM = common factors × remaining factors

$= 2 \times (x + 2y) \times (x - 2y) \times x \times (x^2 - 2xy + 4y^2)$

$= 2x(x - 2y)(x^3 + 8y^3)$

10.1 Highest Common Factor

Activity 1

Take the algebraic expressions $x^2 - 4$ and $x^3 - 8$, discuss the following questions and conclude;

(a) What are the factors of the given expressions?
(b) What are the common factors on them?
(c) What are these common factors called?
(d) Present these factors in a Venn diagram.

Here, factors of the first expression $x^2 - 4$ are $(x + 2)$ and $(x - 2)$.

We should apply the following process in the factorization method:

- Taking common
- Express the given algebraic expressions in the form of a formula
- Factorize it

Factors of the second expression x3 – 8 are (x – 2) and (x2 + 2x + 4).

The common factor is the factor included in both expressions. So, the common factor is $(x - 2)$.

This common factor is the Highest Common Factor(HCF) in the given expressions.

Hence, HCF is $(x - 2)$

The product of common factors of given expression is called Highest Common Factor (HCF).

Example 1

Find HCF of $x^2 - 9$ and $x^3 + 27$ and present it in venn diagram:

Solution,

Here,

First expression $= (x^2 - 9)$

$= (x + 3)(x - 3)$

Second expression $= x^3 + 27$

$= (x)^3 + (3)^3$

$= (x + 3)(x^2 - 3x + 9)$

$\therefore$ HCF = common factors $= (x + 3)$

Example 2

Find HCF of following expressions:

(a) $8x^3 + y^3$ and $16x^4 + 4x^2y^2 + y^4$

(b) $p^3 - q^3$ and $p^3 + q^3$

(c) $x^3y + y^4$, $x^4 + x^2y^2 + y^4$, $2x^3 - 2x^2y + 2xy^2$

(d) $16a^4 - 4a^2 - 4a - 1$, $16a^4 + 16a^3 + 4a^2 - 1$, $16a^4 + 4a^2 + 1$

Solution,

Here,

(a) **First expression** $= 8x^3 + y^3$

$= (2x)^3 + (y)^3$

$= (2x + y)(4x^2 - 2xy + y^2)$

Second expression $= 16x^4 + 4x^2y^2 + y^4$

$= (4x^2)^2 + (y^2)^2 + 4x^2y^2$

$= (4x^2 + y^2)^2 - 2 \cdot 4x^2 \cdot y^2 + 4x^2y^2$ $\quad [\because a^2 + b^2 = (a + b)^2 - 2ab]$

$= (4x^2 + y^2)^2 - 4x^2y^2$

$= (4x^2 + y^2)^2 - (2xy)^2$

$= (4x^2 + y^2 + 2xy)(4x^2 + y^2 - 2xy)$ $\quad [\because a^2 - b^2 = (a + b)(a - b)]$

$= (4x^2 + 2xy + y^2)(4x^2 - 2xy + y^2)$

So, HCF $= (4x^2 - 2xy + y^2)$

(b) **First expression** $= p^3 - q^3$

$= (p - q)(p^2 + pq + q^2)$

Second expression $= p^3 + q^3$

$= (p + q)(p^2 - pq + q^2)$

Now, HCF = 1

Note: If the given expressions have no common factors, then HCF= 1.

(c) Here, first expression $= x^3y + y^4$

$= y(x^3 + y^3)$

$= y(x + y)(x^2 - xy + y^2)$

Second expression $= x^4 + x^2y^2 + y^4$

$= (x^2)^2 + (y^2)^2 + x^2y^2$

$= (x^2 + y^2)^2 - 2x^2y^2 + x^2y^2$ $\quad [\because a^2 + b^2 = (a + b)^2 - 2ab]$

$= (x^2 + y^2)^2 - x^2y^2$

$= (x^2 + y^2)^2 - (xy)^2$

$= (x^2 + y^2 + xy)(x^2 + y^2 - xy)$ $\quad [\because a^2 - b^2 = (a + b)(a - b)]$

$= (x^2 + xy + y^2)(x^2 - xy + y^2)$

Third expression $= 2x^3 - 2x^2y + 2xy^2$

$= 2x(x^2 - xy + y^2)$

So, HCF $= (x^2 - xy + y^2)$

(d) Here, first expression $= 16a^4 - 4a^2 - 4a - 1$

$= 16a^4 - (4a^2 + 4a + 1)$

$= 16a^4 - \{(2a)^2 + 2.2a.1 + (1)^2\}$

$= (4a^2)^2 - (2a + 1)^2$

$= (4a^2 + 2a + 1)(4a^2 - 2a - 1)$

Second expression $= 16a^4 + 16a^3 + 4a^2 - 1$

$= (4a^2)^2 + 2 \cdot 4a^2 \cdot 2a + (2a)^2 - 1$

$= (4a^2 + 2a)^2 - (1)^2$

$= (4a^2 + 2a + 1)(4a^2 + 2a - 1)$

Third expression $= 16a^4 + 4a^2 + 1$

$= (4a^2)^2 + (1)^2 + 4a^2$

$= (4a^2 + 1)^2 - 2 \cdot 4a^2 \cdot 1 + 4a^2$

$= (4a^2 + 1)^2 - 4a^2$

$= (4a^2 + 1)^2 - (2a)^2$

$= (4a^2 + 1 + 2a)(4a^2 + 1 - 2a)$

$= (4a^2 + 2a + 1)(4a^2 - 2a + 1)$

So, HCF $= 4a^2 + 2a + 1$

Example 3

Find HCF of the following expressions:

(a) $5m^3 - 20m,\ m^3 - 3m^2 - 10m,\ m^3 - m^2 - 2m + 8$

(b) $(a - b)^2 + 4ab,\ (a + b)^3 - 3ab\,(a + b),\ a^2 + (2a + b)b$

Solution,

Here,

(a) First expression $= 5m^3 - 20m$

$= 5m\,(m^2 - 4)$

$= 5m\,\{(m)^2 - (2)^2\}$

$= 5m\,(m + 2)\,(m - 2)$

Second expression $= m^3 - 3m^2 - 10m$

$= m(m^2 - 3m - 10)$

$= m\{m^2 - (5 - 2)m - 10\}$

$= m\,(m^2 - 5m + 2m - 10)$

$= m\{m(m - 5) + 2(m - 5)\}$

$= m\,(m - 5)\,(m + 2)$

Third expression $= m^3 - m^2 - 2m + 8$

$= m^3 + 8 - m^2 - 2m$

$= (m)^3 + (2)^3 - m(m + 2)$

$= (m + 2)\,(m^2 - 2m + 4) - m(m + 2)$

$= (m + 2)\,(m^2 - 2m + 4 - m)$

$= (m + 2)\,(m^2 - 3m + 4)$

So, HCF $= (m + 2)$

(b) Here,

First expression $= (a-b)^2 + 4ab$

$= a^2 - 2ab + b^2 + 4ab$

$= a^2 + 2ab + b^2$

$= (a+b)^2$

$= (a+b)(a+b)$

Second expression $= (a+b)^3 - 3ab(a+b)$

$= (a+b)\{(a+b)^2 - 3ab\}$

$= (a+b)(a^2 + 2ab + b^2 - 3ab)$

$= (a+b)(a^2 - ab + b^2)$

Third expression $= a^2 + (2a+b)b$

$= a^2 + 2ab + b^2$

$= (a+b)^2$

$= (a+b)(a+b)$

So, HCF $= (a+b)$

Exercise 10.1

1. (a) What do you mean by HCF of algebraic expression?
   (b) In what condition the HCF of algebric expression becomes 1?
   (c) What is the common factors of the expression $4x^3y$ and $3z^3$?

2. **Find the Highest Common Factor (HCF) of following expressions:**
   (a) $(x + y)^2$ and $(x + y)(x - y)$
   (b) $(x - y)(x^2 + xy + y^2)$ and $(x^2 + xy + y^2)(x^2 - xy + y^2)$
   (c) $(2x - 3y)(4x^2 + 6xy + 9y^2)$ and $4xy(2x - 3y)(2x + 3y)$
   (d) $4a^3b(a - b)(a + b - 1)$ and $16a^2b^2(a + 2b)(a + b - 1)$

3. **Find the HCF of following algebric expressions:**
   (a) $a^2 - ab$ and $a^3b - a^2b^2$
   (b) $3x^2 + 9x$ and $7x + 21$
   (c) $a^3 + 1$ and $a^4 + a^2 + 1$
   (d) $8x^3 + 27y^3$ and $16x^4 + 36x^2y^2 + 81y^4$
   (e) $m^3 + 1 + 2m^2 + 2m$ and $m^3 - 1$
   (f) $8(6x^4 - x^3 - 2x^2)$ and $12(2x^6 + 3x^5 + x^4)$
   (g) $a^2 + 4ab + 4b^2 - c^2$ and $a^2 - 4b^2 + ac - 2bc$
   (h) $x^4 + 4y^4$ and $2x^3y + 4xy^3 + 4x^2y^2$
   (i) $x^2 - 10x + 24 + 6y - 9y^2$ and $x^2 + 3xy - 6x$
   (j) $(1 - x^2)(1 - y^2) + 4xy$ and $1 - 2x + y - x^2y + x^2$
   (k) $8a^3 + 1$ and $16a^4 - 4a^2 + 4a - 1$
   (l) $2a^3 - a^2 + a - 2$ and $a^3 - a^2 + a - 1$

4. **Find the HCF of given expressions:**
   (a) $a^3 + b^3$, $a^3 - a^2b + ab^2$ and $a^4 + a^2b^2 + b^4$
   (b) $x^3 + 2x^2 + 2x + 1$, $x^3 - 1$ and $x^4 + x^2 + 1$
   (c) $x^3 - 4x$, $4x^3 - 10x^2 + 4x$ and $3x^4 - 8x^3 + 4x^2$
   (d) $y^2 + 2y - 8$, $y^2 - 5y + 6$ and $y^2 + 5y - 14$
   (e) $x^2 + 2x + 1$, $x^2 + 5x + 6$ and $2x^2 - 5x + 2$
   (f) $x^2 + 2x - 8$, $x^2 - 2x - 24$ and $x^2 + 5x + 4$

(g) $x^3 + 2x^2 + 4x$, $x^4 + 4x^2 + 16$ and $x^3 - 8$

(h) $8x^3 + 27y^3$, $16x^4 + 36x^2y^2 + 81y^4$ and $4x^3 - 6x^2y + 9xy^2$

(i) $2x^3 - 54$, $24x^4 + 18x^2 + 162$ and $2x^2 + 6x + 18$

(j) $9x^2 - 3y^2 - 8yz - 4z^2$, $4z^2 - 4y^2 - 9x^2 - 12xy$ and $9x^2 + 12xz + 4z^2 - 4y^2$

(k) $2ax^2 + 2ax - 12a$, $3a^2x^2 - 7a^2x - 6a^2$ and $a^3x^2 + 4a^3x - 21a^3$

(l) $x^3 + 64y^3$, $x^4 + 16x^2y^2 + 256y^4$ and $4x^3 - 16x^2y + 64y^2x$

(m) $(a + b)^3 - 3ab(a + b)$, $a^4 + a^2b^2 + b^4$ and $a^4 - 2a^3b + a^2b^2 - b^4$

5. The area of three different rooms are $(x + 3)(x + 6)$, $(x^2 + 8x + 15)$ and $(x^2 + 7x + 12)$ square unit respectively. Find the breadth of the rooms.

Answers

2. (a) $x + y$ (b) $(x^2 + xy + y^2)$
   (c) $(2x - 3y)(2x + 3y)$ (d) $4a^2b(a + b - 1)$
3. (a) $a(a - b)$ (b) $(x + 3)$
   (c) $(a^2 - a + 1)$ (d) $(4x^2 - 6xy + 9y^2)$
   (e) $(m^2 + m + 1)$ (f) $4x^2(2x + 1)$
   (g) $(a + 2ab + c)$ (h) $(x^2 + 2xy + 2y^2)$
   (i) $(x + 3y - 6)$ (j) $(1 - x + y + xy)$
   (k) $(4a^2 - 2a + 1)$ (l) $a - 1$
4. (a) $(a^2 - ab + b^2)$ (b) $x^2 + x + 1$
   (c) $x(x - 2)$ (d) $(y - 2)$
   (e) 1 (f) $(x + 4)$
   (g) $(x^2 + 2x + 4)$ (h) $(4x^2 + 6xy + 9y^2)$
   (i) 2 (j) $(3x + 2y + 2z)$
   (k) $a(x - 3)$ (l) $(x^2 - 4xy + 16y^2)$
   (m) $a^2 - ab + b^2$
5. $(x + 3)$ unit

10.2 Lowest common multiple

Activity 1

Take two expressions $x^3 - 125y^3$ and $x^4 - 15x^2y^2 + 25y^4$ by each student. Factorize these statements. Observe the factors of each expression and find the answer to the following questions in your group:

(a) Write the common factors of given expressions.

(b) Find the factors other than the common factors.

(c) What is the product of common factors and remaining factors called?

(d) Analyze the result between the product of the factors of given algebraic expressions and the product of their HCF and LCM.

To find the common factors of the given expressions, we have to factorize them as:

First expressions $= x^3 - 125y^3$

$= (x)^3 - (5y)^3$

$= (x - 5y)(x^2 + 5xy + 25y^2)$

Similarly, second expressions $= x^4 + 25x^2y^2 + 625y^4$

$= (x^2)^2 + (25y^2)^2 + 25x^2y^2$

$= (x^2 + 25y^2)^2 - 2\cdot x^2\cdot 25y^2 + 25x^2y^2$

$= (x^2 + 25y^2)^2 - 25x^2y^2$

$= (x^2 + 25y^2)^2 - (5xy)^2$

$= (x^2 + 25y^2 + 5xy)(x^2 + 25y^2 - 5xy)$

$= (x^2 + 5xy + 25y^2)(x^2 - 5xy + 25y^2)$

Observing the factors of two algebraic expression, we found,

Common factors $= x^2 + 5xy + 25y^2$

By second question, remaining factors $(x - 5y)$ and $(x2 - 5xy + 25y2)$.

By third question, the product of common factors and remaining factors is:

$= (x^2 + 5xy + 25y^2) \times (x - 5y) \times (x^2 - 5xy + 25y^2)$

$= (x - 5y)(x^2 + 5xy + 25y^2)(x^2 - 5xy + 25y^2)$

$= (x^3 - 125y^3)(x^2 - 5xy + 25y^2)$

The product of common factors and remaining factors is the LCM of given expressions.

The product of common factors and remaining factors of given algebric expression is called Lowest Common Multiple. In short, it is written as LCM.

Activity 2

Take three algebraic expressions $(x-y)^2 + 4xy$, $(x+y)^3 - 3xy(x+y)$ and $x^2 + 2xy + y^2$. Find the LCM of the given expressions and present the result in your class. The work of one group is as follows:

$$\begin{aligned}
\text{First expression} &= (x-y)^2 + 4xy \\
&= x^2 - 2xy + y^2 + 4xy \\
&= x^2 + 2xy + y^2 \\
&= (x+y)^2 = (x+y)(x+y) \\
\text{Second expression} &= (x+y)^3 - 3xy(x+y) \\
&= (x+y)\{(x+y)^2 - 3xy\} \\
&= (x+y)(x^2 + 2xy + y^2 - 3xy) \\
&= (x+y)(x^2 - xy + y^2) \\
\text{Third expression} &= x^2 + 2xy + y^2 \\
&= (x+y)^2 \\
&= (x+y)(x+y) \\
\text{Common factors} &= (x+y)(x+y) \\
\text{Remaining factors} &= (x^2 - xy + y^2)
\end{aligned}$$

LCM = common factors × remaining factors $= (x+y)(x+y)(x^2 - xy + y^2)$

To find the HCF of three expressions, the factors common to the two expressions are also common factors.

Example 1

Find the LCM of following expressions.

(a) $a^3 - y^3$ and $a^4 + a^2y^2 + y^4$

(b) $x^2 - y^2 - 2y - 1$ and $x^2 - 1 + 2xy + y^2$

(c) $x^4 - 8xy^3$ and $3x^2 - 5xy - 2y^2$

Solution,

Here,

(a) First expression $= a^3 - y^3$

$$= (a)^3 - (y)^3 = (a - y)(a^2 + ay + y^2)$$

Second expression $= a^4 + a^2y^2 + y^4$

$$= (a^2)^2 + (y^2)^2 + a^2y^2$$
$$= (a^2 + y^2)^2 - 2a^2y^2 + a^2y^2$$
$$= (a^2 + y^2)^2 - a^2y^2$$
$$= (a^2 + y^2)^2 - (ay)^2$$
$$= (a^2 + y^2 + ay)(a^2 + y^2 - ay)$$
$$= (a^2 + ay + y^2)(a^2 - ay + y^2)$$

Common factors $= (a^2 + ay + y^2)$

Remaining factors $= (a - y)(a^2 - ay + y^2)$

So, Lowest Common Multiples (LCM) = common factors × remaining factors

$$= (a^2 + ay + y^2) \times (a - y) \times (a^2 - ay + y^2)$$
$$= (a - y)(a^2 + ay + y^2)(a^2 - ay + y^2)$$

Let the factors of first expression by set A and factors of second expression by set B and present in venn diagram,

(b) First expression $= x^2 - y^2 - 2y - 1$

$$= x^2 - (y^2 + 2y + 1)$$
$$= (x)^2 - (y + 1)^2$$
$$= (x + y + 1)(x - y - 1)$$

Second expression $= x^2 - 1 + 2xy + y^2$

$$= (x^2 + 2xy + y^2) - 1$$
$$= (x + y)^2 - (1)^2$$

Common factors = $(x + y + 1)$

Remaining factors = $(x - y - 1)\ (x + y - 1)$

$= (x + y + 1)\ (x + y - 1)$

$\therefore$ LCM = common factors $\times$ remaining factors

$= (x + y + 1) \times (x - y - 1) \times (x + y - 1)$

$= (x + y + 1)\ (x - y - 1)\ (x + y - 1)$

(c) First expression $= x^4 - 8xy^3$

$= x(x^3 - 8y^3) = x\{(x)^3 - (2y)^3\}$

$= x(x - 2y)\ (x^2 + 2xy + 4y^2)$

Second expression $= 3x^2 - 5xy - 2y^2$

$= 3x^2 - (6 - 1)\ xy - 2y^2$

$= 3x^2 - 6xy + xy - 2y^2$

$= 3x\ (x - 2y) + y\ (x - 2y)$

$= (x - 2y)\ (3x + y)$

Common factors = $(x - 2y)$

Remaining factors = $x(x^2 + 2xy + 4y^2)\ (3x + y)$

$\therefore$ LCM = common factors $\times$ remaining factors

$= (x - 2y) \times x \times (3x + y) \times (x^2 + 2xy + 4y^2)$

$= x(x - 2y)\ (3x + y)\ (x^2 + 2xy + 4y^2)$

Example 2

Find the LCM of following expressions:

(a) $(x^2 + xy + y^2)^3$, $x^3 - y^3$ and $x^4 + x^2y^2 + y^4$

(b) $8x^3 + 125y^3$, $4x^3 - 10x^2y + 25xy^2$ and $16x^4 + 100x^2y^2 + 625y^4$

(c) $3y^3 + 14y^2 - 5y$, $y^4 + 125y$ and $y^5 + 25y^3 + 625y$

Solution,

Here,

(a) First expression $= (x^2 + xy + y^2)^3$

$= (x^2 + xy + y^2)\ (x^2 + xy + y^2)\ (x^2 + xy + y^2)$

Second expression $= x^3 - y^3$

$= (x)^3 - (y)^3$

$= (x - y)(x^2 + xy + y^2)$

Third expression $= x^4 + x^2y^2 + y^4$

$= (x^2)^2 + (y^2)^2 + x^2y^2$

$= (x^2 + y^2)^2 - 2.x^2y^2 + x^2y^2$

$= (x^2 + y^2)^2 - x^2y^2$

$= (x^2 + y^2)^2 - (xy)^2$

$= (x^2 + y^2 + xy)(x^2 + y^2 - xy)$

$= (x^2 + xy + y^2)(x^2 - xy + y^2)$

Common factors $= (x^2 + xy + y^2)$

Remaining factors $= (x^2 + xy + y^2)(x^2 + xy + y^2)(x - y)(x^2 - xy + y^2)$

$\therefore$ LCM = common factors $\times$ remaining factors

$= (x^2 + xy + y^2) \times (x - y) \times (x^2 - xy + y^2) \times (x^2 + xy + y^2) \times (x^2 + xy + y^2)$

$= (x - y)(x^2 + xy + y^2)^3 (x^2 - xy + y^2)$

(b) First expression $= 8x^3 + 125y^3$

$= (2x)^3 + (5y)^3$

$= (2x + 5y)(4x^2 - 10xy + 25y^2)$

Second expression $= 4x^3 - 10x^2y + 25xy^2 = x(4x^2 - 10xy + 25y^2)$

Third expression $= 16x^4 + 100x^2y^2 + 625y^4$

$= (4x^2)^2 + (25y^2)^2 + 100x^2y^2$

$= (4x^2 + 25y^2)^2 - 2 \cdot 4x^2 \cdot 25y^2 + 100x^2y^2$

$= (4x^2 + 25y^2)^2 - 100\, x^2y^2$

$= (4x^2 + 25y^2)^2 - (10xy)^2$

$= (4x^2 + 25y^2 + 10xy)(4x^2 + 25y^2 - 10xy)$

$= (4x^2 + 10xy + 25y^2)(4x^2 - 10xy + 25y^2)$

$\therefore$ LCM = common factors $\times$ remaining factors

$= (4x^2 - 10xy + 25y^2) \times x \times (2x + 5y) \times (4x^2 + 10xy + 25y^2)$

$= x(2x + 5y)(4x^2 - 10xy + 25y^2)(4x^2 + 10xy + 25y^2)$

(c) **First expression** $= 3y^3 + 14y^2 - 5y$

$= y(3y^2 + 14y - 5)$

$= y\{3y^2 + (15 - 1)y - 5\}$

$= y\,(3y^2 + 15y - y - 5)$

$= y\{3y\,(y + 5) - 1(y + 5)\}$

$= y\,(y + 5)\,(3y - 1)$

Second expression $= y^4 + 125y$

$= y(y^3 + 125)$

$= y\{(y)^3 + (5)^3\}$

$= y(y + 5)\,(y^2 - 5y + 25)$

Third expression $= y^5 + 25y^3 + 625y = y(y^4 + 25y^2 + 625)$

$= y\{(y^2)^2 + (25)^2 + 25y^2\}$

$= y\{(y^2 + 25)^2 - 2 \cdot y^2 \cdot 25 + 25y^2\}$

$= y\{(y^2 + 25)^2 - 25y^2\}$

$= y\{(y^2 + 25)^2 - (5y)^2\}$

$= y\{(y^2 + 25 + 5y)\,(y^2 + 25 - 5y)\}$

$= y(y^2 + 5y + 25)\,(y^2 - 5y + 25)$

$\therefore$ LCM = common factors $\times$ remaining factors

$= y \times (y^2 - 5y + 25) \times (y + 5) \times (3y - 1)\,(y^2 + 5y + 25)$

$= y\,(y + 5)\,(3y - 1)\,(y^2 - 5y + 25)\,(y^2 + 5y + 25)$

Exercise 10.2

1. (a) Define LCM.

   (b) If two expressions are given, then what is the relationship between their LCM and HCF?

2. **Find the lowest common multiple (LCM) of given algebraic expressions:**

   (a) $2x(x + 2)(x - 2)$ and $4x^2(3x + 7)(x - 2)$

   (b) $3x^2y(x - y)(x^2 + 3xy + 9y^2)$ and $10x(x^2 + y^2)(x + y)(x - y)$

   (c) $(2x - y)(x^2 + xy + y^2)(x^2 - xy + y^2)$ and $25xy(2x - y)$

   (d) $8x^3y^2(a + b + 1)(a - b + 1)$ and $5x^3y(a + b + 1)(a + b + 2)$

3. If HCF and LCM of two expressions are $(a + b)^2$ and $3a^2(a + b)(2a + b)$ respectively and the second expression is $3a(a + b)^2$, find the first expression.

4. The two expressions are respectively $(x + 5)$ and $(x + 5)(x^2 - 5x + 25)$ and HCF is $(x + 5)$. Find the LCM of given expression.

5. **Find the LCM of following expression:**

   (a) $x^2 - x + 1$ and $x^4 + x$

   (b) $4x + 16$ and $5x + 20$

   (c) $3x + 27$ and $8x^3 + 72x^2$

   (d) $(x - y)^3$ and $x^3 - y^3$

   (e) $(a + b)^3$ and $a^3 + b^3$

   (f) $x^4 + 4$ and $2x^3 - 4x^2 + 4x$

   (g) $a^4 + a^2 + 1$ and $a^2 - a + 1$

   (h) $x^4 + x^2y^2 + y^4$ and $x^3 - y^3$

   (i) $1 + 4p + 4p^2 - 16p^4$ and $1 + 2p - 8p^3 - 16p^4$

   (j) $x^3 + x^2 + x + 1$ and $x^3 - x^2 + x - 1$

   (k) $y^4 + (2b^2 - a^2)y^2 + b^4$ and $y^3 - ay^2 + b^2y$

   (l) $\frac{x^4}{y^4} + \frac{y^4}{x^4} + 1$ and $\frac{x^3}{y^3} + \frac{y^3}{x^3}$

6. **Find the LCM from following expressions:**

   (a) $x^3 + 1$, $x^4 - x^3 + x^2$ and $x^3 - x^2 + x$

   (b) $a^3 + 1$, $a^4 + a^2 + 1$ and $a^2 + a + 1$

   (c) $x^2 - 3x + 2$, $x^2 - 5x + 6$ and $x^2 - 8x + 12$

   (d) $2x^3 + 16$, $x^2 + 4x + 4$ and $x^2 + 3x + 2$

   (e) $x^6 - 1$, $x^3 - 1$ and $x^4 + x^2 + 1$

(f) $x^6 - 16x^4$, $x^5 + 6x^4 + 8x^3$ and $x^4 + 8x^3 + 16x^2$

(g) $x^4 + 8x^2 + 144$, $x^3 + x(x + 12) + 3x^2$ and $x^3 + 12x + 4x^2$

(h) $x^4 - 8x^2 + 196$, $x^3 + x(x + 14) + 5x^2$ and $2x^2 + 12x + 28$

(i) $x^4 + 10x^2 + 169$, $x^3 + 4x^2 + 13x$ and $x^3 + x(x + 13) + 3x^2$

(j) $(y + 3)^2 - 9y - 27$, $y^3 - 2y^2 - 15y$ and $y^5 - 13y^3 + 36y$.

7. Find LCM:

(a) $m^2 - 10m + 24 + 6n - 9n^2$, $m^2 + 6mn + 9n^2 - 36$ and $m^2 + 3mn - 6m$

(b) $x^4 - 8x^2 - 33 - 14y - y^2$, $x^4 + 2x^2y - 9 + y^2$ and $x^3 + xy + 3x$

(c) $a^4 + b^2(2a^2 - 1) + b^4$, $a^3 - b(a + 1)(a - b) - b^3$ and $a^3 - b(a - 1)(a - b) - b^3$

Answers

2. (a) $4x^2 (x - 2)(x + 2)(3x + 7)$

(b) $30x^2y (x - y)(x + y)(x^2 + y^2)(x^2 + 3xy + 9y^2)$

(c) $25xy (2x - y)(x^2 + xy + y^2)(x^2 - xy + y^2$

(d) $40x^3y^2 (a + b + 1)(a - b + 1)(a + b + 2)$

3. $a(2a + b)(a + b)$

4. $(x + 5)(x^2 - 5x + 25)$

5. (a) $x(x + 1)(x^2 - x + 1)$ (b) $20(x + 4)$ (c) $24x^2 (x + 9)$

(d) $(x - y)^3 (x^2 + xy + y^2)$ (e) $(a + b)^3 (a^2 - ab + b^2)$

(f) $2x (x^2 - 2x + 2)(x^2 + 2x + 2)$ (g) $(a^2 + a + 1)(a^2 - a + 1)$

(h) $(x - y)(x^2 + xy + y^2)(x^2 - xy + y^2)$

(i) $(1 + 2p)(1 - 2p)(1 + 2p + 4p^2)(1 + 2p - 4p^2)$

(j) $(x + 1)(x - 1)(x^2 + 1)$

(k) $y (y^2 + ay + b^2)(y^2 - ay + b^2)$

(l) $\left(\frac{x}{y} + \frac{y}{x}\right)\left(\frac{x^2}{y^2} + 1 + \frac{y^2}{x^2}\right)\left(\frac{x^2}{y^2} - 1 + \frac{y^2}{x^2}\right)$

6. (a) $x^2 (x + 1)(x^2 - x + 1)$

(b) $(a + 1)(a^2 - a + 1)(a^2 + a + 1)$

(c) $(x - 1)(x - 2)(x - 3)(x - 6)$

(d) $2(x + 2)^2 (x + 1)(x^2 - 2x + 4)$

(e) $(x^2 - 1)(x^4 + x^2 + 1)$

(f) $x^4 (x^2 - 16)(x^2 + 6x + 8)$

(g) $x (x^2 + 4x + 12)(x^2 - 4x + 12)$

(h) $2x (x^2 + 6x + 14)(x^2 - 6x + 14)$

(i) $x(x^2 + 4x + 13)(x^2 - 4x + 13)$

(j) $y(y^2 - 9)(y^2 - 4)(y - 5)(y - 6)$

7. (a) $m(m + 3n - 6)(m - 3n - 4)(m + 3n + 6)$

(b) $x(x^2 + y + 3)(x^2 - y - 11)(x^2 + y - 3)$

(c) $(a - b)(a^2 + b^2 - b)(a^2 + b^2 + b)$

Lesson 11

Linear Equations

11.0 Review

The rate of a non-veg meal in a hotel is Rs. 200. Ronish needs Rs.50 additional to buy a non-veg meal other than the money he has. Express this statement can be expressed in mathematical language as, x + Rs. 50 = Rs. 200. Then, find the answer to the following questions in a group and answer.

(a) What are the variables and constants in the above statement?

(b) How much money does Ronish have at the beginning?

(c) Which mathematical symbols connect the left and right-hand sides of the statement?

(d) What is the power of x in the above mathematical sentence?

Discuss the above questions in a group and find the following conclusions:

In the first question, x is a variable, and Rs 50 and Rs 200 are two constants.

In the second question, Ronish has (x)=Rs 200 - Rs 50 = Rs 150 at the beginning.

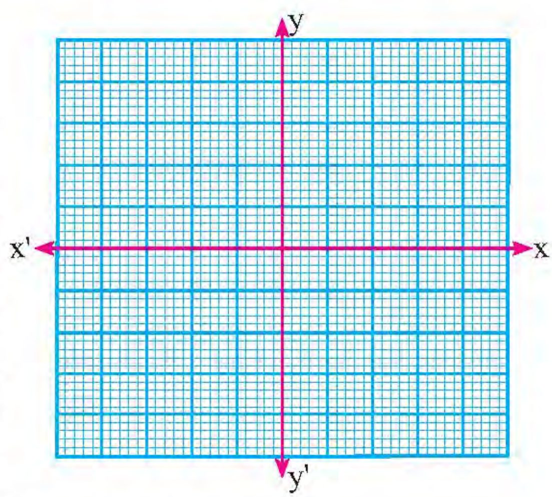
In the third question, the left-hand and right-hand side of the expressions is connected by the equality (=) sign.

In the fourth question, the power of x is 1. So, the above mathematical sentence is the linear equation.

Therefore, we get x = Rs. 150 to solve the equation x + Rs 50 = Rs. 200.

11.1 Simultaneous linear equations with two variables

Activity 1



Take two equations with two variables:

$5x + 2y = 12,\ 3x + 4y = 10$

Present these two equations in a graph and find the point of intersections of given lines. Can we find the common point (point of intersection) using other methods? Discuss in your group. We use the method of solving the simultaneous equations with two variables by the graphical method in previous classes. Now, we will discuss other methods to solve the simultaneous linear equations.

The first-degree equation is called a linear equation. Both the equations are linear, so, it gives straight lines. The solution to the equations gives the value of two variables x and y, satisfying both equations. If the variables of both equations give a fixed value, then these equations are called simultaneous linear equations. In simultaneous linear equations, there are two variables. So, these equations are called simultaneous linear equations with two variables.

11.2 Methods of solving simultaneous linear equations with two variables

Activity 2

Milan and Aashish have to share three balls. The possibilities for sharing are presented below:

Let, the number of balls for Milan = x

Number of balls for Aashish = y

Now, present the possibilities in the table;

x	3	2	1	0
y	0	1	2	3

In all the conditions, Milan and Aashish got a total of three balls.

So, $x + y = 3$ ....(i)

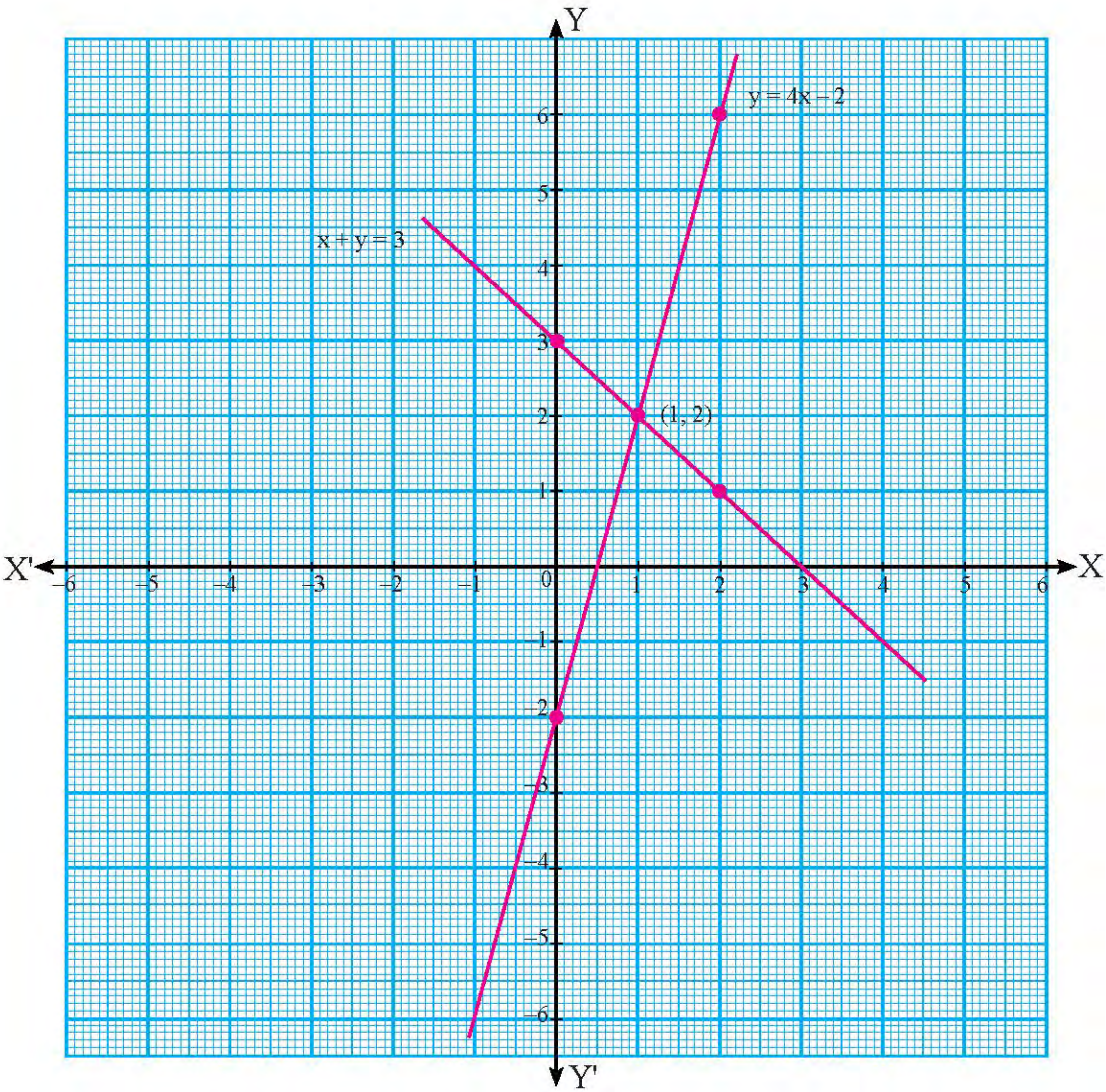
If the number of balls with Aashish is equal to four times the number of balls with Milan minus two,

$y = 4x - 2$ .... (ii)

We can present it in the table as,

x	1	2	0
y	2	6	-2

The two lines $x + y = 3$ and $y = 4x - 2$ are intersected at point (1, 2) is the solution of the above equations (i) and (ii). Because point (1, 2) satisfy both the equations.



11.2.1 Substitution method

Activity 3

Take a simultaneous linear equation of two variables. i.e

$5x + 2y = 240$…….. (i)

$3x + 4y = 200$ .... (ii)

To solve these equations by substitution method and find the value of variables, discuss the following questions in the group and answer these questions.

(a) From equation (i) what is the value of x in the form of y?

(b) Find the value of y in the form of x from equation (i).

(c) How can we find the value of y by replacing the value of x from equation (i) with the value of x in equation (ii)? We can get the following conclusion discussing the above questions:

Here,

(a) Express the value of y in the form of x from equation (i),

$$5x + 2y = 240$$

or, $5x = 240 - 2y$

$$x = \frac{240 - 2y}{5}$$

(b) Likewise, express the value of x in the form of y from (i),

$$2y = 240 - 5x$$

or, $y = \dfrac{240 - 2x}{2}$

(c) For question (c), observe the following steps,

Now, replace the value of x in equation (ii),

$$3\left(\frac{240 - 2y}{5}\right) + 4y = 200$$

or, $\dfrac{720 - 6y + 20y}{5} = 200$

or, $720 + 14y = 200 \times 5$

or, $14y = 1000 - 720$

or, $14y = 280$

or, $y = \dfrac{280}{14}$

$\therefore y = 20$

Now, replace the value of y in equation (ii),

We can replacing the value of y in equation (i) or, (ii) in each case, the value of x and y becomes same.

or, $3x + 4 \times 20 = 200$

or, $3x = 200 - 80$

or, $3x = 120$

or, $x = \frac{120}{3}$

$x = 40$

Note: Both the equations (i) and (ii) are valid if we replacing the value of x and y. Put $x = 40$ and $y = 20$ in both equations and check the results:

Put the values of x and y in equation (i) and check,

$5x + 2y = 240$

or, $5 \times 40 + 2 \times 20 = 240$

or, $200 + 40 = 240$

or, $240 = 240$

$\therefore$ LHS = RHS

Put the value of x and y in equation (ii) and check,

$3x + 4y = 200$

or, $3 \times 40 + 4 \times 20 = 200$

or, $120 + 80 = 200$

or, $200 = 200$

$\therefore$ LHS = RHS

So, the value of x and y satisfies both the equations.

11.2.2 Elimination method

Activity 4

Bipana pays Rs 100 for 4 small and 3 large-size copies, and Ramila pays Rs. 90 for 5 small and 2 large-size copies. Answer the following questions from the group discussion.

(a) Write the above statements in mathematical sentences.

(b) How can we find the unit price of large and small size copies by the elimination method?

(c) How can we make the coefficient of any one variable same after converting the above statements into mathematical sentences?

(d) Can we solve these equations by the replacement method as well?

(e) Dose the result become the same using another method?

We can get the following conclusion discussing the above questions:

According to the first question,

Let the price of small copies is Rs. x and the price of large copies is Rs. y ; we can make equation from given conditions as:

$4x + 3y = 100$ .... (i)

$5x + 2y = 90$ .... (ii)

Multiplying equation (i) by 2 and equation (ii) by 3, to replace value of y,

$[4x + 3y = 100] \times 2$

$8x + 6y = 200$ .... (iii)

and $[5x + 2y = 90] \times 3$

$15x + 6y = 270$ .... (iv)

$\because$ Make same coeffecient of any variable by multiplying fixed numbers on both equations. Performed addition if the signs are different and subtraction if the signs are same.

Subtracting equation (iv) from equation (iii),

$$\begin{array}{r} 15x + \cancel{6y} = 270 \\ 8x + \cancel{6y} = 200 \\ - \quad - \quad - \\ \hline 7x = 70 \end{array}$$

or, $x = \dfrac{70}{7}$

$\therefore$ $x = 10$

$\because$ The coefficient of y in both equations are equal, and same in sign and substration is performed to eliminate.

Again, replacing the value of x in equation (ii),

$5x + 2y = 90$

or, $5 \times 10 + 2y = 90$

or, $2y = 90 - 50$

or, $2y = 40$

$y = \dfrac{40}{2}$

$\therefore$ $y = 20$

Hence, the price of small copies is Rs. 10 per pieces and the price of large copies is Rs. 20 per pieces.

We can use replacement method to solve the equations. The value of x and y becomes same, if we use any method to solve the equations.

To solve the equations by elimination method, the coefficient of x on both equations should be made the same if we eliminate x and the coefficient of y on both equations should be made the same if we eliminate y.

Then, the sign of the variables should be different for the variable to be eliminated.

The method of finding the value of one variable by eliminating another variable in simultaneous linear equations is called elimination method.

Example 1

Solve the simultaneous equations and check the result using elimination method.

$11x + 17y - 67 = 0$ and $17x + 11y - 73 = 0$

Solution,

Here, given equations are:

$11x + 17y - 67 = 0$ .... (i)

$17x + 11y - 73 = 0$ .... (ii)

Multiplying equation (i) by 11 and equation (ii) by 17, then subtracting (ii) from (i),

$$\begin{array}{rl} 121x + \cancel{187y} - 737 & = 0 \\ 289x + \cancel{187y} - 1241 & = 0 \\ - \quad - \quad + & \\ \hline -168x + 504 & = 0 \end{array}$$

or, $-168x = -504$

or, $x = \dfrac{-504}{-168} = 3$

$\therefore x = 3$

Again, replacing the value of x in equation (i),

$11x + 17y - 67 = 0$

or, $11 \times 3 + 17y - 67 = 0$

or, $33 + 17y = 67$

or, $17y = 67 - 33$

or, $y = \dfrac{34}{17}$

$\therefore y = 2$

Now put $x = 3$ and $y = 2$ and check the result from equation (i),

$11x + 17y - 67 = 0$

or, $11 \times 3 + 17 \times 2 - 67 = 0$

or, $33 + 34 - 67 = 0$

or, $67 - 67 = 0$

or, $0 = 0$

$\therefore$ LHS = RHS

Again from equation (ii),

$17x + 11y - 73 = 0$

or, $17 \times 3 + 11 \times 2 - 73 = 0$

or, $51 + 22 - 73 = 0$

or, $73 - 73 = 0$

$0 = 0$

$\therefore$ LHS = RHS

The value of x and y satisfy both the equations. So, the result is correct.

Exercise 11.1

1. **Solve the following simultaneous linear equations using substitution method:**

   (a) $3x + 5y = 31$ and $2x - y = 12$

   (b) $5x + 6y = 27$ and $3x + 4y = 17$

   (c) $3x - 2y = 11$ and $x + 3y = 11$

   (d) $3x - 2y = 8$ and $x + 2y = 8$

   (e) $4x - 3y + 1 = 0$ and $3x + 2y - 12 = 0$

   (f) $5x - y + 1 = 0$ and $2x - 5y + 51 = 0$

   (g) $9x - 8y = 12$ and $2x + 3y = 17$

   (h) $y = 5x - 23$ and $3x - 2y = 4$

   (i) $2x - y = 7$ and $x + y = 5$

   (j) $3x + 2y = 15$ and $5x - 3y - 25 = 0$

2. **Solve the following pair of simultaneous linear equations and check the result using eliminations method.**

   (a) $x + y = 16$, $x - y = -4$

   (b) $3x - 2y = 4$, $5x - y = 23$

   (c) $5x - 2y = 2$, $2x + 3y = 16$

   (d) $2x + 5y + 7 = 0$, $2x - 2y = 14$

   (e) $9x - 8y = 12$, $2x + 3y = 17$

   (f) $3x + 4y = 17$, $5x + 6y = 27$

(g) $7x + 8y = -1$

$10x + 15y = -5$

(h) $4x - 16 = 3y$

$5y = 12 - 3x$

(i) $3x - 3y + 6 = 0$

$4y - 2 - 2x = 0$

(j) $3x = 4y + 18$

$5x = 7y + 31$

3. Solve the following pairs of simultaneous linear equations:

(a) $3x + 4y = 2$

$5x + 3y + 4 = 0$

(b) $2x + 5y = 120$

$8x - 9y + 100 = 0$

(c) $\frac{6x}{5} + \frac{7y}{5} = 1$

$\frac{7x}{3} + \frac{8y}{3} = 2$

(d) $\frac{x}{2} + \frac{y}{3} = 6$

$\frac{3x}{8} + 1 = \frac{2y}{3}$

Answers

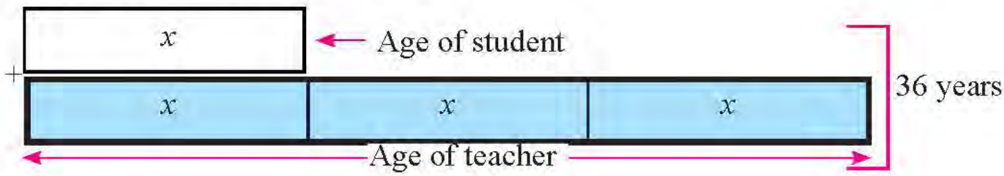
1. (a) $x = 7, y = 2$ (b) $x = 3, y = 2$ (c) $x = 5, y = 2$
   (d) $x = 4, y = 2$ (e) $x = 2, y = 3$ (f) $x = 2, y = 11$
   (g) $x = 4, y = 3$ (h) $x = 6, y = 7$ (i) $x = 4, y = 1$
   (j) $x = 5, y = 0$
2. (a) $x = 6, y = 10$ (b) $x = 6, y = 7$ (c) $x = 2, y = 4$
   (d) $x = 4, y = -3$ (e) $x = 4, y = 3$ (f) $x = 3, y = 2$
   (g) $x = 1, y = -1$ (h) $x = 4, y = 0$ (i) $x = -3, y = -1$
   (j) $x = 2, y = -3$
3. (a) $x = -2, y = 2$ (b) $x = 10, y = 20$ (c) $x = 2, y = -1$
   (d) $x = 8, y = 6$

11.2.3 Word problems related to simultaneous equation

Activity 1

The sum of present age of a teacher and a student is 36 years. The age of teacher is three times the age of his student. Find the present age of the teacher and his students discussing in your group.

The conclusion of one group is:



$4x = 36$

$x = \frac{36}{4}$

$x = 9$

So, age of student (x) = 9 years

Age of teacher ($3x$) = $3 \times 9 = 27$ years

Hence, present age of teacher = 27 years

present age of students = 9 years

The conclusion of second group (Alternative method)

Let, present age of teacher = x years

present age of student = y years

From the first condition,

$x + y = 36$

$x = 36 - y$ .... (i)

From the second condition,

$x = 3y$ .... (ii)

From the equation (i) and (ii)

$36 - y = 3y$

or, $36 = 4y$

or, $y = \frac{36}{4}$

$\therefore y = 9$

Replacing the value of y in equation (ii),

$x = 3 \times 9 = 27$

Hence, present age of teacher = 27 years

present age of student = 9 years

The conclusion of third group

Let, present age of student = x years

present age of teacher = $3x$ years

By question,

$$x + 3x = 36$$

or, $4x = 36$

or, $x = \frac{36}{4}$

$x = 9$

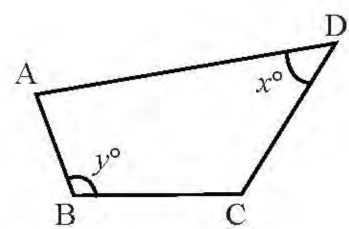
So, present age of student = 9 years

present age of teacher = 3×9 years = 27 years.

Example 1

If the sum of opposite angles of a parallelogram is 150° and their difference is 48°, find the measurement value of angles.

Solution,



Here, opposite angles are x and y. (by figure)

From first condition, $x + y = 150°$

$x = 150° - y$ .... (i)

From the second condition,

$x - y = 48°$ ... (ii) [$\because$ When $x > y$]

From the equation (i) and (ii),

$150° - y - y = 48°$

or, $150° - 2y = 48°$

or, $150° - 48° = 2y$

or, $102° = 2y$

$y = \frac{102°}{2} = 51°$

put, $y = 51°$ in equation (i), we get

$x = 150° - 51° = 99°$

So, the angles are 99° and 51°.

Alternative Method

Here, by question, one angle is 48° more than the other angle.

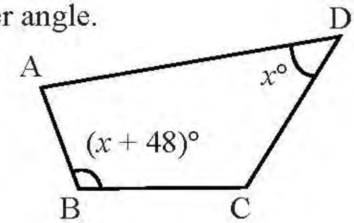
or, $x + 48 + x = 150$

or, $2x = 150 - 48$

or, $x = 51°$

So, small angle is $= x = 51°$

Other angle is $= x + 48 = 51 + 48 = 99°$



Example 2

Manish and Sima bought equal number of copies with same shape and size at the beginning of class 9. At the end of the session, Manish and sima finished 25 and 31 copies respectively and number of copies remained with Manish is two times the copies remained with Sima. Find the number of copies that Sima and Manish buy at the beginning:

Solution,

Here,

Suppose, number of copies with Manish at beginning $= x$

Number of copies with Sima $= y$

From the first condition,

$x = y$ .... (i)

From the second condition,

$x - 25 = 2(y - 31)$

or, $x - 25 = 2y - 62$

or, $x = 2y - 62 + 25$

$x = 2y - 37$ .... (ii)

Replacing the value of x in equation (ii) from (i),

$y = 2y - 37$

or, $37 = 2y - y$

or, $y = 37$

Putting value of y in equation (i),

$x = 37$

So, number of copies with Sima at beginning $(y) = 37$

Number of copies with Manish at the beginning $(x) = 37$

Solving by model drawing method,

x	31	Sima
$2x$	25	Manish

$$2x + 25 = x + 31$$

or, $2x - x = 31 - 25$

or, $x = 6$

So, number of copies with Sima at the beginning is $x + 31 = 6 + 31 = 37$.

Number of copies with Manish at the beginning = 37

Alternative Method

Let, number of copies with Sima and Manish at the beginning = $\boxed{\quad x \quad}$

The conditions after finishing 25 by Sima and 31 copies by Manish is,

$$\overbrace{\boxed{x-31}\boxed{\quad 31 \quad}}^{x}$$

$$\boxed{x-25}\boxed{\quad x \quad}$$

$$2(x - 31) = x - 25$$

or, $2x - 62 = x - 25$

or, $2x - x = 62 - 25$

or, $x = 37$

So, number of copies with Sima and Manish at the beginning is = 37.

Example 3

The present age of a mother is 3 times as her son has now. After 12 years The age of mother will be one year less than the two times of her son's age. Find the present age of mother and his son:

Solution,

Here, present age of mother = x years

present age of son = y years

From the first condition,

$$x = 3y \text{ .... (i)}$$

From the second condition,

$$x + 12 = 2(y + 12) - 1$$

or, $x + 12 = 2y + 24 - 1$

or, $x = 2y + 23 - 12$

or, $x = 2y + 11$ ..... (ii)

Replacing the value of x in equation (i) we have,

$3y = 2y + 11$

or, $3y - 2y = 11$

$\therefore$ y = 11

Putting the value of y in equation (i), we get,

$$x = 3y = 3 \times 11 = 33$$

Hence, present age of mother (x) = 33 years

and present age of son (y) = 11 years

Alternative Method

Let age of son = x years

age of mother = $3x$ years

From first condition,

$\{(x + 12) \times 2\} - 1 = 3x + 12$

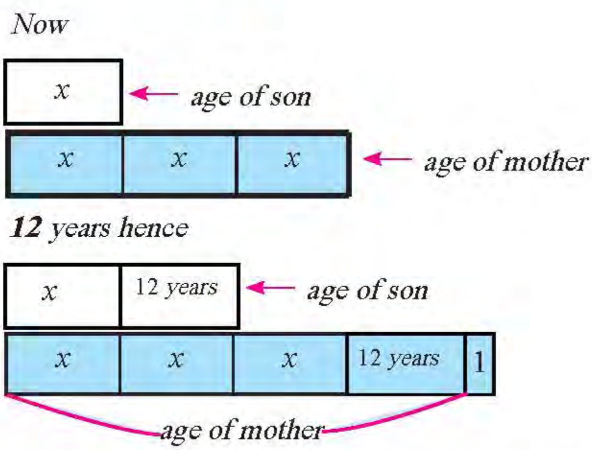
or, $2x + 23 = 3x + 12$

$\therefore x = 11$

So, age of son (x) = 11 years

age of mother ($3x$) = 33 years

Alternative Method



According to the question, the age of mother in 12 years time will be 1 year less than 2 times of his son's age. So, added 1 to make the mother's age 2 times than his son.

Now, $2(x + 12) = 3x + 12 + 1$

or, $2x + 24 = 3x + 13$

or, $x = 11$

Hence, present age of son $= x = 11$ years

present age of mother $= 3x = 3 \times 11 = 33$ years

Example 4

The result of a fraction multiplying its numerator by 4 and subtracting 2 from denominator becomes 2. The result becomes $\frac{9}{7}$ if 15 is added on the numerator and 2 is subtracted from two times of the dinominator. Find the fraction:

Solution,

Here, the fraction is $\frac{x}{y}$ where, x is numerator and y is denominator.

From the first condition,

$$\frac{x \times 4}{y - 2} = 4$$

or, $4x = 2(y - 2)$

or, $x = \frac{2(y-2)}{4} = 4$

or, $x = \dfrac{y-2}{2}$ .... (i)

From the second condition,

$$\frac{x+15}{2y-2} = \frac{9}{7}$$

or, $7x + 105 = 18y - 18$

or, $7x = 18y - 18 - 105$

or, $7x = 18y - 123$

$x = \dfrac{18y + 123}{7}$ .... (ii)

Putting the value of x in equation (ii) we get,

$$\frac{y-2}{2} = \frac{18y+123}{7}$$

or, $7y - 14 = 36y - 246$

or, $246 - 14 = 36y - 7y$

or, $232 = 29y$

or, $y = \dfrac{232}{29} = 8$

Putting the value of y in equation (i) then,

$$x = \frac{y-2}{2} = \frac{8-2}{2} = \frac{6}{2} = 3$$

So, the required fraction is $= \dfrac{x}{y} = \dfrac{3}{8}$

Example 5

The digit in ones place of two digits number is 3 times the digit in tens place. The sum of the number formed by intercharging its digit and the starting number is 88. Then find the starting number:

Solution,

Here, let two digits number is $= 10x + y$, where x and y are digits in tens and ones places respectively.

From the first condition,

$3x = y$

or, $y = 3x$ .... (i)

From second condition,

$$(10x + y) + (10y + x) = 88$$

or, $10x + y + 10y + x = 88$

or, $11x + 11y = 88$

or, $11(x + y) = 88$

or, $x + y = 8$ .... (ii)

Put the value of x in equation (ii) we have,

or, $x + 3x = 8$

or, $4x = 8$

or, $x = \frac{8}{4} = 2$

Put the value of x in equation (i),

$y = 3x = 3 \times 2 = 6$

Hence the starting number $= 10x + y = 10 \times 2 + 6 = 26$.

Alternative Method

Sum of the starting number and the number formed by interchanging its digits is 88.

Let the possible numbers are 17 and 71, 26 and 62, 35 and 53 and 44 and 44.

From first condition,

The digit in ones place is 3 times the digit in tens place. So, the number 26 only satisfy the condition.

So, required number = 26.

Example 6

The price of 6 kg apples and 5 kg mangos is Rs. 560. Likewise, the price of 9 kg apples and 7 kg mangos is Rs. 820. Find the price of 1 kg apple and 1 kg mango each:

Solution,

Here,

Let the price of 1 kg apple is Rs. x and price of 1 kg mango is Rs. y,

From the first condition,

$$6x + 5y = 560 \text{ .... (i)}$$

From the second condition,

$$9x + 7y = 820 \text{ .... (ii)}$$

Multiplying equation (i) by 3 and equation (ii) by 2 and subtracting (ii) from (i),

$$\begin{array}{r} 18x + 15y = 1680 \\ 18x + 14y = 1640 \\ - \quad - \qquad - \\ \hline y = 40 \end{array}$$

Putting the value of y in equation (i),

$6x + 5y = 560$

or, $6x + 5 \times 40 = 560$

or, $6x = 560 - 200$

or, $x = \dfrac{360}{6} = 60$

So, price of 1 kg apple is Rs. 60 and price of 1 kg mango is Rs. 40.

A rule for declaring some winners is made in a essay competition of a school.

Example 7

Total participants of the competition is 63. Each of the winner in that competition get Rs. 100 and each participant gets Rs. 25. The total distributed amount on that competition was Rs. 3000. Find the nuber of winners and number of participants:

Solution,

Here,

Let total number of winners is x and total number of participants only is y.

From the first condition,

$x + y = 63$

$x = 63 - y$ .... (i)

From the second condition,

$100x + 25y = 3000$ ... (iii)

Putting the value of x in equation (ii), we get,

$100 (63 - y) + 25y = 3000$

or, $6300 - 100y + 25y = 3000$

or, $6300 - 3000 = 75y$

or, $3300 = 75y$

$$y = \frac{3300}{75} = 44$$

from equation (i),

$$x = 63 - 44 = 19$$

Hence, total number of winner is 19 and total number of participants only is 44.

Alternative Method

$$\begin{array}{ccc} \overbrace{\boxed{\quad 63 - x \quad}\boxed{\quad x \quad}}^{63} & & \\ \times 25 \qquad \times 100 & =?= & 3,000 \end{array}$$

$25(63 - x) + 100x = 3,000$

or, $1575 - 25x + 100x = 3,000$

or, $75x = 3,000 = 1575$

or, $75x = 1425$

or, $x = 19$

Hence, number of winners $= x = 19$

number of participants only $= 63 - x = 63 - 19 = 44$

Exercise 11.2

1. (a) How can we write three times and more than 1 the sum of two positive integers x and y?

   (b) If the present age of Ram and Sita are x and y years respectively, write what their ages in 10 years later will be.

   (c) The present age of the two brothers is 15 and 11 years respectively. Write what their ages were in ten years ago.

   (d) How can we represent the two-digit number with y and x in ones and tens places respectively? What is the opposite number of the given number?

2. The sum of two positive integers is 128 and their difference is 16. Find the numbers.

3. A number is 5 times the other. Find the numbers if the sum of them is 72.

4. The sum of two acute angles of a triangle is 105° and their difference is 15°. Find the angles.

5. Bishal and Sumita buy an equal number of chocolate at the same price. After eating 5 and 12 chocolates by Bishal and Sumita respectively, Bishal has two times and less than two chocolates than the Susmita has. Find the number of chocolates they have at in the beginning.

6. Five years ago, the age of father was 5 times as old as his son. At present, the sum of their ages is 46 years. Find their present ages.

7. 15 years ago, a father was 4 times as old as his daughter. After five years he will be twice as old as his daughter. Find their present ages.

8. One year after the mother will be 4 times as old as her son. Two years ago the age of the mother was three times the age of her son will be after four years. Find their present ages.

9. Three years ago, the age of Ramesh and Nimesh are in the ratio of 4:3. Three years after the ratio of their ages will be 11:9. Find their present ages.

10. Ten years ago, the ratio of the age of the father and his son was 11:3. Five years after the age of father will be 10 years older than two times the age of his son. Find after how many years the age of the son will be equal to the present age of his father.

11. The sum of numerator and denominator is equal to three less than two times the denominator of a fraction. Find the fraction if 1 is subtracted from its numerator and denominator the numerator will be half of its denominator.

12. A two-digit number is three times the sum of its digits. If 45 is added to the number, the digits interchange their places. Find the number.

13. Find a two-digit number if the sum of the number and the number formed by interchanging its digit is 121 and their difference is 27.

14. The sum of the digits of a number between 10 and 100 is 9. The seven times the number is equal to the four times the number formed by interchanging its digits. Compare the digits of the number in percentage.

15. The total cost of 5 kg of onion and 7 kg of sugar is Rs 810. If the cost of 5 kg of onion is the same as the cost of 2 kg of sugar, find the cost of per kg onion and sugar.

16. A rural municipality provides rectangular land for a school. The ratio of length and breadth of the ground is 11:4. The rural municipality needs Rs. 75,000 for fencing wire of costs Rs 100 per meter for covering one round of the ground. Find the length and breadth of the ground.

17. The breadth of the playground of a school is one-third of its length. If the perimeter of the playground is 32 meters then find their length and breadth.

18. Sambridi goes to Rastra Bank to exchange Rs. 2,000 into paper notes of Rs 50 and Rs 100. The Bank provides only 25 paper notes, find how many paper notes of each category she found.

Project Work

Go to a nearby shop and ask how much the cost of half a dozen or a dozen or a full cartoon of any two of the goods of daily consumption. Make simultaneous equation based on these price and the unit price of those goods. What is the difference in the price of a unit if a person buys large quantity of goods and just a unit of goods or 1 kg? Prepare a report based on this.

Answers

1. (a) $3(x + y) + 1$ (b) $(x + 10)$ year and $(y + 10)$ year
   (c) $(15 - m)$ year and $(11 - m)$ year (d) $10x + y$, $10y + x$
2. 72, 56 3. 12, 60 4. 60, 45
5. 21, 21 6. 35 year, 11 year 7. 55 year, 25 year
8. 47 year 11 year 9. 19 year, 15 year 10. 40 year
11. $\frac{4}{7}$ 12. 27 13. 74
14. 100 % bigger or 50% smaller
15. Onion Rs. 36 per kg Sugar Rs. 90 per kg
16. 275 *m*, 100 *m*
17. 12 *m*, 4 *m*
18. 10 of Rs. 50 and 15 of Rs. 100

Lesson 12 Indices

12.0 Review

The distance between the Earth and the Sun is 1,49,60,00,00,000 km. Likewise, the velocity of light is 30,00,00,000 m/s. Based on this fact, discuss the following questions and present the conclusions in your class:

(a) How can we write the distance between the Earth and the Sun in short form?

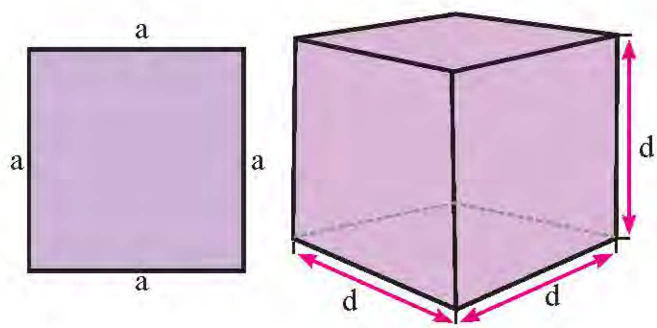
(b) How can we express the velocity of light in short form?

(c) Which format is used to write large numbers in short form?

12.1 Indices

Activity 1

Make different groups in your class with appropriate numbers. Take one square cardboard and one solid cube to each group and discuss the following questions and present the conclusions in class:



(a) What is the area of square cardboard?

(b) What is the volume of the cube? How can we calculate?

(c) Write the base and index from area and volume on (a) and (b).

The area of square cardboard = $a \times a = a^2$ where a is base and 2 is the index. Likewise, the volume of cube = $d \times d \times d = d^3$ where d is the base and 3 is the index.

> If any number or variable is multiplied by the same number or variable two or more than two times, this multiplication can be expressed in an index. Likewise, if a is multiplied by n times then
>
> $a \times a \times a \times \dots (n \text{ times}) = a^n$

12.2 Laws of indices

(a) Multiplication Law of indices

$a^m \times a^n = a^{m+n}$ where m and n are positive integers.

(b) Division law of Indices

$\frac{a^m}{a^n} = a^{m-n}$ where $m > n$

$a^m \div a^n = \frac{1}{a^{n-m}}$ where $n > m$

(c) Power law of indices

$(a^m)^n = a^{m \times n}$, $(ab)^m = a^m b^m$, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

(d) Law of Negative Indices

$a^{-m} = \frac{1}{a^m}$

or, $a^m = \frac{1}{a^{-m}}$

(e) Law of zero index

$a^{\circ} = 1$ $b^{\circ} = 1$, $x^{\circ} = 1$, $(abx)^{\circ} = 1$

(f) Root law of indices

$\sqrt[n]{a^m} = a^{\frac{m}{n}}$

12.3 Simplification of problems related to indices

Activity 2

Discuss the following questions and present the conclusions.

(a) What is the difference between 10^4 and $\frac{1}{10 \times 10 \times 10 \times 10}$?

(b) Does $(6)^3$ and $(6)^{-3}$ have the same values?

(c) Does $\sqrt[3]{64}$ and $\sqrt{64}$ give the same value?

In the first question, in 10^4, 10 is the base, and 4 is the index. In $\frac{1}{10^4} = 10^{-4}$, 10 is the base and the index is – 4

In the second question, $(6)^3 = 216$ and $6^{-3} = \frac{1}{6^3} = \frac{1}{216}$ don't have the same value.

In the third question $\sqrt[3]{64} = (4^3)^{\frac{1}{3}} = 4$ and $\sqrt{64} = (8^2)^{\frac{1}{2}} = 8$. So, it has different values.

$a^m \neq \frac{1}{a^m}$ similarly a^m and $(a)^{-m}$ gives different values. $\sqrt[n]{a^m}$ and $\sqrt[m]{a^n}$ also gives different results.

Example 1

Find the value of :

(a) $(16)^{\frac{3}{2}}$ (b) $\left(\frac{27}{64}\right)^{\frac{-2}{3}}$ (c) $\sqrt[3]{\sqrt{64}}$

Solution,

Here,

(a) $(16)^{\frac{3}{2}} = (4^2)^{\frac{3}{2}} = 4^{2\times\frac{3}{2}} = 4^3 = 64$

(b) $\left(\frac{27}{64}\right)^{\frac{-2}{3}} = \left(\frac{3^3}{4^3}\right)^{\frac{-2}{3}} = \frac{3^{3\times\frac{-2}{3}}}{4^{3\times\frac{-2}{3}}} = \frac{3^{-2}}{4^{-2}} = \frac{4^2}{3^2} = \frac{16}{9}$

(c) $\sqrt[3]{\sqrt{64}} = \sqrt[3]{\sqrt{8^2}} = \sqrt[3]{8} = \sqrt[3]{2^3} = 2^{\frac{3}{3}} = 2$

Example 2

Simplify:

(a) $\left(\frac{27}{8}\right)^{\frac{1}{3}} \times \left\{\left(\frac{243}{32}\right)^{\frac{1}{5}} \div \left(\frac{2}{3}\right)^{-2}\right\}$

(b) $(8x^3 y^9)^{\frac{1}{3}} \div (16x^4y^{12})^{\frac{1}{4}}$

(c) $\sqrt{(x+y)^{-3}} \times (x+y)^{\frac{2}{3}}$

Solution,

Here,

(a) $\left(\frac{27}{8}\right)^{\frac{1}{3}} \times \left\{\left(\frac{243}{32}\right)^{\frac{1}{5}} \div \left(\frac{2}{3}\right)^{-2}\right\}$ $\left[\left\{\left(\frac{a}{b}\right)^{n} = \left(\frac{b}{a}\right)^{-n}\right\}\right]$

$= \left(\frac{3}{2}\right)^{3 \times \frac{1}{3}} \times \left\{\left(\frac{3}{2}\right)^{5 \times \frac{1}{5}} \div \left(\frac{3}{2}\right)^{2}\right\} = \frac{3}{2} \times \left\{\frac{3}{2} \times \left(\frac{2}{3}\right)^{2}\right\} = \frac{3}{2} \times \frac{3}{2} \times \left(\frac{3}{2}\right)^{-2}$

$= \left(\frac{3}{2}\right)^{1 + 1 - 2} = \left(\frac{3}{2}\right)^{0} = 1$ $\quad [\because a^{o} = 1]$

(b) $(8x^3y^9)^{\frac{1}{3}} \div (16x^4y^{12})^{\frac{1}{4}}$

$= \{2^3\, x^3\, y^9\}^{\frac{1}{3}} \div (2^4\, x^4\, y^{12})^{\frac{1}{4}} = 2^{3 \times \frac{1}{3}} x^{3 \times \frac{1}{3}} \div 2^{4 \times \frac{1}{4}} \times x^{4 \times \frac{1}{4}} \times y^{12 \times \frac{1}{4}}$

$= 2xy^3 \div 2xy^3 = \frac{2xy^3}{2xy^3} = 2^{1-1}\, x^{1-1}\, y^{3-3} = 2^0\, x^0\, y^0 = 1 \times 1 \times 1 = 1$

(c) $\sqrt{(x + y)^{-3}} \times (x + y)^{\frac{2}{3}}$ $\quad \left[\because \sqrt[n]{a^m} = a^{\frac{m}{n}}\right]$

$= (x + y)^{-\frac{3}{2}} \times (x + y)^{\frac{2}{3}} = (x + y)^{-\frac{3}{2}+\frac{2}{3}} = (x + y)^{\frac{-9 + 4}{6}} = (x + y)^{\frac{-5}{6}} = \frac{1}{(x + y)^{\frac{5}{6}}}$

Example 3

Simplify:

(a) $\left(\frac{x^a}{x^b}\right)^{a + b} \times \left(\frac{x^b}{x^c}\right)^{b + c} \times \left(\frac{x^c}{x^a}\right)^{c + a}$

(b) $\left(\frac{x^l}{x^m}\right)^{l^2 + lm + n^2} \times \left(\frac{x^m}{x^n}\right)^{m^2 + mn + n^2} \times \left(\frac{x^n}{x^l}\right)^{n^2 + nl + l^2}$

Solution,

Here,

(a) $\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a}$

$= (x^{a-b})^{a+b} \times (x^{b-c})^{b+c} \times (x^{c-a})^{c+a} \left[\because \frac{x^m}{x^n} = x^{m-n}\right]$

$= x^{a^2-b^2} \times x^{b^2-c^2} \times x^{c^2-a^2}$ $[\because (a-b)(a+b) = a^2 - b^2]$

$= x^{a^2-b^2+b^2-c^2+c^2-a^2}$ $[\because x^m \times x^n = x^{m+n}]$

$= x^0 = 1$

(b) $\left(\frac{x^l}{x^m}\right)^{l^2+lm+m^2} \times \left(\frac{x^m}{x^n}\right)^{m^2+mn+n^2} \times \left(\frac{x^n}{x^l}\right)^{n^2+nl+l^2}$

$= (x^{l-m})^{l^2+lm+m^2} \times (x^{m-n})^{m^2+mn+n^2} \times (x^{n-l})^{n^2+nl+l^2}$

$[\because x^m \div x^n = x^{m-n}]$

$= x^{(l-m)(l^2+lm+m^2)} \times x^{(m-n)(m^2+mn+n^2)} \times x^{(n-l)(n^2+nl+l^2)}$

$[\because (x^m)^n = x^{mn}]$

$= x^{(l^3-m^3)} \times x^{(m^3-n^3)} \times x^{(n^3-l^3)}$ $[\because a^3 - b^3 = (a-b)(a^2+ab+b^2)]$

$= x^{l^3-m^3+m^3-n^3+n^3-l^3}$ $[\because x^m \times x^n \times x^p = x^{m+n+p}]$

$= x^0 = 1$ $[\because a^0 = 1]$

Example 4

Prove that:

(a) $\sqrt[bc]{\frac{x^{b/c}}{x^{c/b}}} \times \sqrt[ca]{\frac{x^{c/a}}{x^{a/c}}} \times \sqrt[ab]{\frac{x^{a/b}}{x^{b/a}}} = 1$

(b) $\frac{1}{1+a^{x-y}+a^{z-y}} + \frac{1}{1+a^{y-z}+a^{x-z}} + \frac{1}{1+a^{z-x}+a^{y-x}} = 1$

Solution,

Here,

(a) L.H.S $= \sqrt[bc]{\dfrac{x^{b/c}}{x^{c/b}}} \times \sqrt[ca]{\dfrac{x^{c/a}}{x^{a/c}}} \times \sqrt[ab]{\dfrac{x^{a/b}}{x^{b/a}}}$

$$= \left(\frac{x^{b/c}}{x^{c/b}}\right)^{1/bc} \times \left(\frac{x^{c/a}}{x^{a/c}}\right)^{1/ac} \times \left(\frac{x^{a/b}}{x^{b/a}}\right)^{1/ab} \quad \left[\because \sqrt[n]{a^m} = a^{\frac{m}{n}}\right] \quad \left[\because (x^m)^n = x^{m\times n}\right]$$

$$= \frac{x^{b/c \times 1/bc}}{x^{c/b\times 1/bc}} \times \frac{x^{c/a \times 1/ca}}{x^{a/c\times 1/ca}} \times \frac{x^{a/b \times 1/ab}}{x^{b/a\times 1/ab}}$$

$$= \frac{x^{1/c^2}}{x^{1/b^2}} \times \frac{x^{1/a^2}}{x^{1/c^2}} \times \frac{x^{1/b^2}}{x^{1/a^2}} \quad \left[\because \frac{a^m}{a^n} = a^{m-n}\right] \quad \left[\because a^0 = 1\right]$$

$$= x^{1/c^2 - 1/b^2} \times x^{1/a^2 - 1/c^2} \times x^{1/b^2 - 1/a^2}$$

$$= x^{1/c^2 - 1/b^2 + 1/a^2 - 1/c^2 + 1/b^2 - 1/a^2}$$

$$= x^0 = 1 = \text{R.H.S.}$$

(b) L.H.S

$$= \frac{1}{1 + a^{x-y} + a^{z-y}} + \frac{1}{1 + a^{y-z} + a^{x-z}} + \frac{1}{1 + a^{z-x} + a^{y-x}} \quad \left[\because a^{m-n} = \frac{a^m}{a^n}\right]$$

$$= \frac{1}{1 + \frac{a^x}{a^y} + \frac{a^z}{a^y}} + \frac{1}{1 + \frac{a^y}{a^z} + \frac{a^x}{a^z}} + \frac{1}{1 + \frac{a^z}{a^x} + \frac{a^y}{a^x}}$$

$$= \frac{1}{\frac{a^y + a^x + a^z}{a^y}} + \frac{1}{\frac{a^z + a^y + a^x}{a^z}} + \frac{1}{\frac{a^x + a^z + a^y}{a^x}}$$

$$= \frac{a^y}{a^y + a^x + a^z} + \frac{a^z}{a^y + a^z + a^x} + \frac{a^x}{a^y + a^z + a^x}$$

$$= \frac{a^y + a^z + a^x}{a^y + a^z + a^x} = 1 = \text{R.H.S}$$

Example 5

Simplify:

$$\frac{\left(p^2-\frac{1}{q^2}\right)^x \times \left(p-\frac{1}{q}\right)^{y-x}}{\left(q^2-\frac{1}{p^2}\right)^y \times \left(q+\frac{1}{p}\right)^{x-y}}$$

Solution,

Here,

$$\frac{\left(p^2-\frac{1}{q^2}\right)^x \times \left(p-\frac{1}{q}\right)^{y-x}}{\left(q^2-\frac{1}{p^2}\right)^y \times \left(q+\frac{1}{p}\right)^{x-y}}$$

$$= \frac{\left(p+\frac{1}{q}\right)^x\left(p-\frac{1}{q}\right)^x \times \left(p-\frac{1}{q}\right)^{y-x}}{\left(q+\frac{1}{p}\right)^y\left(q-\frac{1}{p}\right)^y \times \left(q+\frac{1}{p}\right)^{x-y}}$$

$$= \frac{\left(p+\frac{1}{q}\right)^x\left(p-\frac{1}{q}\right)^{x+y-x}}{\left(q+\frac{1}{p}\right)^{y+x-y}\left(q-\frac{1}{p}\right)^y}$$

$$= \frac{\left(\frac{pq+1}{q}\right)^x\left(\frac{pq-1}{q}\right)^y}{\left(\frac{pq+1}{p}\right)^x\left(\frac{pq-1}{p}\right)^y}$$

$$= \left(\frac{pq+1}{q} \times \frac{p}{pq+1}\right)^x \times \left(\frac{pq-1}{q} \times \frac{p}{pq-1}\right)^y$$

$$= \left(\frac{p}{q}\right)^x \times \left(\frac{p}{q}\right)^y$$

$$= \left(\frac{p}{q}\right)^{x+y}$$

Example 6

If, $a + b + c = 0$ then prove that,

$$\frac{1}{1 + x^a + x^{-b}} + \frac{1}{1 + x^b + x^{-c}} + \frac{1}{1 + x^c + x^{-a}} = 1$$

Solution,

Here,

$a + b + c = 0$

or, $a + b = -c$ .... (i)

$$\text{L.H.S} = \frac{1}{1 + x^a + x^{-b}} + \frac{1}{1 + x^b + x^{-c}} + \frac{1}{1 + x^c + x^{-a}}$$

$$= \frac{1}{1 + x^a + \frac{1}{x^b}} + \frac{1}{1 + x^b + x^{a+b}} + \frac{1}{1 + x^c + x^{-a}} \quad [\because \text{From equation (i)}]$$

$$= \frac{1}{\frac{x^b + x^{a+b} + 1}{x^b}} + \frac{1}{1 + x^b + x^{a+b}} + \frac{1}{1 + x^c + x^{-a}}$$

$$= \frac{x^b}{x^b + 1 + x^{a+b}} + \frac{1}{x^b + 1 + x^{a+b}} + \frac{x^{a+b}}{x^{a+b}(1 + x^c + x^{-a})}$$

$[\because$ Multiply x^{a+b} n$]$

$$= \frac{x^b + 1}{x^b + 1 + x^{a+b}} + \frac{x^{a+b}}{x^{a+b} + x^{a+b+c} + x^{a+b-a}}$$

$$= \frac{x^b + 1}{x^b + 1 + x^{a+b}} + \frac{x^{a+b}}{x^{a+b} + 1 + x^b} \quad [\because x^{a+b+c} = x^0 = 1]$$

$$= \frac{x^b + 1}{x^b + 1 + x^{a+b}} + \frac{x^{a+b}}{x^b + 1 + x^{a+b}}$$

$$= \frac{x^b + 1 + x^{a+b}}{x^b + 1 + \mathrm{x}^{\mathrm{a+b}}} = 1 = \text{R.H.S.}$$

Exercise 12.1

1. Find the value of:

(a) $3^5 \times 3^{-5}$ (b) $9^4 \times \frac{1}{9^3}$ (c) $5^3 \times \frac{1}{5^2}$

(d) $(64)^{\frac{2}{3}}$ (e) $\left(\frac{81}{3}\right)^{\frac{1}{3}}$ (f) $\sqrt[3]{\frac{1}{125}}$

(g) $\left(\frac{27}{64}\right)^{\frac{-2}{3}}$ (h) $\left\{\left(\frac{216}{125}\right)^{-\frac{1}{3}}\right\}^{2}$ (i) $\left(\frac{1}{128}\right)^{\frac{1}{7}} + \left(\frac{1}{64}\right)^{\frac{1}{6}}$

(j) $\left(\frac{1}{16}\right)^{\frac{1}{4}} \times (8)^{\frac{4}{3}}$ (k) $\left(\frac{1}{3}\right)^{-3} + \left(\frac{1}{3}\right)^{-1} + \left(\frac{1}{2}\right)^{-3} + \left(\frac{1}{8}\right)^{0}$

2. Simplify:

(a) $(125\,m^3 \div 27b^{-3})^{-\frac{2}{3}}$ (b) $(64a^3 \div 125b^{-3})^{-\frac{2}{3}}$

(c) $x^{a-b} \times x^{b-c} \times x^{c-a}$ (d) $x^{b^2-c^2} \times x^{c^2-a^2} \times x^{a^2-b^2}$

(e) $x^{ab} \times x^{bc} \times x^{ca} \times x^{-ab} \times x^{-bc} \times x^{-ca}$ (f) $\frac{x^{2m+3n} \times x^{3m+6n}}{x^{2m+3n} \times x^{4m-4n}}$

(g) $\frac{a^{9n+3} \times a^{-4n}}{a^{2n+10} \times a^{3n-7}}$ (h) $\frac{7^{n+1} \times 7^{n^2-1}}{7^{n^2-n} \times 7^{2n+2}}$

3. Find the value of:

(a) $\left(\frac{27}{8}\right)^{\frac{-1}{3}}\left[\left(\frac{81}{16}\right)^{\frac{1}{4}} \div \left(\frac{4}{25}\right)^{\frac{-1}{2}}\right]$ (b) $\left(\frac{25}{16}\right)^{\frac{-1}{3}}\left[\left(\frac{125}{64}\right)^{\frac{1}{3}} \div \left(\frac{8}{27}\right)^{\frac{-1}{3}}\right]$

(c) $\left(\frac{27}{8}\right)^{\frac{1}{3}}\left[\left(\frac{243}{32}\right)^{\frac{1}{5}} \div \left(\frac{2}{3}\right)^{-2}\right]$ (d) $\frac{3^4 \times 27^3 \times 9^5}{81^6 \times 3^3 \times 9^{-2}}$

(e) $\frac{2^4 \times 8^3 \times 4^5}{16^6 \times 2^3 \times 4^{-2}}$

4. Simplify:

(a) $\frac{1}{1+x^{a-b}} + \frac{1}{1+x^{b-a}}$ (b) $\frac{1}{1-a^{x-y}} + \frac{1}{1-a^{y-x}}$

(c) $\dfrac{(x^{a+b})^2 \times (x^{b+c})^2 \times (x^{c+a})^2}{(x^a . x^b . x^c)^4}$

(d) $\dfrac{(a^{x+y})^3 \times (a^{y+z})^3 \times (a^{z+x})^3}{(a^x . a^y . a^z)^6}$

(e) $\dfrac{27^{3a+1} \times (243)^{\frac{-4a}{5}}}{9^{a+5} \times 3^{3a-7}}$

(f) $\dfrac{(32)^{\frac{2n}{5}} \times 2^{2n+1}}{4^{n+1} \times 2^{2(n-2)}}$

5. Simplify:

(a) $\left(\dfrac{x^b}{x^{-a}}\right)^{b-a} \times \left(\dfrac{x^a}{x^{-c}}\right)^{a-c} \times \left(\dfrac{x^c}{x^{-b}}\right)^{c-b}$

(b) $\left(\dfrac{x^{-b}}{x^{-a}}\right)^{a+b} \times \left(\dfrac{x^{-c}}{x^{-b}}\right)^{c+b} \times \left(\dfrac{x^{-a}}{x^{-c}}\right)^{c+a}$

(c) $\left(\dfrac{x^m}{x^{-n}}\right)^{m^2-mn+n^2} \times \left(\dfrac{x^n}{x^{-l}}\right)^{n^2-nl+l^2} \times \left(\dfrac{x^l}{x^{-m}}\right)^{l^2-ml+m^2}$

(d) $\left(\dfrac{x^{a+b}}{x^c}\right)^{a-b} \times \left(\dfrac{x^{b+c}}{x^a}\right)^{b-c} \times \left(\dfrac{x^{c+a}}{x^b}\right)^{c-a}$

(e) $\sqrt[a+b]{x^{a^2-b^2}} \times \sqrt[b+c]{x^{b^2-c^2}} \times \sqrt[c+a]{x^{c^2-a^2}}$

(f) $\left(\dfrac{x^{a^2+b^2}}{x^{-ab}}\right)^{a-b} \times \left(\dfrac{x^{b^2+c^2}}{x^{-bc}}\right)^{b-c} \times \left(\dfrac{x^{c^2+a^2}}{x^{-ca}}\right)^{c-a}$

(g) $\left(\dfrac{x^b}{x^c}\right)^{\frac{1}{bc}} \times \left(\dfrac{x^c}{x^a}\right)^{\frac{1}{ca}} \times \left(\dfrac{x^a}{x^b}\right)^{\frac{1}{ab}}$

(h) $\left\{(a^x.a^y)^{x-y}\left(\dfrac{a^y}{a^z}\right)^{y+z}\right\} \times \left(\dfrac{a^z}{a^x}\right)^{z+x}$

6. Prove that:

(a) $\sqrt[bc]{\dfrac{x^b}{x^c}} \times \left(\dfrac{x^c}{x^a}\right)^{\frac{1}{ca}} \times \sqrt[ab]{\dfrac{x^a}{x^b}} = 1$

(b) $\sqrt[c]{\dfrac{x^a}{x^b}} \times \sqrt[a]{\dfrac{x^b}{x^c}} \times \sqrt[b]{\dfrac{x^c}{x^a}} = 1$

(c) $\sqrt[bc]{\dfrac{x^{b/c}}{x^{c/b}}} \times \sqrt[ca]{\dfrac{x^{c/a}}{x^{a/c}}} \times \sqrt[ab]{\dfrac{x^{a/b}}{x^{b/a}}} = 1$

(d) $$\frac{1}{1+x^{a-b}+x^{c-b}}+\frac{1}{1+x^{b-c}+x^{a-c}}+\frac{1}{1+x^{c-a}+x^{b-c}}=1$$

(e) $$\frac{\left(p+\frac{1}{q}\right)^m\times\left(p-\frac{1}{q}\right)^m}{\left(q+\frac{1}{p}\right)^m\times\left(q-\frac{1}{p}\right)^m}=\left(\frac{p}{q}\right)^{2m}$$

(f) $$\frac{\left(x^2+\frac{1}{y^2}\right)^a\times\left(x-\frac{1}{y}\right)^{b-a}}{\left(y^2+\frac{1}{x^2}\right)^b\times\left(y+\frac{1}{x}\right)^{a-b}}=\left(\frac{x}{y}\right)^{a+b}$$

(g) $$\frac{\left(a^2-\frac{1}{b^2}\right)^x\times\left(a-\frac{1}{b}\right)^{y-x}}{\left(b^2-\frac{1}{a^2}\right)^y\times\left(b+\frac{1}{a}\right)^{x-y}}=\left(\frac{a}{b}\right)^{x+y}$$

7. If, $p^3+q^3+r^3=1$, prove that:

$$\left(\frac{x^p}{x^{-q}}\right)^{p^2-pq+q^2}\times\left(\frac{x^q}{x^{-r}}\right)^{q^2-qr+r^2}\times\left(\frac{x^r}{x^{-p}}\right)^{r^2-rp+p^2}=x^2$$

8. If, $x^2+y^2+z^2=2(xy+yz+zx)$, then prove that:

$$\left(\frac{a^x}{a^y}\right)^{x-y}\times\left(\frac{a^y}{a^z}\right)^{y-z}\times\left(\frac{a^z}{a^x}\right)^{z-x}=1$$

9. If, $g+h+f=0$ then, prove that:

$$\frac{1}{1+m^g+m^{-h}}+\frac{1}{1+m^h+m^{-f}}+\frac{1}{1+m^f+m^{-g}}=1$$

10. If, $r+s+t=0$ then, prove that:

$$\frac{1}{1+x^r+x^{-s}}+\frac{1}{1+x^s+x^{-t}}+\frac{1}{1+x^t+x^{-r}}=1$$

11. If, $xyz=1$ then, prove that:

$$\frac{1}{1+x+y^{-1}}+\frac{1}{1+y+z^{-1}}+\frac{1}{1+z+x^{-1}}=1$$

Answer

1.	(a)	1	(b)	9	(c)	5
	(d)	16	(e)	3	(f)	$\frac{1}{5}$
	(g)	$\frac{16}{9}$	(h)	$\frac{25}{36}$	(i)	1
	(j)	8	(k)	39		
2.	(a)	$\frac{9}{25m^2b^2}$	(b)	$\frac{25}{16a^2b^2}$	(c)	1
	(d)	1	(e)	1	(f)	x^{10n-m}
	(g)	1	(h)	$\frac{1}{49}$		
3.	(a)	$\frac{2}{5}$	(b)	$\frac{2}{3}$	(c)	1
	(d)	1	(e)	1		
4.	(a)	1	(b)	1	(c)	1
	(d)	1	(e)	1	(f)	8
5.	(a)	1	(b)	1	(c)	$x^{2(x^3+m^3+n^3)}$
	(d)	1	(e)	1	(f)	1
	(g)	1	(h)	1		

Mixed Exercise

1. Factorize the following expressions:

(a) $216a^3 + \frac{1}{8}$ (b) $8x^3 - 125y^3$

(c) $(x+2)^3$ (d) $(x-3)^3$

(e) $16m^4 - 65m^2n^2 + 49n^4$ (f) $9x^4 - \frac{2x^2}{y^2} + \frac{1}{9y^4}$

(g) $64 - 144a + 108a^2 - 27a^3$ (h) $a^4 - 5a^2b^2 + 4b^4$

2. Find Highest Common Factor (HCF) of the given expressions:

(a) $x^2 + 2x - 8 - y^2 - 6y, x^2 + 2xy + y^2 - 16, x^2 + xy + 4x$

(b) $9(a + b)^2 + a + b - 8, a^2 + 2ab + b^2 - 1, a^2 + ab + a$

(c) $x^2 - 10x + 24 + 6y - 9y^2, x^2 + 6xy + 9y^2 - 36$ and $x^2 + 3xy - 6x$

(d) $\frac{x^4}{y^4} + 1 + \frac{y^4}{x^4}, \frac{x^3}{y^3} + \frac{y^3}{x^3}$ र $\frac{x^3}{y^2} - x + \frac{y^2}{x^2}$

3. Find Lowest Common Multiple (LCM) of the given expression:

(a) $a^5 + 2a^4 - 9a^3 - 18a^2, a^5 - 2a^4 + 8a^2 - 4a^3, a^4 - 4a^2$

(b) $x^4 - 8x^2 - 33 - 14y - y^2, (x^2 + y^2)^2 - 9, x^3 + xy + 3\text{x}$

(c) $x^2 + 2xy + y^2 - z^2, y^2 + 2yz + z^2 - x^2, z^2 + 2zx + x^2 - y^2$

(d) $a(a + c) - b(b + c), \text{b}(a + b) - c(c + a)\ c(b + c) - a(a + b)$

4. The HCF and LCM of two algebraic expressions are $2u + 5v$, and $3u(2u + 5v)(2u - 5v)$ respectively and if one of the expressions is $4u^2 - 25v^2$, then find the other expression.

5. Find HCF and LCM of the given expression:

(a) $(p + 3)^3, p^2 + 6p + 9, 2p^3 + 18p^2 + 54p + 54$

(b) $q^2 - 6q - 40 + 14r - r^2, q^2 - 2qr + r^2 - 16, q^2 - qr + 4r$

6. Solve the following pair of linear equations by replacement method:

(a) $2x + 3y = 8, 3x + 2y = 7$

(b) $\frac{2}{3}x + 2y = 1, \frac{x}{3} - \frac{y}{3} = 1$

(c) $5x - 2y = 10, 4x - 3y = -6$

7. Solve the following pair of linear equations by elimination method:

(a) $\frac{x}{6} - y = -6$

$\frac{3x}{4} - 1 = y$

(b) $\frac{x + y}{2} = \frac{3x - 5y}{4}$

$\frac{x - 3y}{2} = \frac{x - 1}{3}$

8. The present age of A is 2 times and 4 years more than the present age of B. The present age of B is $\frac{2}{5}$ times and 2 years more than the present age of A. Find the difference between the present age of A and B.

9. The ratio of the present age of Ram and Harish is 5:7. Before 8 years, the ratio between their age was 7:13, then find their present age.

10. Find a two-digit number where the digit of one's place is three times the digit of the tens place and if 36 is added to that number, the place value of the digits interchanged.

11. Bikash plans to establish a new office. He asks the price of the table and chair in a shop. If the price of three tables and two chairs is Rs. 1900 and the price of two tables and four chairs is Rs 1,800, then calculate the price of one table and four chairs.

12. Simplify:

$$\left(\frac{x^m}{x^n}\right)^{m-p+n} \times \left(\frac{x^n}{x^p}\right)^{n-m+p} \times \left(\frac{x^p}{x^m}\right)^{p-n+m}$$

13. Simplify:

$$\frac{1}{1+x^{p-q}+x^{p-r}} + \frac{1}{1+x^{q-p}+x^{q-r}} + \frac{1}{1+x^{r-p}+x^{r-q}}$$

14. Simplify:

$$\frac{\left(9x^2-\frac{1}{9y^2}\right)^{3x}\left(3x-\frac{1}{3y}\right)^{3y-3x}}{\left(9y^2-\frac{1}{9x^2}\right)^{3y}\cdot\left(3y+\frac{1}{3y}\right)^{3x-3y}}$$

15. If, $\frac{1}{ab}+\frac{1}{bc}+\frac{1}{ca}=0$, prove that:

$(1+x^a+x^{-b})^{-1}+(1+x^b+x^{-c})^{-1}+(1+x^c+x^{-a})^{-1}=1$

Answers

1. (a) $\left(6a + \frac{1}{2}\right)(36a^2 - 3a + \frac{1}{4})$

   (b) $(2x - 5y)(4x^2 + 10xy + 25y^2)$

   (c) $(x + 2)(x + 2)(x + 2)$

   (d) $(x - 3)(x - 3)(x - 3)$

   (e) $(4m^2 + 3mn - 7n^2)(4m^2 - 3mn - 7n^2)$

   (f) $\left(3x^2 - \frac{1}{3y^2}\right)\left(3x^2 - \frac{1}{3y^2}\right)$

   (g) $(4 - 3a)(4 - 3a)(4 - 3a)$

   (h) $(a - 2b)(a + 2b)(a + b)(a - b)$

2. (a) $(x + y + 4)$ (b) $(a + b + 1)$

   (c) $x + 3y - 6$ (d) $\left(\frac{x^2}{y^2} - 1 + \frac{y^2}{x^2}\right)$

3. (a) $a^2(a + 2)(a - 2)^2(a^2 - 9)$

   (b) $x(x^2 + y + 3)(x^2 - y - 11)(x^2 + y - 3)$

   (c) $(x + y + z)(x + y - z)(y + z - x)(z + x - y)$

   (d) $(a + b + c)(a - b)(b - c)(c - a)$

4. $2u^2 + 15uv$

5. (a) HCF = $(p + 3)^2$ LCM = $2(p + 3)^3$

   (b) HCF = $(q - r + 4)$ LCM = $q(q - r + 4)(q + r - 10)(q - r - 4)$

6. (a) $x = 1, y = 2$

   (b) $x = 2, y = -1$

   (c) $x = 6, y = 10$

7. (a) $x = 12, y = 8$ 8. 22 years

9. 15 years, 21 years

10. The number of Binya searches = 62

11. Rs. 500, Rs. 200 12. 1 13. 1

14. $\left(\frac{x}{y}\right)^{3(x+y)}$

Lesson 13

Triangle

13.0 Review

Draw a pair of triangles in a sheet of paper. Measure all the sides and angles of these triangles and tabulate them. What types of triangles are formed in terms of sides and angles? How can we differentiate whether these triangles are congruent or similar or not? Discuss within pair and write the conclusion. Present the pair task in news print paper with figure in the classroom.

13.1 Properties of triangles and their verification

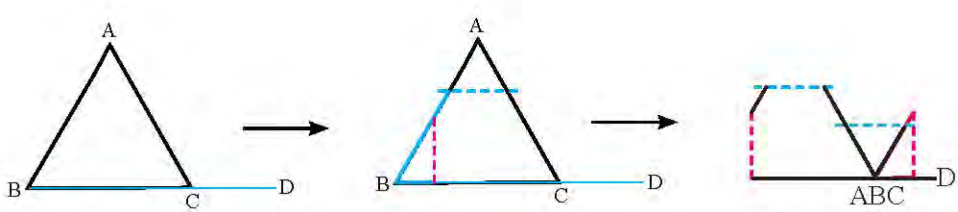
Divide the students in proper groups on the basis of class size. Draw a pair of triangles by each set of students, such as: equilateral, isosceles, right angled, and scalene.

Activity 1

Verify experimentally that if any side of a triangle is produced the exterior angle so formed is equal to the sum of the two opposite interior angles.

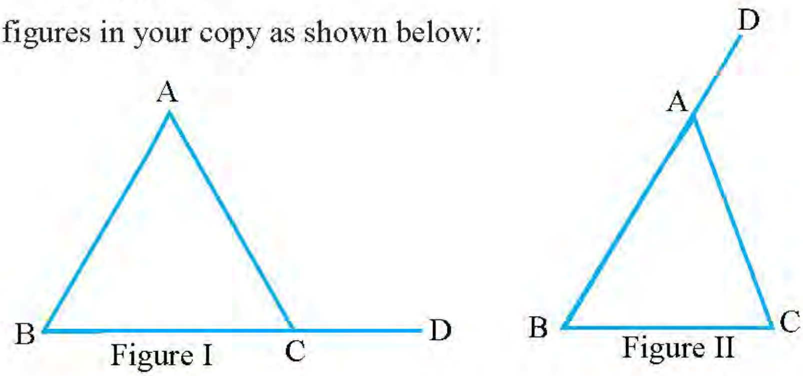
(I) Paper cutting

Draw a figure in a paper and cut it as shown below. Put the two pieces of opposite interior angles in the exterior angle of a triangle. Then present the relation between the exterior angle and the sum of its opposite interior angles of the triangle in your class.



(II) Measuring angles

Draw the figures in your copy as shown below:



Measure the angles using protractor and tabulate them as shown in the following table.

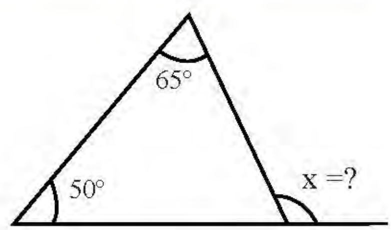
Figure	Opposite interior angles		Exterior angles	Sum of two opposite interior angles	Results
I.	∠ABC =	∠BAC =	∠ACD =	∠ABC + ∠BAC =	
II.	∠ABC =	∠ACB =	∠CAD =	∠ABC + ∠ACB =	

Present the results in your class based on the above table.

Conclusion: If any side of a triangle is produced, the exterior angle so formed is equal to the sum of the two opposite interior angles.

Example 1

Find the value of x from the following figure:



Solution

Here,

$x = 50° + 65°$

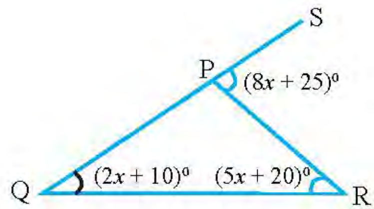
$= 115°$

[∵ An exterior angle of a triangle is equal to the sum of two opposite interior angles.]

$\therefore \quad x = 115°$

Example 2

Find the value of x from the following figure:



Solution

Here,

From figure, $\angle SPR = (8x + 25)°$

$\angle PQR = (2x + 10)°$ र $\angle PRQ = (5x + 20)°$

We know that, $\angle PQR + \angle PRQ = \angle SPR$

or, $(2x + 10) + (5x + 20) = (8x + 25)$

or, $2x + 10 + 5x + 20 = 8x + 25$

or, $7x + 30 = 8x + 25$

or, $x = 30 - 25 = 5$

Therefore, $\angle SPR = 8x + 25 = 8 \times 5 + 25 = 40 + 25 = 65°$

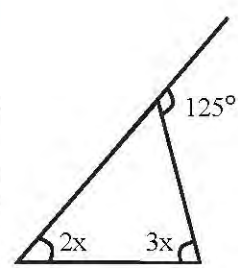
or, $\angle PQR = 2x + 10 = 2 \times 5 + 10 = 10 + 10 = 20°$

and $\angle PRQ = 5x + 20 = 5 \times 5 + 20 = 25 + 20 = 45°$

Example 3

The exterior angle of a triangle is 125° and the two opposite interior angles are in the ratio 3:2, then find all interior angles of the triangle.

Solution



Here, exterior angle = 125°

Let us assume that the two non-adjacent interior angles be $3x$ and $2x$.

Now, $3x + 2x = 125°$ [$\because$ An exterior angle of a triangle is equal to the sum of two opposite interior angles.]

or, $5x = 125°$

or, $x = \dfrac{125°}{5} = 25°$

Then, the three angles of the triangle are:

First angle $= 3x = 3 \times 25° = 75°$

Second angle $= 2x = 2 \times 25° = 50°$

Third angle $= 180° - 125° = 55°$

Exercise 13.1

1. Write the reason whether the following statements are true or false.

(a) There are two right angles in a triangle.

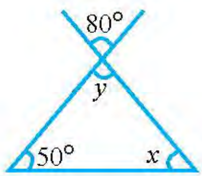
(b) There are all acute angles in a triangle.

(c) If the value of an exterior angle of a triangle is $130°$, then the adjacent angle is also obtuse angle.

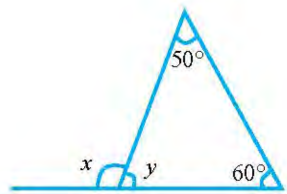
(d) If interior angle is acute angle, then the adjacent exterior angle is obtuse angle.

2. Find the value of x and y from the following figures:

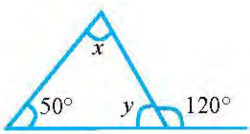
(a)



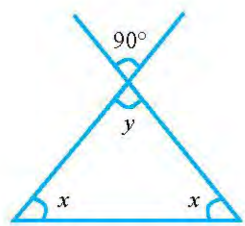
(b)



(c)

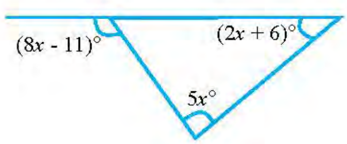


(d)

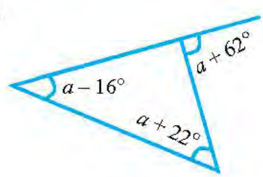


3. Find the value of x, y, z and a from the following figures:

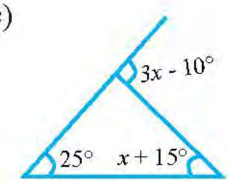
(a)



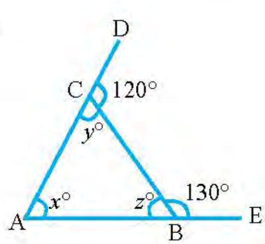
(b)



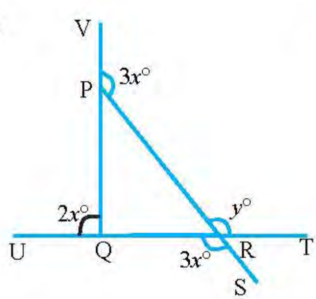
(c)



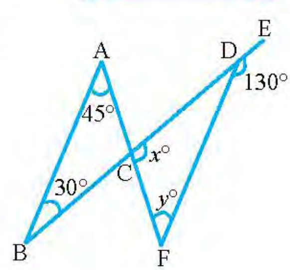
(d)



(e)

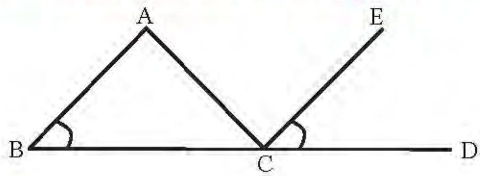


(f)



4. The exterior angle of a triangle is $120°$ and the ratio of two opposite interior angles are 3:2, then find the measurement of all the interior angles of the triangle.

5. **What is the measurement value of $\angle ACE$, if $\angle BAC : \angle ABC : \angle ACB$ = 2:1:1 and $\angle ABC = \angle ECD$ in the given figure?**



Answer

1. Show the answer to your teacher.
2. (a) $x = 50°, y = 80°$ (b) $x = 110°, y = 70°$ (c) $x = 70°, y = 60°$
   (d) $x = 45°, y = 90°$
3. (a) $17°$ (b) $56°$ (c) $25°$
   (d) $x = 70°, y = 60°, z = 50°$ (e) $45°, 135°$ (f) $x = 105°, y = 25$
4. $40°, 60°, 80°$ 5. $90°$

13.2 Verification of properties of isosceles triangle

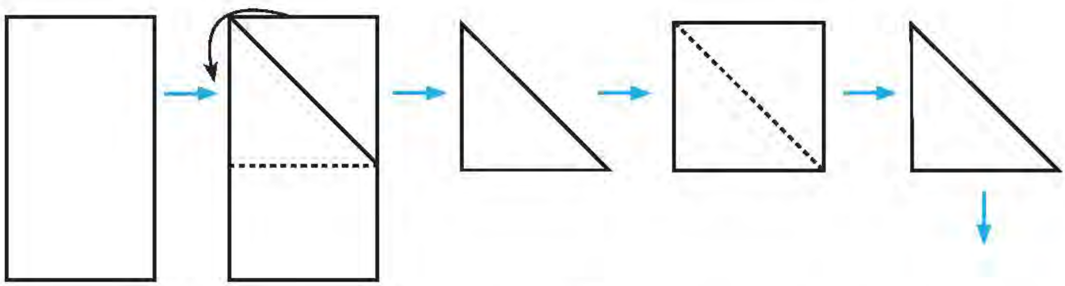
Divide the suitable groups and draw a pair of isosceles triangle by each groups. Measure the sides and angles of both isosceles triangles using scale and protractor. Find the relationship of between sides and angles, and draw the conclusion. Discuss in your group and present the conclusion in the class.

Activity 1

(a) Verify experimentally that the bisector of vertical angle of an isosceles triangle is perpendicular to the base and bisects it.

(I) Folding paper

Take a rectangular sheet of paper. Fold the paper by overlapping the adjacent sides and make square. Then, fold the square piece of paper diagonally and cut it. It formors an isosceles triangle shown as in the following figure.

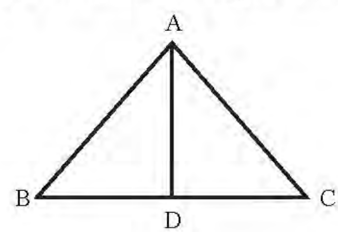


Again, fold the isosceles triangular piece of paper ABC as shown in figure and measure the sides and angles formed.

(a) BD and CD

(b) $\angle BDA$ and $\angle CDA$

Draw the conclusion and present in the class.



(II) Measuring sides and angles

Divide the class in a suitable group. Draw two different size isosceles triangles ABC in each group.

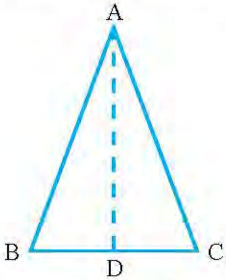


Figure I

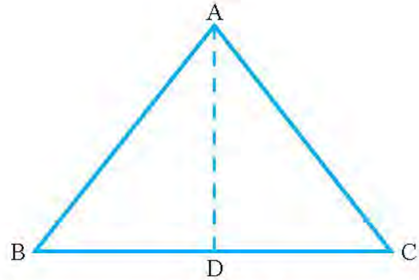


Figure II

Draw angle bisector of vertex A using compass or protractor. The angle bisector AD meets opposite side BC at point D.

Now, measure the angles $\angle ADB$ and $\angle ADC$ and sides BD and CD; and tabulate them as shown in the following table.

Figure	$\angle$ADB	$\angle$ADC	BD	DC	Results
I.					
II.					

Conclusion: The bisector of vertical angle of an isosceles triangle is perpendicular to the base and bisects it.

Activity 2

(b) Verify experimentally that the perpendicular bisector of base bisects the vertical angle of isosceles triangle.

Draw two different size isosceles triangles ABC in each group with your friend.

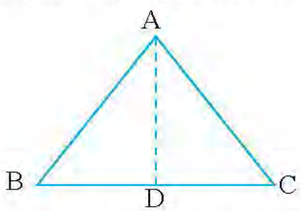


Figure I

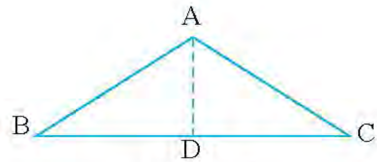


Figure II

Draw perpendicular bisector of base BC using compass. The perpendicular bisector cut at point D of base BC.

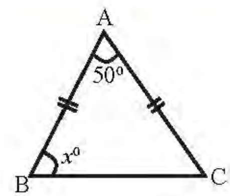
Now, measure the angles $\angle BAD$ and $\angle CAD$; and tabulate them as shown in the following table.

Figure	$\angle BAD$	$\angle CAD$	Result
I.			
II.			

Conclusion: Perpendicular bisector of base bisects the vertical angle of isosceles triangle.

Example 1

Find the value of x from the given figure:



Solution

Here, $\angle BAC = 50^\circ$, AB = AC

Since, the base angles of isosceles triangle are equal.

$\angle ABC = \angle ACB = x$

$\angle ABC + \angle ACB + \angle BAC = 180^\circ$ [$\because$ Sum of interior angle of triangle]

or, $x + x + 50^\circ = 180^\circ$

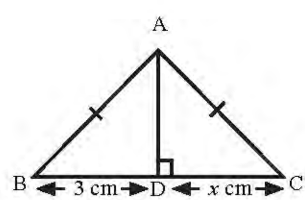
or, $2x = 130^\circ$

or, $x = \dfrac{130^\circ}{2}$

or, $x = 65^\circ$

Example 2

Find the value of x from the given figure:



Solution:

Here, AB = AC, AD $\perp$ BC, DC = x cm, BD = 3 cm.

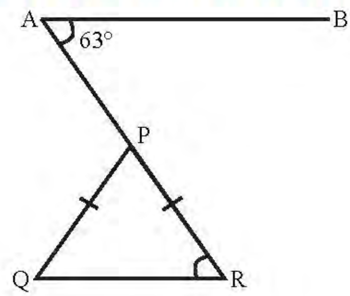
In the isosceles triangle ABC, the perpendicular bisects the base BC into two equal parts.

Thus, BD = DC

or, 3 cm = x $\quad \therefore x = 3$ cm

Example 3

If PQ = PR and ∠BAR = ∠PRQ in the given figure, then find the value of ∠APQ.



Solution

Here, PQ = PR and ∠BAR = ∠PRQ

$\therefore \angle PRQ = 63^{\circ}$

Now, in isosceles triangle PQR

$\therefore \angle PRQ = \angle PQR = 63^{\circ}$

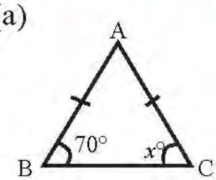
In ΔPQR, $\angle APQ = \angle PRQ + \angle PQR = 63^{\circ} + 63^{\circ} = 126^{\circ}$

$\therefore \angle ARQ = 126^{\circ}$

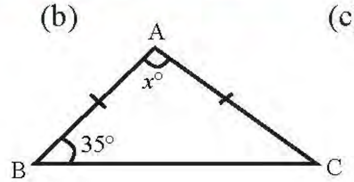
Exercise 13.2

1. Find the value of x from the following figures:

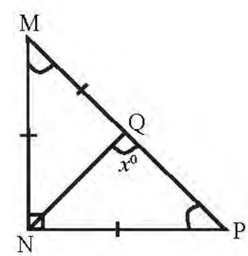
(a)



(b)

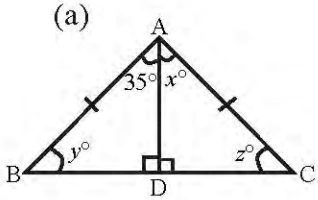


(c)

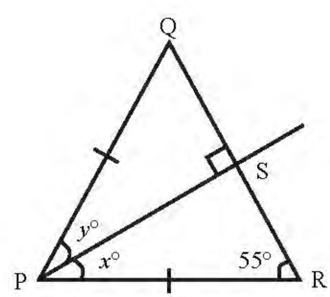


2. Find the value of x°, y° and z° from the given figures:

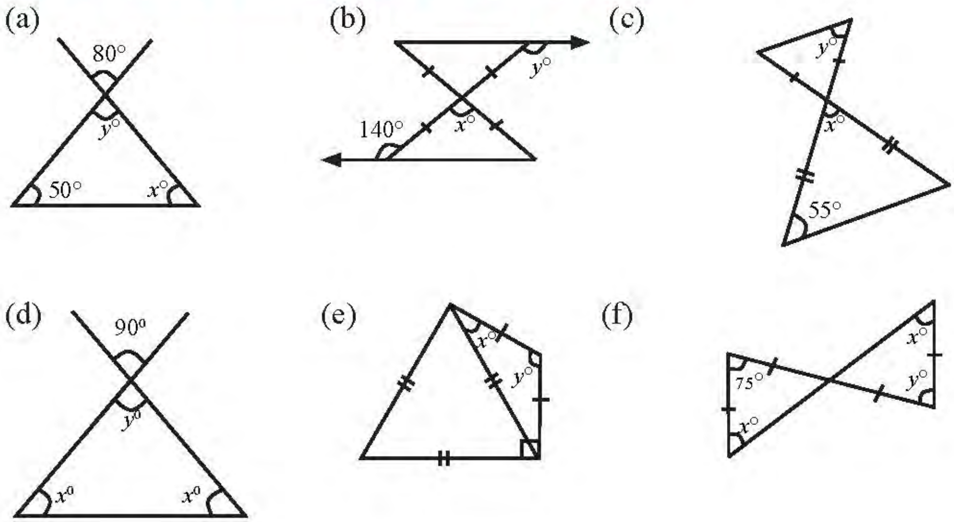
(a)



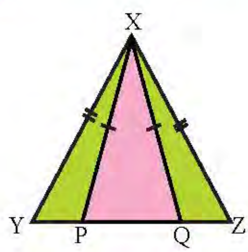
(b)



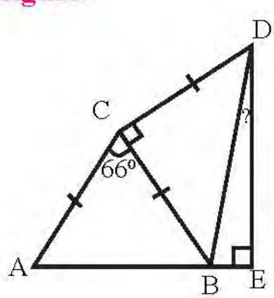
3. **Find the value of x and y from the given figures:**



4. **In the below figure, if XY = $3y$, XZ = $7x$, XP = $9x$ and XQ = $13 + 2y$, then find the value of x and y.**



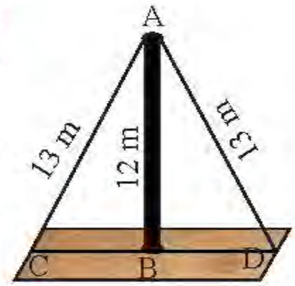
5. **If triangle ABC and BCD are isosceles triangle, then find the value of ∠BDE from the below figure.**



6. **Two equal length of wire are tied at the ground from the top of a pole. If the length of the wire is 13m and height of the pole is 12m, then**

(a) Find the distance between the pole and wire in the ground

(b) Are the area of ΔABC and ΔABD equal? Justify.



Answer

1. (a) $70°$ (b) $110°$ (c) $112.5°$
2. (a) $x = 35°$, $y = 55°$, $z = 55°$ (b) $x = 35°$, $y = 35°$
3. (a) $x = 50°$, $y = 80°$ (b) $x = 100°$, $y = 140°$
   (c) $x = 70°$, $y = 55°$ (d) $x = 45°$, $90°$
   (e) $x = 30°$, $120°$ (f) $x = 52.5°$, $75°$
4. $x = 3$, $y = 7$
5. $12°$
6. 5 m

13.3 Relation among sides and angles of triangle

Activity 1

Divide the class in a suitable group. Collect three stick and connect the ends of each stick to form a triangle by each group.

Are all the figures of each group triangle? If the triangle in formed then what type of triangle is it? If not, why the figure not a triangle? Discuss the results and present the conclusion in the class.

> The type of triangle depends on the length of the sides of the triangle.

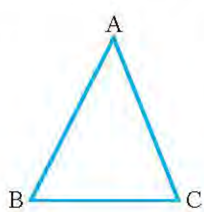
(a) Relation between the sum of any two sides of a triangle and the third side

Activity 2

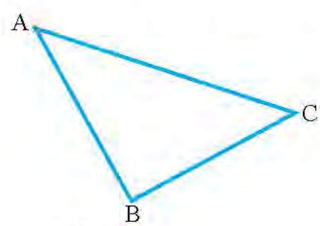
Experimentally verify that the sum of any two sides of a triangle is greater than the third side.

(a) Divide the class into a suitable group.

(b) Draw a different shape and size triangle by each student.



lrq -s_



lrq -v_

(c) Measure all the sides of the triangle and tabulate the sum of any two sides as shown in the following table:

Figure	AB	BC	AC	AB + AC	AB + BC	BC + AC	Results
I.							
II.							

(d) Discuss and draw the conclusion in your group, and write the conclusion.

Conclusion: The sum of any two sides of a triangle is greater than the third side.

If the sum of any two sides of a triangle is equal or less than the third side, then what may be the result?

Relation between the sides and angles of a triangle

Activity 3

Experimentally verify that the angle opposite to the longer side is greater than the angle opposite to the shorter side

(I) Using Machaon Strip

Make a triangle ABC using Mechano strip by each group.

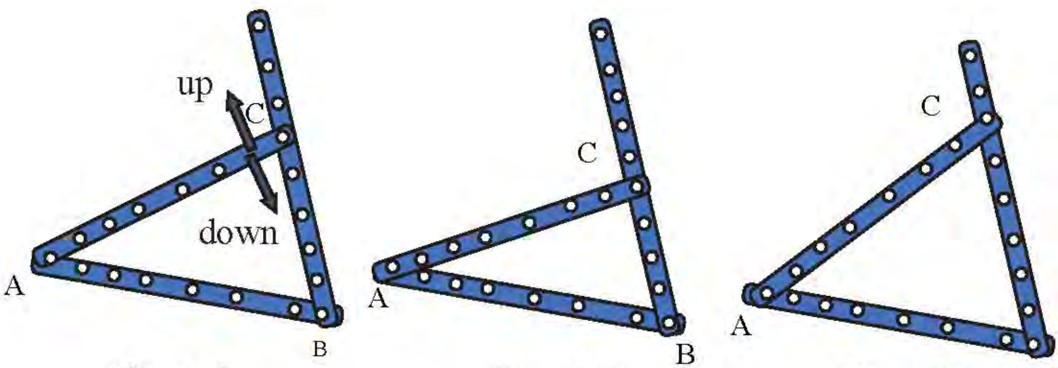


Figure I Figure II Figure III

Move the point C of the triangle ABC down as shown in the above figure II.

Move the point C of the triangle ABC up as shown in the above figure III.

Measure the ∠CAB and side BC in both cases, and tabulate them as follows:

Figure	AB	AC	BC	∠CAB	Relation of BC and ∠CAB	Result
I.						
II.						
III.						

Draw the conclusion from the above table about the relation between a angle and its opposite side.

(II) Measuring sides and angles

Draw the figure of three types of triangles according to the angle (acute, right and obtuse angled triangle).

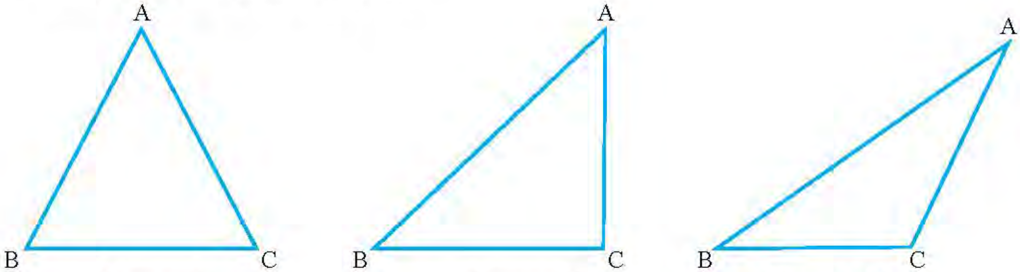


Figure I   Figure II   Figure III

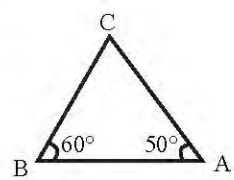
Measure all the angles and their opposite sides and tabulate them as follows:

Figure	AB	∠ACB	BC	∠BAC	AC	∠ABC	Result
I.							
II.							
III.							

Conclusion: The angle opposite to the longer side is greater than the angle opposite to the shorter side.

Example 1

In triangle ABC, ∠BAC = 50° and ∠ABC = 60° then find the longest and shortest side of the triangle ABC.



Solution

Here, $\angle BAC = 50°$, $\angle ABC = 60°$

We know that, $\angle BAC + \angle ABC + \angle ACB = 180°$ [∵ Sum of interior angle of triangle.]

or, $50° + 60° + \angle ACB = 180°$

or, $110° + \angle ACB = 180°$

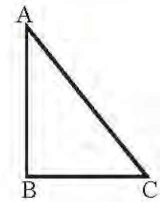
or, $\angle ACB = 180° - 110°$

or, $\angle ACB = 70°$

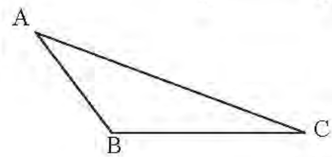
Hence, the longest side is AB and shortest side is BC which are the opposite sides of angles 70° and 50° respectively.

Exercise 13.3

1. Measure all the sides of triangle ABC, and find the largest and smallest angle of the triangle based on the length of the sides.



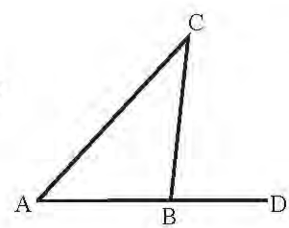
2. Measure all the angles of triangle ABC, and find the longest and shortest sides of the triangle based on the size of the angles.



3. The measurements of three sides of a triangle are 5 cm, 6 cm and x. If x is the longest side, then which one is the correct answer among the following options?

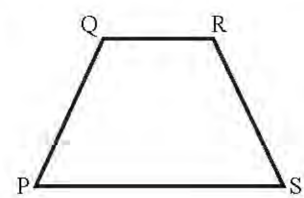
   (a) 11 cm (b) <11 cm (c) >11 cm (d) $\leq$ 11 cm

4. If $\angle ABC > \angle DBC$, then prove that AC > BC from the given figure.



5. Prove that PQ + QR + RS > SP in the given figure.

   [Hints: Join points P and R]



Answer

1 - 2. Show to your teacher

3. (b) <11

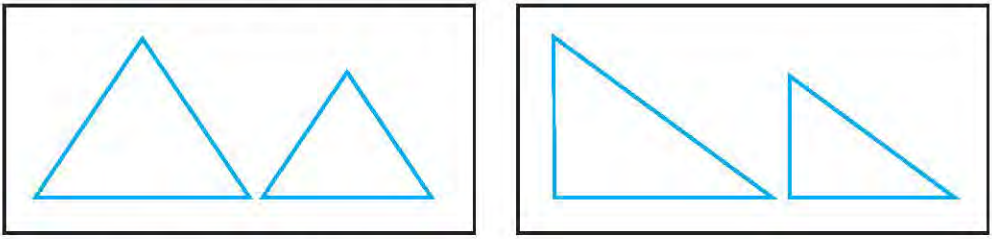
4. 13 cm

5 - 6. Show to your teacher

13.4 Similar triangles

Activity 1

Observe the following figures and discuss.



(a) What types of above pair figures?

(b) Are the sides of each pair figures equal?

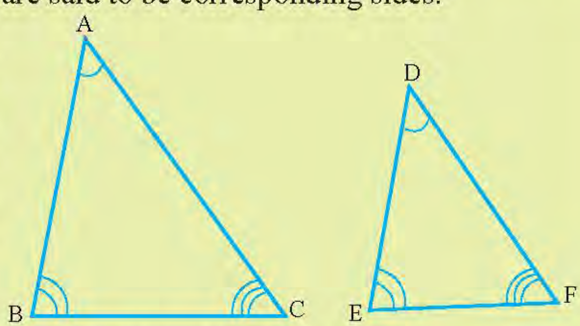
(c) Similarly, are the angles of each pair figures equal?

The given pair figures are same shape but not same size. Such types of figures are called similar figures.

The geometrical figure which are in same shape are called the similar figures.

In similar triangles, equal angles are said to be corresponding angles and opposite sides of equal angles are said to be corresponding sides.

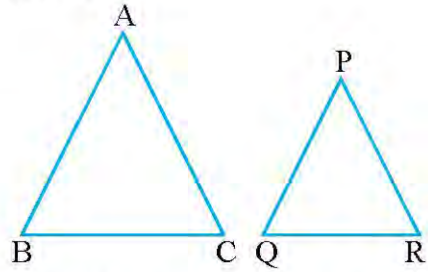
Given triangles ABC and DEF are similar, and angles A, B and C are equal with D, E and F respectively. Then the angles A, B and C are corresponding angles with D, E and F respectively. Similarly, the sides AB, BC and AC are corresponding sides of DE, EF and DF respectively.



Activity 2

Draw two triangles in a cardboard as shown in the figure.

Cut the cardboard to make triangle with a help of scissor.



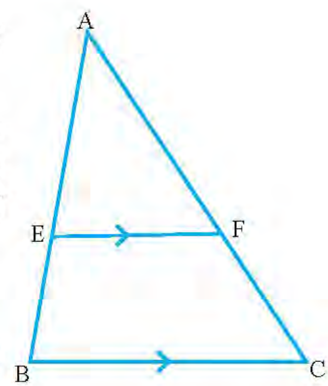
Overlap the corresponding vertices of these two triangles and observe. If all the vertices are completely overlapped, then we can say that the corresponding angles are equal. Thus, the two triangles are similar.

All three angles of a triangle are equal with all the three angles of another triangle, then the two triangles are similar. In the given triangles ABC and PQR, $\angle$A and P, $\angle$B and $\angle$Q, $\angle$C and $\angle$R are corresponding angles.

If corresponding angles of two triangles are equal, then the triangles are similar.

Relation of corresponding angles of similar triangles

Activity 3



Take a triangle ABC. Draw a parallel line EF with BC using set square.

There are two triangles ABC and AEF. Are the triangles ABC and AEF similar? Measure the sides AB, BC and AC of triangle ABC, and the sides AE, EF and AF of triangle AEF.

And then, find the value of ratio:

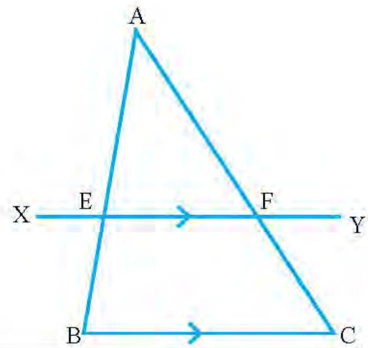
$\frac{AE}{AB}$, $\frac{EF}{BC}$, $\frac{AF}{AC}$ and the relation of sides; and discuss in class.

If two triangles are similar to each other, then the ratio of corresponding sides are equal.

In the above figure $\frac{AE}{AB} = \frac{EF}{BC} = \frac{AF}{AC}$. It can also be written as $\frac{AB}{AE} = \frac{BC}{EF} = \frac{AC}{AF}$.

Theorem 1

Line drawn parallel to the any side of a triangle cuts the remaining two sides in the same ratio.



Here,

Given: Line XY is drawn parallel to the base BC of a triangle ABC and meets the points E and F of sides AB and AC respectively.

To prove: $\frac{AE}{EB} = \frac{AF}{FC}$

Proof:

	Fact		Reason
1.	ΔABC and ΔAEF	1.	
(i)	$\angle ABC = \angle AEF$ (A)	(i)	Corresponding angles (XY $\parallel$ BC)
(ii)	$\angle ACB = \angle AFE$ (A)	(ii)	Corresponding angles (XY $\parallel$ BC)
(iii)	$\angle BAC = \angle EAF$ (A)	(iii)	Common angle
2.	$\Delta ABC \sim \Delta AEF$	2.	All the corresponding angles of two triangles are equal to each other
3.	$\frac{AB}{AE} = \frac{AC}{AF}$ or, $\frac{AE + EB}{AE} = \frac{AF + FC}{AF}$ or, $\frac{AE}{AE} + \frac{EB}{AE} = \frac{AF}{AF} + \frac{FC}{AF}$ or, $1 + \frac{EB}{AE} = 1 + \frac{FC}{AF}$ or, $\frac{EB}{AE} = \frac{FC}{AF}$ or, $\frac{AE}{EB} = \frac{AF}{FC}$	3.	The ratio of corresponding sides of similar triangles

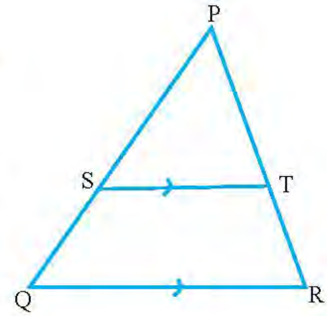
Proved

Example 1

Given triangles, PQR and PST are similar. If PQ = 8 cm, PR = 6cmand PT = 4cm, then

(a) Find the length of side PS.

(b) If ST = 6 cm, then find the length of side QR.



Solution

Here, the triangles PQR and PST are similar.

Thus,

$$\frac{PQ}{PS} = \frac{PR}{PT} = \frac{QR}{ST}$$

$$\text{or, } \frac{8\text{ cm}}{PS} = \frac{6\text{ cm}}{4\text{ cm}} = \frac{QR}{6\text{ cm}}$$

(a) From first and second ratio

$$\frac{8\text{ cm}}{PS} = \frac{6\text{ cm}}{4\text{ cm}}$$

$$\text{or, } PS = \frac{8\text{ cm} \times 4\text{ cm}}{6\text{ cm}} = 5.34\text{ cm}$$

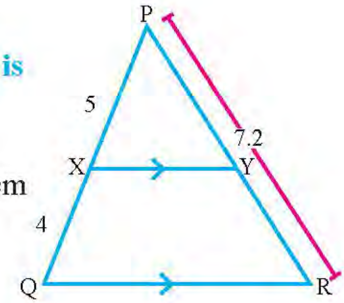
(b) From second and third ratio

$$\frac{6\text{ cm}}{4\text{ cm}} = \frac{QR}{6\text{ cm}}$$

$$\text{or, } QR = \frac{6\text{ cm} \times 6\text{ cm}}{4\text{ cm}} = 9\text{ cm}$$

Example 2

In the given figure XY// QR, the ratio of PX and XQ is 5:4 and PR = 7.2 cm, then what is the value of PY?



Solution

From the figure, XY||QR, PX: XQ = 5: 4 and PR = 7.2 cm

Then PY =?

Let, PY = x

Thus, YR = 7.2 cm – x cm

We know that, $\frac{PX}{XQ} = \frac{PY}{YR}$

or, $\frac{5}{4} = \frac{x}{7.2 - x}$

or, 36 – 5x = 4x

or, 9x = 36

$\therefore$ x = PY= 4 cm

Alternative method

$\frac{PX}{PQ} = \frac{PY}{PR}$ [ΔPXY and ΔPQR are similar triangles]

or, $\frac{5}{9} = \frac{PY}{7.2}$

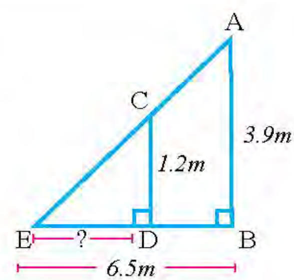
or, 9PY = 36

or, 9 PY = 36

or, PY = 4 cm

Example 3

The height of Dipika is 1.2 meter. She stands in front of the light pole of 3.9 m height. If the length of the shadow of the pole is 6.5 m, then find the length of shadow of Dipika at that time.



Solution

Let AB be a height of pole, CD be the height of Dipika

BE be the length of shadow of pole and DE be the length of the shadow of Dipika.

Then, AB = 3.9 m, BE = 6.5 m, CD = 1.2 m, ED =?

ΔCDE and ΔABE are similar.

[$\because$ Both right angled triangle and $\angle$E is common]

or, $\frac{AB}{CD} = \frac{BE}{DE}$ [$\because$ Ratio of corresponding angles of similar triangles]

or, $\frac{3.9}{1.2} = \frac{6.5}{DE}$

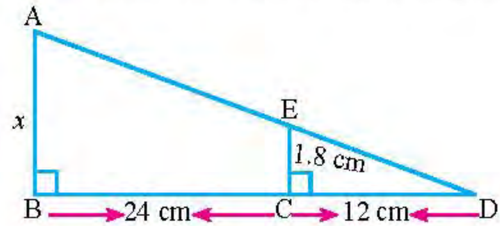
or, 3.9 DE = 6.5 × 1.2

or, $DE = \dfrac{6.5 \times 1.2}{3.9} = 2$ m

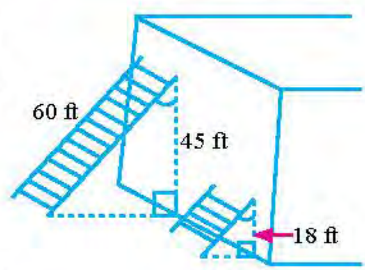
Hence, the length of the shadow of Dipika is 2 meter.

Exercise 13.4

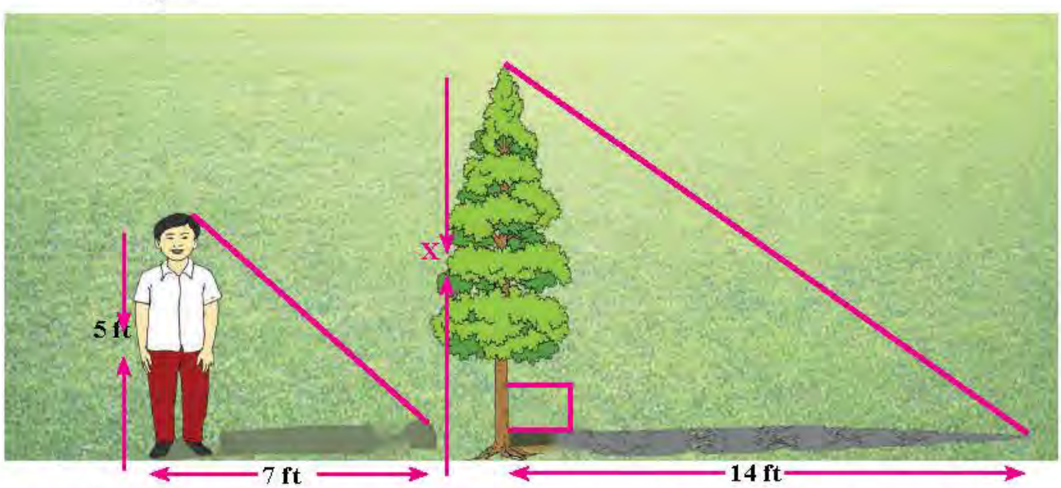
1. In triangle BC || DE, AD: BD = 2: 3 and EC = 12 cm, , then what is the value of AE?
2. In triangle XYZ, if YZ || PQ, XQ: QZ = 3.5 and PY = 7.5 cm, then find the value of XY.
3. Find the value of x from the given figure:



4. The angles made by two ladders are the same in the given figure. Find the length of the smaller ladder.

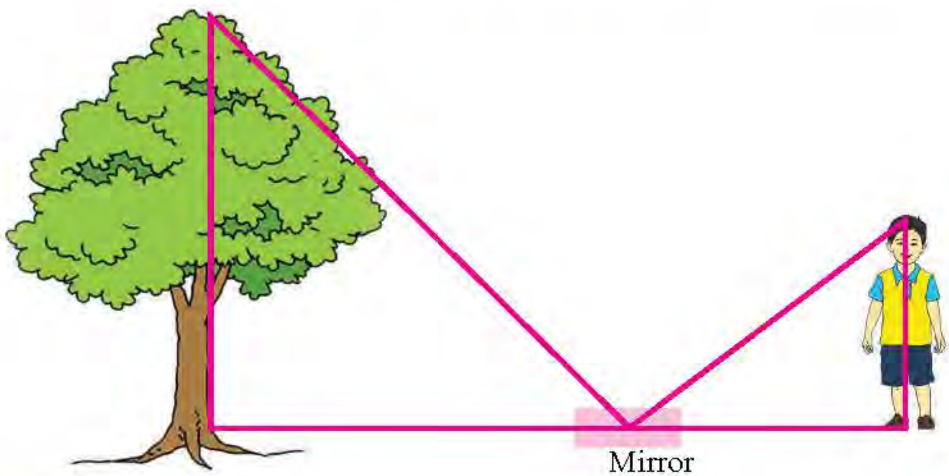


5. (a) Find the height of tree from the adjoining figure?



b) If the length of the shadow of a pole of height 20 m is 30m, then what is the height of the another pole whose length of shadow is 7.5 m.

6. Angel put a mirror on the ground in between him and a tree of his school ground. He moves here and there until he saw the whole tree. At this time, how can we find the height of the tree discussing your friends?



Project work

Divide all the students in a suitable group. Stand on the playground and take the height and length of your shadow with the help of your friends. At the same time, measure the length of the shadow of a pole standing on the playground. Find the height of the pole from this information and present the result in your class.

Answer

1. 8 cm	2. 12 cm	3. 5.4 cm	4. 24 ft
5. (a) 10 ft	(b) 5 m	6 . Show to your teacher	

Lesson 14 Quadrilateral

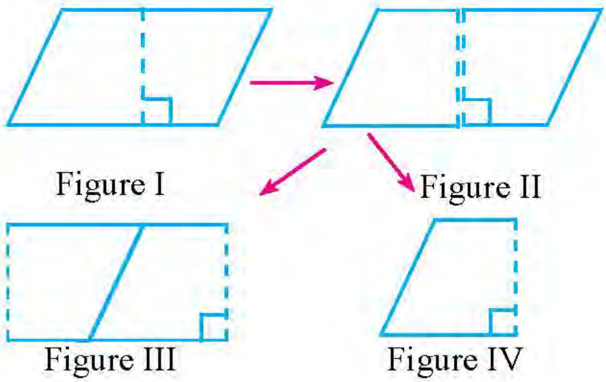
14.0 Review

Activity 1

(a) Divide the students in a appropriate group.

(b) Prepare a parallelogram using Metacard in each group.

(c) Cut the parallelogram in the middle part as shown in the figure below.



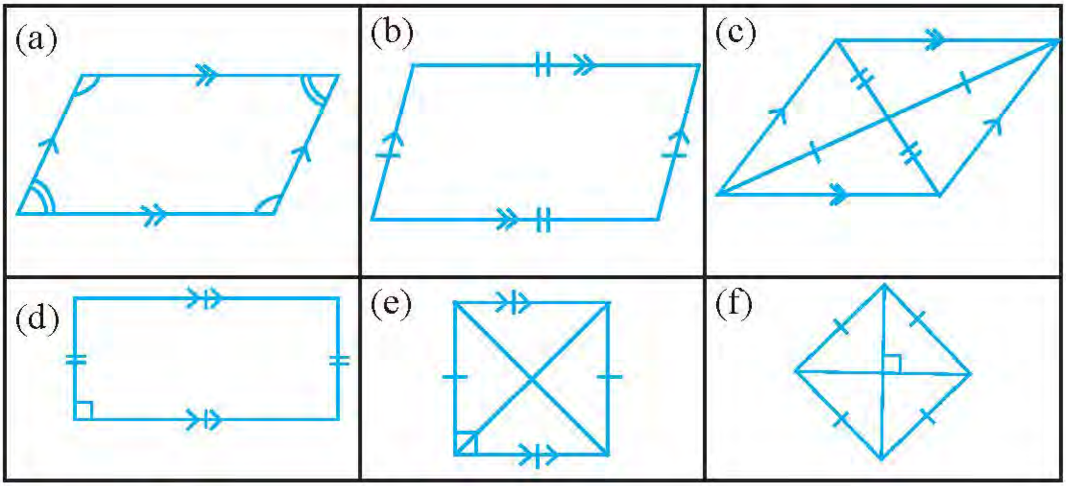
(d) Join the two pieces of parallelogram as shown in Figure III. What type of figure is made? Discuss and draw the conclusion about the sides and angles.

(e) Again, overlap the two pieces of Figure II as shown in Figure IV. What is the relation of angles between the Figure I and Figure IV? Draw the conclusion in your group.

14.1 Verification of properties of parallelogram

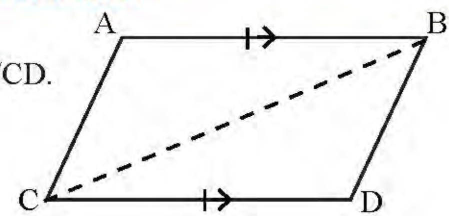
Activity 2

Hang the figures in your classroom wall in different places as given below. Observe the figures by each student in the bench one by one. Draw the conclusion; and write the conclusion in your copy based on the name, sides, and angles. Discuss in your bench, write the properties of each figure with separate figure, and present in your class.



Theorem 1

The line segments joining the ends towards same side of two equal and parallel line segments are also equal and parallel to each other.



Given: Two line segments AB = CD and AB//CD.
Points A and C, with B and D are joining.
To prove, AC = BD and AC // BD
Construction: Join points B and C

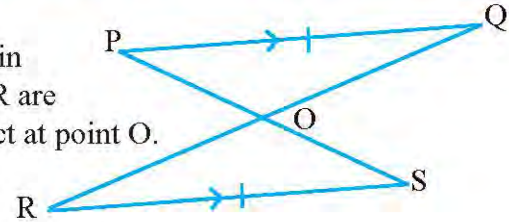
Proof:

	Statements		Reasons
1.	In ΔABC and ΔBCD	1.	
(i) (ii) (iii)	AB = CD (S) $\angle ABC = \angle BCD$ (A) BC = BC (S)	(i) (ii) (iii)	Given Alternate angles AB//CD Common side
2.	$\Delta ABC \cong \Delta BCD$	2.	SAS axiom
3.	AC = BD	3.	Corresponding sides of congruent triangles
4.	$\angle ACB = \angle CBD$	4.	Corresponding angles of congruent triangles are equal
5.	AC // BD	5.	Alternate angles
6.	AC = BD and AC // BD	6.	From statements 3 and 6

Proved that

Theorem 2

The line joining the opposite ends of two equal and parallel line segments bisect each other.



Given: Line segments PQ = RS, PQ//RS in which opposite ends P and S, and Q and R are joined. Line segments PS and QR intersect at point O.

To prove: PO = OS and QO = OR or PS and QR are bisect each other.

Proof:

S. No.	Statements	S. No.	Reasons
1.	In ΔPOQ and ΔROS	1.	
(i) (ii) (iii)	$\angle OPQ = \angle OSR$ (A) PQ = RS (S) $\angle PQO = \angle SRO$ (A)	(i) (ii) (iii)	Alternate angles PQ//RS Given Alternate angles PQ//RS
2.	$\Delta POQ \cong \Delta ROS$	2.	ASA axiom
3.	PO = OS, QO = OR	3.	Corresponding sides of congruent triangles

Proved

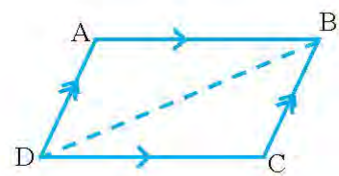
Theorem 3

The opposite sides and angles of a parallelogram are equal.

Given: ABCD is a parallelogram in which AB//DC and AD//BC.

To prove:

1. AB = DC, AD = BC
2. $\angle ABC = \angle ADC$, $\angle DAB = \angle BCD$



Construction: Join the points B and D.

Proof

S. No.	Statements	S. No.	Reasons
1.	In ΔABD and ΔBCD	1.	
(i) (ii) (iii)	$\angle ABD = \angle BDC$ (A) BD = BD (S) $\angle ADB = \angle DBC$ (A)	(i) (ii) (iii)	AB//DC Alternate angles Common sides AD//BC Alternate angles
2.	$\Delta ABD \cong \Delta BCD$	2.	ASA axiom
3.	AB = DC and AD = BC	3.	Corresponding sides of congruent triangles
4.	$\angle DAB = \angle BCD$ $\angle ABD = \angle BDC$ $\angle ADB = \angle DBC$	4.	Corresponding angles of congruent triangles
5.	$\angle ABD + \angle DBC = \angle BDC + \angle ADB$	5.	Equal axiom
6.	$\angle ABC = \angle ADC$	6.	From statement 5
7.	AB = DC, AD = BC and $\angle ABC = \angle ADC$, $\angle DAB = \angle BCD$	7.	From statements 3, 4 and 6

Proved

Theorem 4

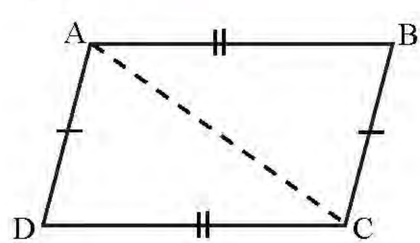
The quadrilateral having opposite sides equal is a parallelogram.

Given: In quadrilateral ABCD, AB = CD and AD = BC.

To prove: ABCD is a parallelogram.

That is, AB//CD and AD//BC

Construction: Join the points A and C



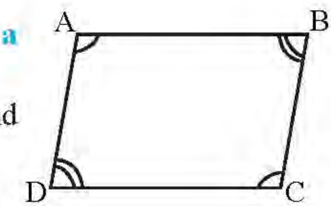
Proof:

S. No.	Statements	S. No.	Reasons
1.	In ΔABC and ΔACD	1.	
(i) (ii) (iii)	$AB = CD$ (S) $AD = BC$ (S) $AC = AC$ (S)	(i) (ii) (iii)	Given Given Given
2.	$\Delta ABC \cong \Delta ACD$	2.	SSS axiom
3.	$\angle ACB = \angle DAC$ $\angle BAC = \angle ACD$	3.	Corresponding angles of congruent triangles
4.	AB // CD , AD //BC	4.	Being equal alternate angles

Proved

Theorem 5

The quadrilateral having opposite angles equal is a parallelogram.



Given: In quadrilateral ABCD $\angle ABC = \angle ADC$ and $\angle DAB = \angle BCD$

To prove: ABCD is a parallelogram.

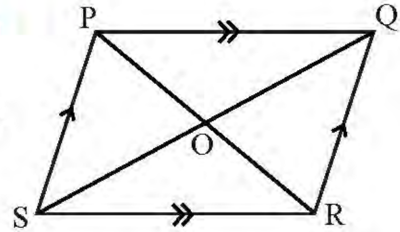
Proof:

S. No.	Statements	S. No.	Reasons
1.	$\angle ABC + \angle BCD + \angle CDA + \angle DAB = 360^0$	1.	The sum of interior angles of quadrilateral
2.	$\angle ABC + \angle BCD + \angle ABC + \angle BCD = 360°$ or, $2\angle ABC + 2\angle BCD = 360^0$ or, $\angle ABC + \angle BCD = 180^0$	2.	Being $\angle ABC = \angle CDA$ and $\angle DAB = \angle BCD$
3.	AB // CD	3.	Being sum of co-interior angles
4.	Similarly $\angle BCD + \angle CDA = 180^0$	4.	Reasons like: 1, 2, 3
5.	BC // AD	5.	Being sum of co-interior angles
6.	AB // CD , BC//AD	6.	From statements 3 and 5
7.	ABCD is a parallelogram	7.	From statement 6

Proved

Theorem 6

The diagonals of a parallelogram bisect to each other.



Given: PQRS is a parallelogram in which the diagonals PR and QS bisect in O.

To prove: PO = OR and SO = OQ

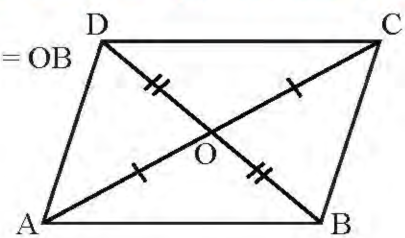
Proof:

S. No.	Statements	S. No.	Reasons
1.	In ΔPOQ and ΔROS	1.	
(i) (ii) (iii)	$\angle OPQ = \angle ORS$ (A) PQ = RS (S) $\angle OQP = \angle OSR$ (A)	(i) (ii) (iii)	Alternate angles PQ//SR Opposite sides of parallelogram Alternate angles PQ//SR
2.	$\Delta POQ \cong \Delta ROS$	2.	ASA axiom
3.	PO = OR, QO = OS	3.	Corresponding sides of congruent triangles

Proved

Theorem 7 (Converse of Theorem 6)

If the diagonals of a quadrilateral bisect each other, the quadrilateral is a parallelogram.



Given: In quadrilateral ABCD, AO = OC and DO = OB

To prove: ABCD is a parallelogram.

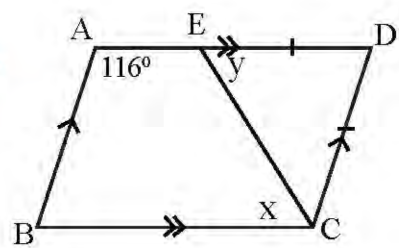
Proof:

S. No.	Statements	S. No.	Reasons
1.	In ΔAOB and ΔDOC	1.	
(i) (ii) (iii)	$AO = OC$ (S) $\angle AOB = \angle COD$ (A) $OB = OD$ (S)	(i) (ii) (iii)	Given Vertically opposite angles Given
2.	$\Delta AOB \cong \Delta DOC$	2.	SAS axiom
3.	$AB = DC$	3.	Corresponding sides of congruent triangles
4.	$\angle OBA = \angle ODC$	4.	Corresponding angles of congruent triangles
5.	$AB \parallel DC$	5.	Alternate angles
6.	$AD \parallel BC$, $AD = BC$	6.	$AB \parallel DC$ and $AB = DC$
7.	ABCD is a parallelogram	7.	Opposite sides are equal and parallel

Proved

Example 1

Find the value of x and y from the given figure:



Solution

We have,

(i) $\angle BAE + \angle EDC = 180°$

[$\because AB \parallel CD$ Co-interior angles]

or, $\angle EDC = 180° - \angle BAE = 180° - 116° = 64°$

$\angle DCE = \angle DEC = y$ [$\because DE = DC$]

(ii) $\angle DEC + \angle ECD + \angle EDC = 180°$ [$\because$ Interior angles of ΔCDE]

or, $y + \angle ECD + 64° = 180°$

or, $y + y = 180° - 64°$ [$\because CD = DE$]

or, $2y = 116°$

$y = 58°$

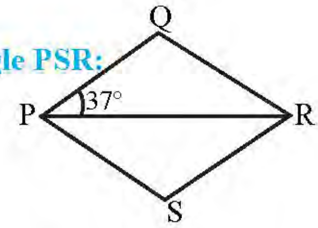
$\angle BCD = \angle BAD$ [$\because$ Opposite angles $\square$ ABCD]

Again, $x + y = 116°$

or, $x = 116° - y = 116° - 58° = 58°$

Example 2

From the given rhombus PQRS, find the value of angle PSR:



Solution

Here $\angle QPR = \angle QRP = 37°$

$[\because$ In rhombus PQRS, PR = RQ$]$

$\angle QPR + \angle QRP + \angle PQR = 180°$ $[\because$ Interior angles of a triangle PQR$]$

or, $37° + 37° + \angle PQR = 180°$

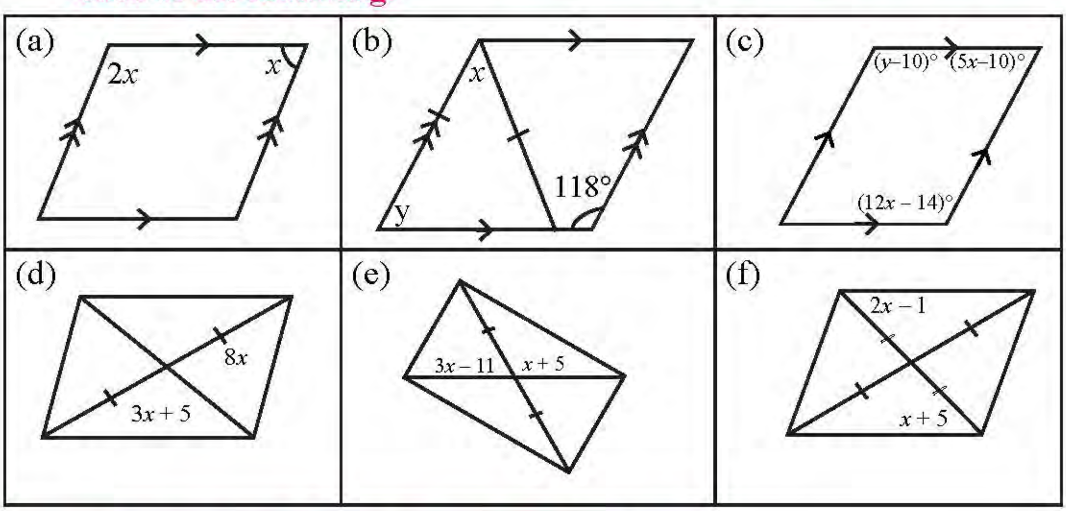
or, $\angle PQR = 180° - 74° = 106°$

Again, $\angle PSR = \angle PQR = 106°$ $[\because$ Opposite angles of rhombus$]$

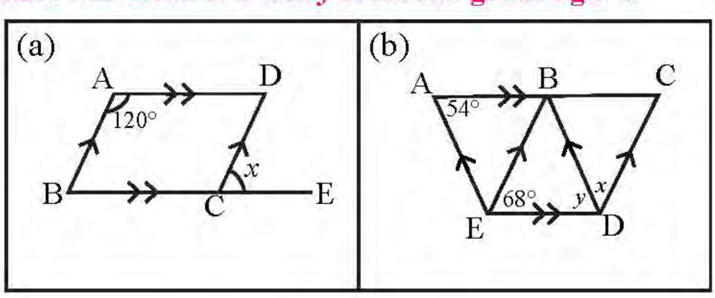
Thus, $\angle PSR = 106°$

Exercise 14

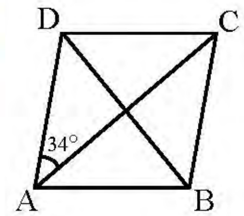
1. What may be the value of x and y for being the parallelogram of each of the following:



2. Find the value of x and y from the given figure:

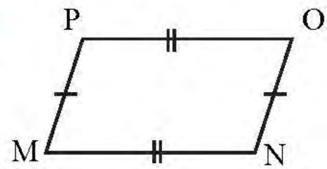


3. **ABCD is a rhombus. If $\angle DAC = 34°$, then find the value of $\angle DAB$, $\angle ABC$, $\angle BCA$ and $\angle CDA$.**

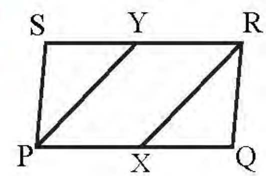


4. If opposite angles of a parallelogram are $(63 - 3x)°$ and $(4x - 7)°$ then find the value of all angles of the parallelogram.

5. **In the given parallelogram MNOP, MN = OP and PM = ON then prove that MNOP is a parallelogram.**

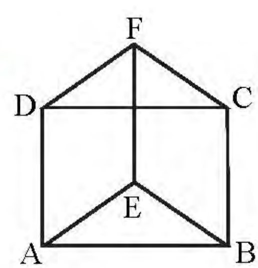


6. **PQRS is a parallelogram. Points X and Y are the midpoint of Sides PQ and RS then prove that PXRY is a parallelogram.**



7. **In the given figure AD = EF, AD//EF, EF = BC, EF//BC and**

   Then prove that ABCD is a parallelogram.



8. In a quadrilateral SLOW, SL = LO = OW = SW then prove that the quadrilateral SLOW is a parallelogram.

9. MOAT is a quadrilateral in which diagonals MA and OT intersect at point R and MR = RA with TR = OR. Then prove that quadrilateral MOAT is a parallelogram.

Project work

Divide the students in a suitable group. Go to your school play ground and make a parallelogram and diagonals with a help of rope by each group. Observe the parallelogram and measure the opposite sides and diagonals. Find the relation of opposite sides and parts of diagonals, and present the result in your class.

Answer

1. (a) $60°$ (b) $x = 56°, y = 62°$ (c) $x = 12°, y = 140°$

(d) $x = 1$ (e) $x = 8$ (f) $x = 6$ 2. (g) 60

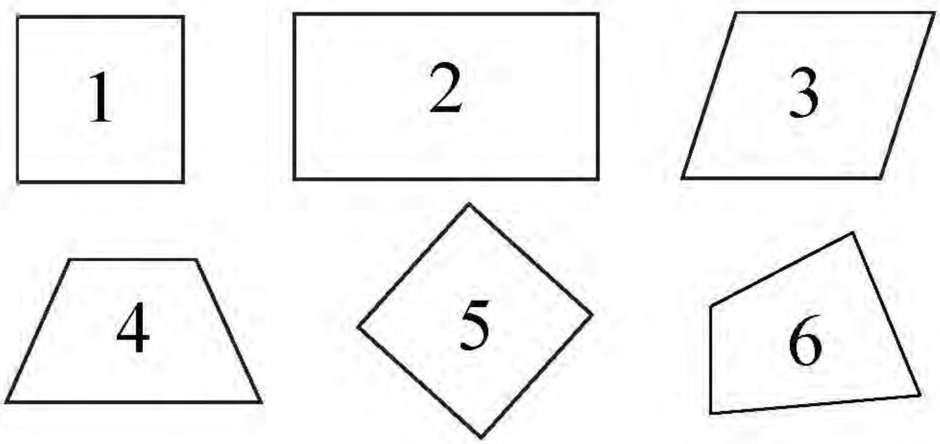
(h) $58°, 54°$ 3. $112°, 112°, 68°, 68°$

4. $33°$ and $147°$ 5-9. Show to your teacher

Lesson 15 Construction

15.0 Review

Divide the students having 6/6 students in each group. Name each student: 1, 2, 3, 4, 5, and 6 of each group. Take the specified quadrilateral as naming 1, 2, 3, 4, 5, and 6 by each students from the teacher as shown in the following figure.



Now, divide the whole students in 6 new groups having the same number, i.e 1/1, 2/2, 3/3, and so on will sit in the new group. Study the quadrilateral of your own group and answer the following questions.

a) What is the name of your group quadrilateral?

b) What are the properties of the quadrilateral?

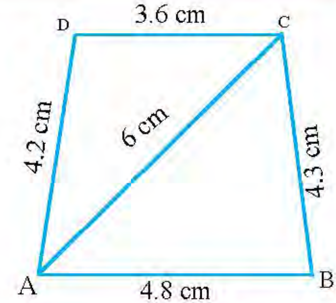
Discuss and draw the common conclusion in new your group.

After drawing the conclusion in your new group, present the result in of all 6 types of quadrilaterets your own old group one. Finally, discuss about the result in the class.

15.1 Construction of scalene quadrilateral

(a) When the measure of four sides and a diagonal are given

Construct a quadrilateral ABCD having AB = 4.8 cm, BC = 4.3 cm, CD = 3.6 cm, AD = 4.2 cm and diagonal AC = 6 cm.



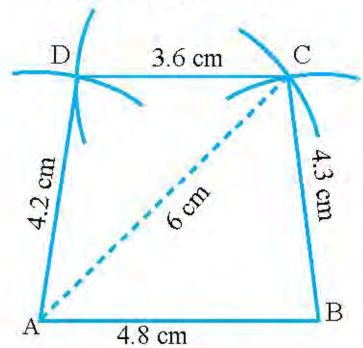
Solution

At first, draw a rough figure of quadrilateral ABCD.

Steps

1. Draw straight line AB = 4.8 cm.
2. Cut from point A with an arc 6 cm
3. 3. Cut from point B with an arc 4.3 cm and name the intersecting point C.
4. Join points B and C, and A and C.
5. Again, cut from point A with an arc 4.2 cm.
6. Cut from point C with an arc 3.6 cm and name the intersecting point D.
7. Join points A and D, and then C and D.

   Then the required quadrilateral ABCD is constructed.

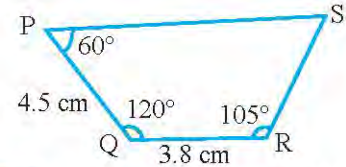


(b) When two adjacent sides and three angles are given

Construct a quadrilateral PQRS of measurement PQ = 4.5 cm, <PQR = PQ = 4.5 cm, $\angle PQR = 120°$, QR = 3.8 cm, $\angle QRS = 105°$ and $\angle QPS = 60°$

Solution

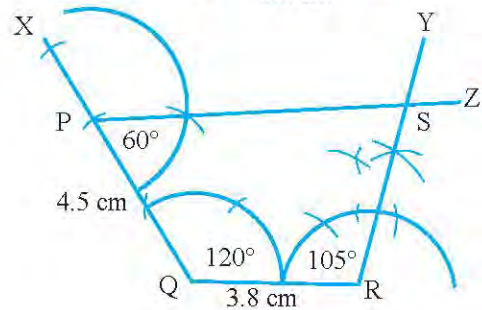
First of all, draw a rough figure of quadrilateral PQRS on the basis of given information.



Steps

1. Draw line segment QR = 3.8 cm.
2. Draw an angle RQX = 120° at point Q using compass.
3. Cut from point Q with an arc 4.5 cm at point P.
4. Draw an angle of <QRY = 105° at point R.
5. Again, draw an angle of <QPZ = 60° at point P in which line RY and PZ intersect at point S.

   Then the required quadrilateral is PQRS.

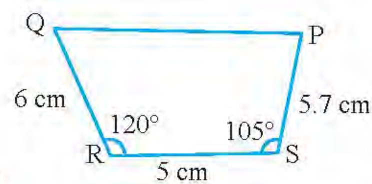


(c) When three sides and two angles are given

Construct a quadrilateral PQRS having QR = 6 cm, RS = 5 cm, PS = 5.7 cm, $\angle S$ = 105°, $\angle R$ = 120°

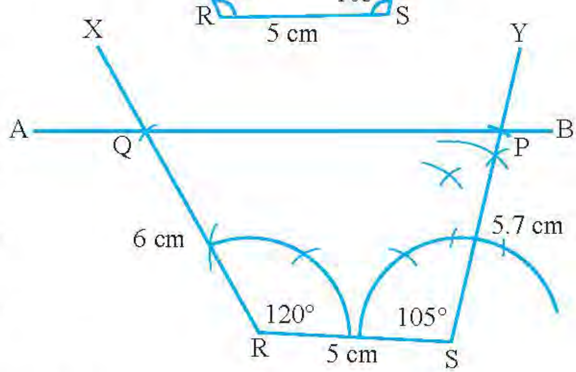
Solution

At first, draw a rough figure of quadrilateral PQRS based on the given information.



Steps

1. Draw line segment RS = 5 cm.
2. Draw two angles of 120° and 105°at points R and S respectively.
3. Cut from point R with an arc 6 cm at point Q.
4. Cut from point S with an arc 5.7 cm at point P.
5. Now, join points P and Q.
   Then the required quadrilateral is PQRS.



Exercise 15.1

Construct quadrilaterals from the following information.

1. Construct quadrilateral ABCD having AB = AD = 3 cm, BC = 2.5, AC = 4 cm and BD = 5 cm.

2. Construct a quadrilateral MNOP having MO = MP = 6 cm, NO = 7.5 cm, OP = 5 cm and NP = 10 cm

3. Construct a quadrilateral PQRS having PQ = 3.5 cm, QR = 2.5 cm, RS = 4 cm, $\angle Q = 75^o$ and $\angle R = 120^o$

4. Construct quadrilateral ABCD having AB = 3.6 cm, BC = 3.3 cm, AD = 2.7 cm diagonals AC = 4.6 cm BD = 4 cm

5. Construct a quadrilateral PQRS having PQ = 5 cm, QR = 6.5 cm, $\angle P = \angle R = 100^\circ$ and $\angle S = 75^\circ$

6. Construct a quadrilateral ABCD having AB = 3.8 cm, BC = 3.4 cm, CD = 4.5 cm, AD = 5 cm and $\angle B = 60^\circ$

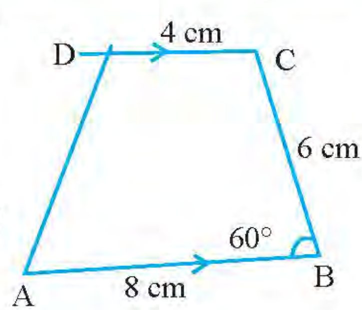
Answer

Show the construction to your teacher.

15.2 Construction of trapezium

(a) When length of three sides and measurement of an angle are given

Construct a trapezium ABCD having AB || CD, AB = 8 cm, BC = 6 cm, CD = 4 cm and $\angle ABC = 60^\circ$



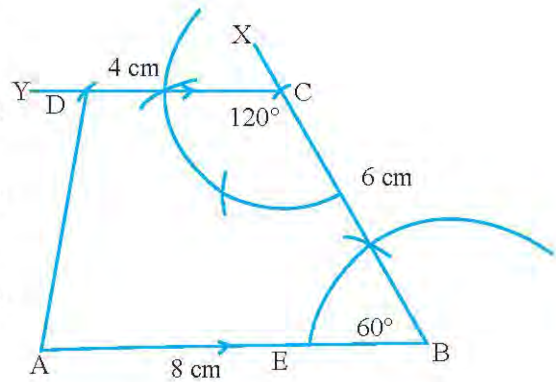
Solution

At first, draw a rough figure of trapezium ABCD based on the given information.

Steps

1. Draw a line segment AB of length.
2. Draw and angle ABX = 60° and cut from point B with an arc 6 cm.
3. Again, draw an angle BCY = 120° at point C using compass.
4. Cut with an arc CD = 4 cm from point C.
5. Now, join points A and D.

   Therefore, the required trapezium is ABCD.

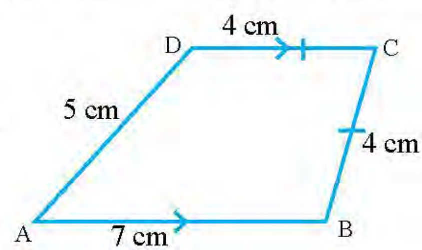


(b) When all the four sides are given and the parallel sides are marked

Construct a trapezium ABCD having AB = 7 cm, BC = CD = 4 cm, DA = 5 cm and AB || CD.

Solution

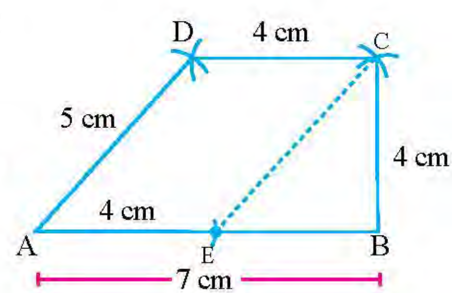
Draw a rough diagram of trapezium ABCD from the given information.



Steps

1. Draw a line segment AB = 7 cm.
2. Draw lines AE = DC = 4 cm in which point E lies on line segment AB.
3. Cut from point E with an arc 5 cm.
4. Again, cut from point B with an arc 4 cm that meet the point C.
5. Cut from point C with an arc 4 cm.
6. Again, cut from point A with an arc 5 cm that meet the point D.
7. Join B and C, C and D and D and A.

   Therefore, the required trapezium is ABCD.

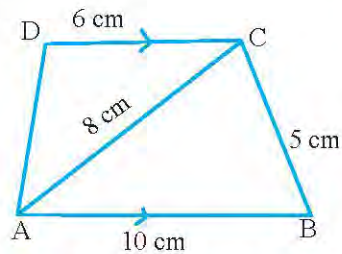


(c) When three sides and one diagonal are given

Construct a trapezium having AB = 10 cm, BC = 6 cm, AC = 8 cm and AB || CD

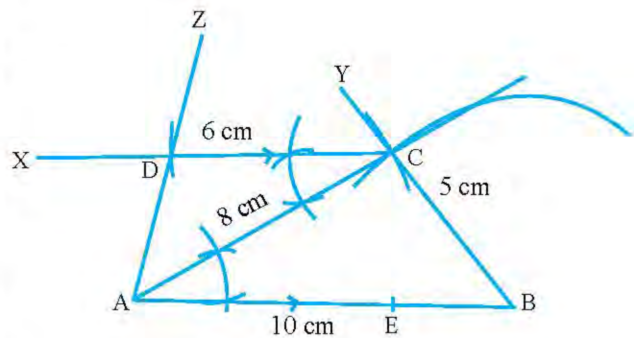
Solution

Draw a rough diagram of trapezium based on the given information.



Steps

1. Draw a line segment AB = 10 cm.
2. Cut from points A and B with an arc of 8 cm and 5 cm respectively and intersect at point C. Draw lines AE = DC = 4 cm.
3. Join the points A and C and B and C.
4. Draw equal angles BAC and ACX with the help of compass.



5. Cut from point C with an arc 6 cm that meet at point D.
6. Join the points D and A.

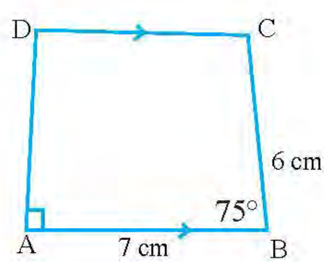
   Therefore, the required trapezium is ABCD.

(d) When two sides and two angles are given

Construct a trapezium ABCD having AB = 7 cm, BC = 6 cm $\angle BAD = 90^\circ$ and $\angle ABC = 75^\circ$

Solution

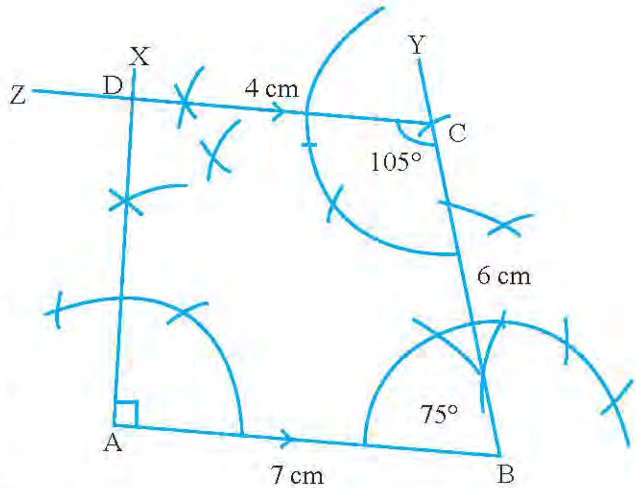
Draw a rough diagram of trapezium ABCD from the given information.



Steps

1. Draw a line segment AB = 7 cm
2. Draw angles BAX = 90° and ABY = 75° at points A and B respectively.
3. Cut from point B with an arc 6 cm at point C
4. Draw an angle 105° at point C that cut at point D.

   Therefore, the required trapezium is ABCD.



Exercise 15.2

Construct trapeziums from the following information:

1. Construct a trapezium ABCD having AB = 6 cm, BC = 4 cm, CD = 3.2 cm, $\angle B = 75°$ and DC // AB
2. Construct a trapezium ABCD having AB // DC, AB = 7 cm, BC = 5 cm, AD = 6.5 cm and $\angle B = 60°$
3. Construct a trapezium ABCD having AB // CD, AB = 8 cm, BC = 6 cm, CD = 4 cm and $\angle C = 120°$
4. Construct a trapezium PQRS having PQ = 10 cm, QR = 4 cm, RS = 6 cm and SP = 3 cm with PQ // SR

Answer

Show your construction to your teacher.

15.3 Construction of rhombus

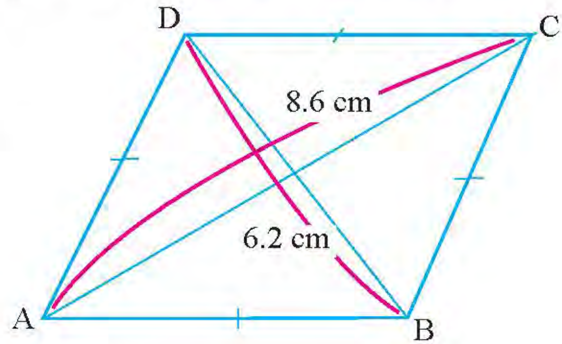
(a) When the length of two diagonals are given

Construct a rhombus ABCD having diagonals AC = 8.6 cm and BD = 6.2 cm.

Solution

Rough diagram

Draw a rough rhombus from the given information.

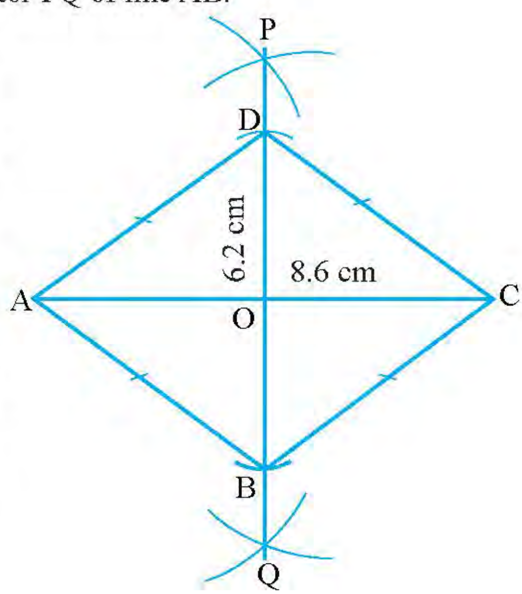


Steps

1. Draw a line segment AC having length 8.6 cm.
2. And then, draw perpendicular bisector PQ of line AB.

   The intersecting point of PQ and AB is O.
3. Cut from point O with an arc 3.1 cm (half of BD) into both sides at points B and D as well cut from point O with an arc 4.3 cm (half of AC) into both sides at points A and C.
4. Join the points A and B, B and C, C and D, and A and D respectively.

   Therefore, the required rhombus is ABCD.



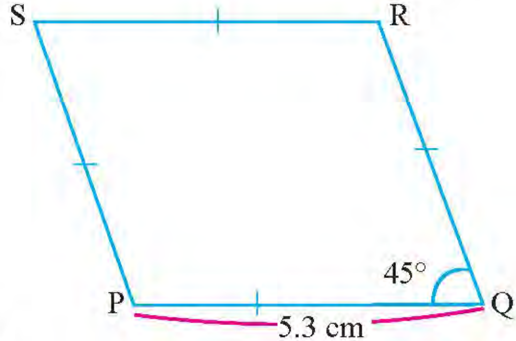
(b) When a side length and an vertex angle are given

Construct a rhombus PQRS having side PQ = 5.3 cm and vertex angle PQR = 45°.

Solution

Rough diagram

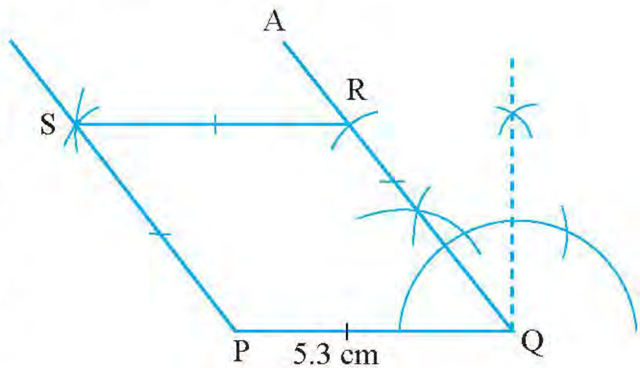
First of all, draw a rough diagram according to the given information.



Steps

1. Draw a line segment PQ of length 5.3 cm.
2. Draw an angle PQR =45° at point Q.
3. Cut at point R from point Q with an arc 5.3 cm.
4. Again, cut from the points P and R with the same arc 5.3 cm that intersect at point S.
5. Joining the points P and S, and R and S with a help of scale.

   Therefore, the required rhombus is PQRS.



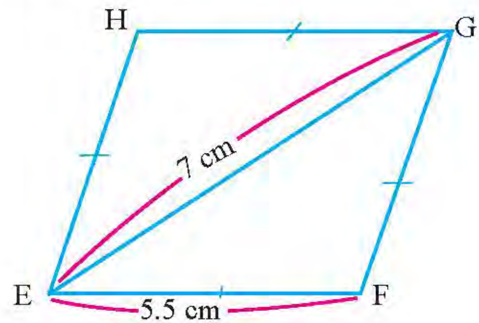
(c) A side length and length of a diagonal are given

Construct a rhombus EFGH having a side EF = 5.5 cm and diagonal EG = 7 cm.

Solution

Rough diagram

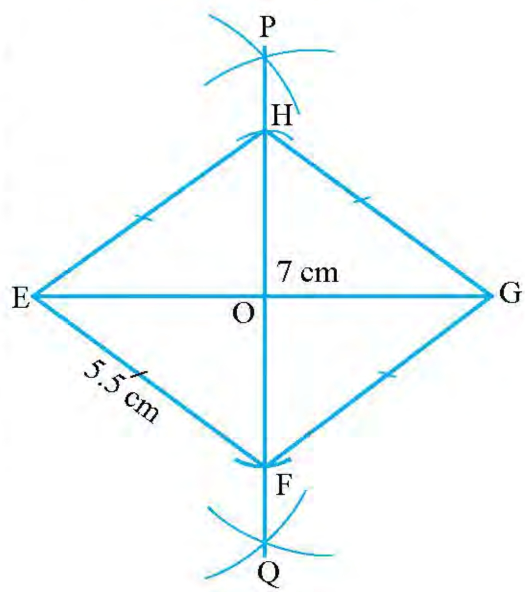
At first, draw a rough diagram of rhombus EFGH based on the given information.



Steps

1. Draw a line segment EG = 7 cm.
2. Draw a perpendicular bisector PQ of the line EG using compass.
3. Cut from point E with an arc 5.5 cm towards OP and OQ, and name the intersecting points H and F respectively.
4. Joining the points E and F, F and G, G and H, and H and E using scale.

   Therefore, the required rhombus is EFGH.



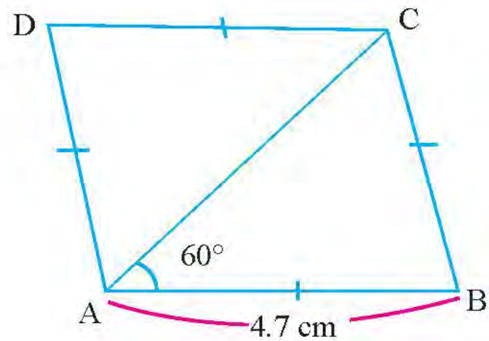
(d) A side length and an angle between the side and diagonal are given

Construct a rhombus ABCD having side AB = 4.7 cm and <CAB = 60°.

Solution

Rough diagram

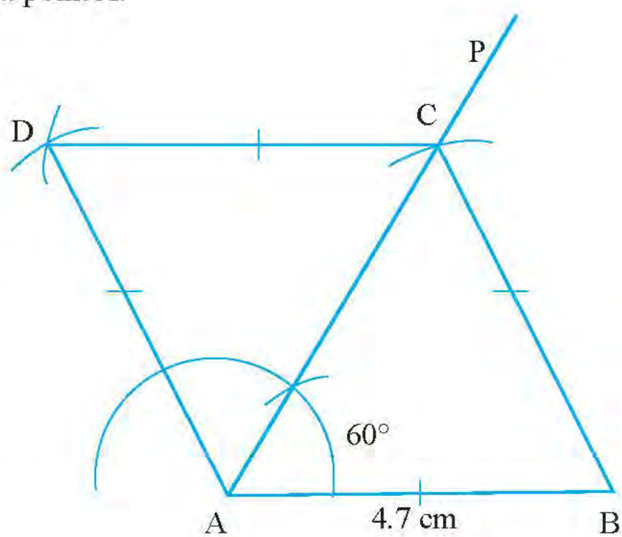
Draw a rough diagram of rhombus ABCD based on the given information.



Steps

1. Draw line segment AB having length 4.7 cm.
2. Draw an angle <PAB = 60° at point A.
3. Cut from the point A with an arc 4.7 cm at point C of the line AP using compass.
4. Again, cut with the same arc from point A towards up and cut from point C towards left those intersect at a point D.
5. Join the points B and C, C and D, and A and D.

   Therefore, the required rhombus is ABCD.



Exercise 15.3

Construct rhombus from the following conditions.

1. (a) Rhombus PQRS having diagonal PR = 9.4 cm and QS = 7.2 cm

   (b) Rhombus WXYZ having diagonal WY = 6.6 cm and XZ = 4.8 cm

   (c) Rhombus ABCD having diagonal AC = 5.9 cm and BD = 6.8 cm

2. (a) Rhombus EFGH having side EF = 6.3 cm and vertex $\angle E = 75°$

   (b) Rhombus ABCD having side BC = 5.5 cm and vertex $<C = 60°$

   (c) Rhombus IJKL having side IJ = 5.1 cm and vertex $\angle I = 105°$ c)

   (d) Rhombus WXYZ having side WX = 6.2 cm and vertex $\angle Y = 45°$

3. (a) Rhombus ABCD having side AB = 5.8 cm and diagonal BD = 7.3 cm

   b) Rhombus PQRS having side QR = 4.3 cm and diagonal PR = 6.5 cm

   c) Rhombus EFGH having side EF = 5.1 cm and diagonal EG = 8.2 cm

4. (a) Rhombus ABCD having side AB = 5.3 cm and $\angle ABD = 30°$

   (b) Rhombus PQRS having side QR = 6.1 cm and $\angle PQR = 75°$

   (c) Rhombus EFGH having side GH = 4.6 cm and $\angle GEF = 60°$

   (d) Rhombus WXYZ having side WX = 5.6 cm and $\angle WXZ = 45°$

Answer

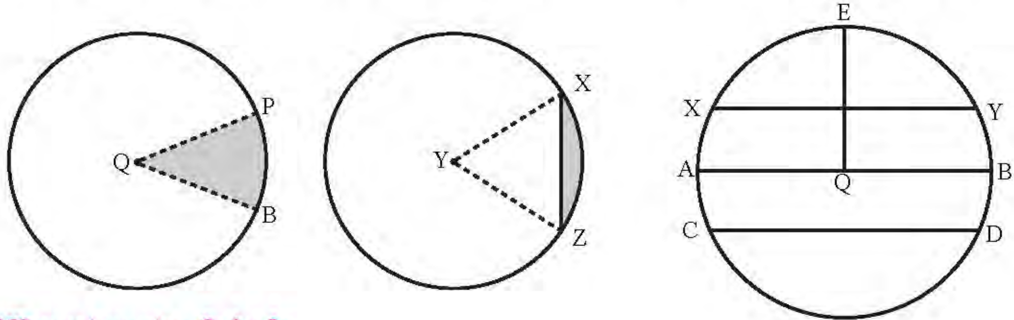
Show the construction to your teacher.

Lesson 16

Circle

16.0 Review

Make a pair with your nearby friend, and discuss and identify about the different parts of circle based on the following figure. Present the result in your class.



Different parts of circle

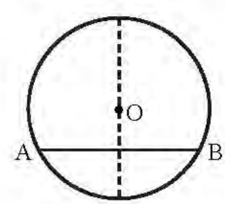
(i) Center
(ii) Radius
(iii) Circumference
(iv) Chord
(v) Diameter
(vi) Arc
(vii) Semi-circle
(viii) Sector
(ix) Segment

16.1 Theorems related to chord of a circle

The line joining two circumference points of a circle is called chord. The diameter of any circle is also a chord. The largest chord is diameter.

Draw a circle with centre O in a tressing paper or plane paper as shown in the given figure.

Draw a chord AB. And then, fold the circle through centre as shown in the given figure.



Measure whether the folding (dotted) line have cut in equal part or not the AB line.

Theorem 1

The perpendicular drawn from the centre of a circle to a chord bisects the chord.

Experimental verification:

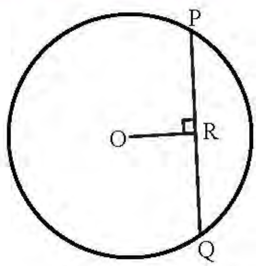


Figure I

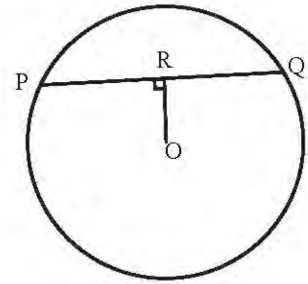


Figure II

Construct two circles having different radius and centre as shown in the above figure.

Draw a chord PQ in each circle and also draw a perpendicular OR from the center O in the chord using a set square.
Measure the segment PR and QR and tabulate in the following table.

Figure	PR	RQ	glthf
I.			
II.			

Conclusion: Therefore, the perpendicular drawn from the centre of a circle to a chord bisects the chord.

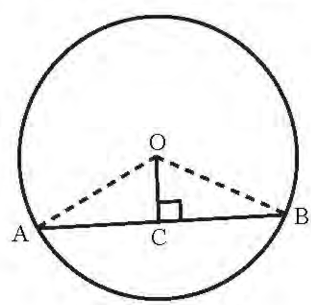
Theoretical proof

Given

In the given figure, O is a center of a circle and AB is a chord in which OC $\perp$ AB.

To prove:

AC = BC
Construction: Join AO and BO



Proof:

S. No.	Statements	S. No.	Reasons
1.	In ΔOCA and ΔOCB	1.	
(i) (ii) (iii)	$\angle OCA = \angle OCB$ (r) $OA = OB$ (h) $OC = OC$ (s)	(i) (ii) (iii)	Given $OC \perp AB$ Radius of the same circle Common side
2.	$\Delta OCA \cong \Delta OCB$	2.	By RHS theorem
3.	$AC = BC$	3.	Corresponding sides of congruent triangle

proved

Theorem 2

The line joining the centre of a circle and the mid-point of a chord is perpendicular to the chord.

Experimental verification

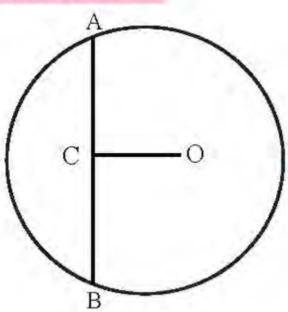


Figure I

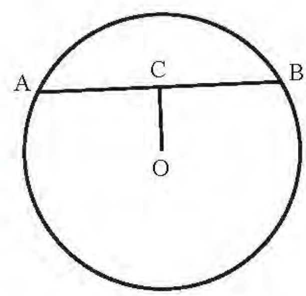


Figure II

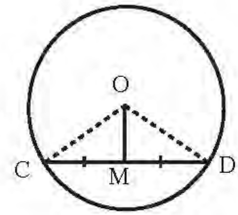
Draw two different size circles having centre O. Draw a chord AB of the circle, and mark the mid-point of chord AB with the help of ruler. And then, join the centre of the circle O and mid-point C.

Measure the angles $\angle OCA$ and $\angle OCB$ using protractor and tabulate in the following table.

Figure	$\angle OCA$	$\angle OCB$	Results
I.			
II.			

Conclusion: Therefore, the line joining the centre of a circle and the mid-point of a chord is perpendicular to the chord.

Theoretical proof



Given

In the given figure, O is a center of a circle and CD is a chord in which CM = MD.

To prove: OM ⊥ CD

Construction: Join OC and OD.

Proof:

S. No.	Statements	S. No.	Reasons
1.	In ΔOMC and ΔOMD	1.	
(i) (ii) (iii)	OC = OD (s) OM = OM (s) CM = DM (s)	(i) (ii) (iii)	Radius of the same circle Common side Given CM = DM
2.	ΔOMC ≅ ΔOMD	2.	By SSS theorem
3.	∠OMC = ∠OMD	3.	Corresponding angles of congruent triangle
4.	∠OMC + ∠OMD = 180° ∠OMC + ∠OMC = 180° or, 2∠OMC = 180° or, ∠OMC = 90°	4.	The sum of adjacent angles in a straight line
5.	OM ⊥ CD	5.	From statement 4 ∠OMC = 90°

proved

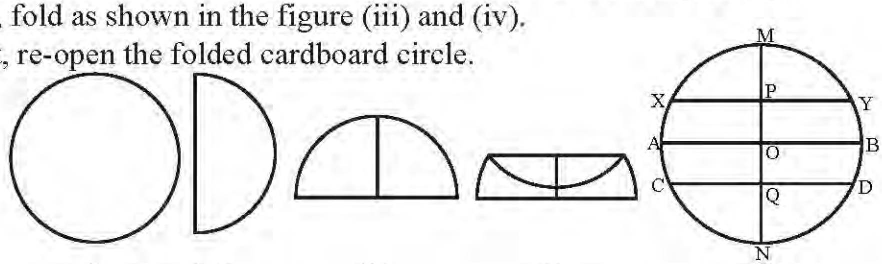
Activity 1

Make a circle of cardboard.

Fold the cardboard circle into two halves as shown in the figure (ii).

Again, fold as shown in the figure (iii) and (iv).

At last, re-open the folded cardboard circle.

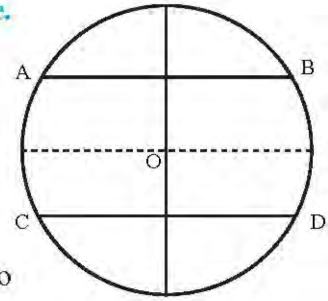


(i) (ii) (iii) (iv) (v)

Now, what may be the relation of chords XY, AB, CD and MN? Observe and draw the conclusion whether the mid-points of XY, AB and CD lie on the vertical line MN or not.

Theorem 3

Equal chords of a circle are equidistant from the centre.



Activity 2

Draw a circle having centre O as in the given figure.

Again, draw two equal chords AB and CD in this figure.

Cut and separate the circle with the help of scissor.

Now, fold at point that divides the chord AB and CD into two equal parts. In this case, check and observe whether the points of the chords are equidistant or not from the centre of the circle.

Experimental verification

Draw two different sized circles having centre O. Draw a pair of equal chords PQ and AB to both sides of the centre if the circle. Then, draw perpendiculars to both the chords PQ and AB from the centre of the circle.

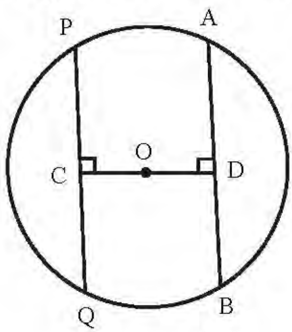


Figure I

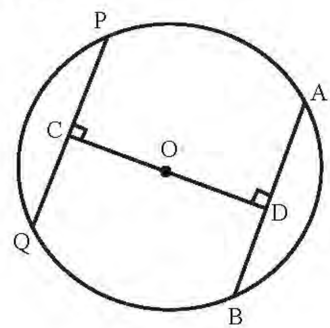


Figure II

Now, measure the distance between centre and chords using ruler and tabulate them as follows:

Figure	OC	OD	Result
I.			
II.			

Conclusion: Therefore, the equal chords of a circle are equidistant from the centre.

Theorem 4 (Converse of theorem 3)

Chords which are equidistant from the centre of a circle are equal.

Experimental verification

Draw two different sized circles having centre O as shown in the following figure. Draw two chords PQ and AB to both sides which are equidistant from the centre. Then, draw perpendiculars to both the chords PQ and AB from the centre of the circle with the help of set square. That is, draw OC$\perp$ PQ and OD$\perp$AB using set squares, in which OC = OD.

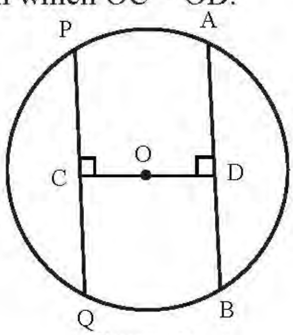


Figure I

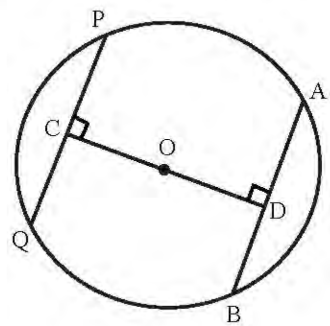


Figure II

Now, measure the length of chords PQ and AB using ruler and tabulate them as follows:

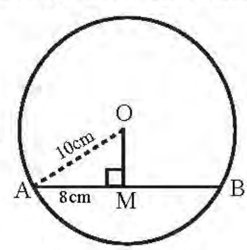
Figure	PQ	AB	Results
I.			
II.			

Conclusion: The chords which are equidistant from the centre of a circle are equal.

Example 1

If radius and chord of a circle are 10 cm and 16 cm respectively, then find the distance between centre and the chord.

Solution



In figure, O is a centre of a circle. OM is perpendicular to chord AB.

Here, OA = 10 cm and AB.

$AM = \frac{1}{2} AB = 8$ cm [∵ The perpendicular drawn from centre to the chord]

Now, ΔOMA, $OA^2 = OM^2 + AM^2$ [∵ Pythagoras theorem]

or, $102 = OM^2 + 82$

or, $100 = OM^2 + 64$

or, $100 - 64 = OM^2$

or, $36 = OM^2$

or, $\sqrt{36} = OM$

or, $OM = 6$ cm

∴ The distance between centre and the chord is 6 cm.

Example 2

In given figure, O is centre of a circle and AB and AC are chords. If $\angle BAO = \angle CAO$, then prove that $AB = AC$.

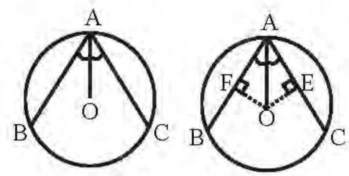
Solution

Given: O is centre of the given circle and AB and AC are chords. And $<BAO = <CAO$.

To prove: $AB = AC$

Construction: $OF \perp AB$ and $OE \perp AC$

$AB = AC$



Proof:

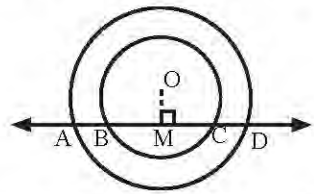
S. No.	Statements	S. No.	Reasons
1.	In ΔOFA and ΔOEA	1.	
(i) (ii) (iii)	$OA = OA$ (s) $\angle OAF = \angle OAE$ (a) $\angle AFO = \angle AEO$ (a)	(i) (ii) (iii)	Common side Given Both 90°
2.	$\Delta OFA \cong \Delta OEA$	2.	By SAA theorem
3.	$AF = AE$	3.	Corresponding sides of congruent triangles
4.	$AF = BF$, $AE = EC$	4.	Having $OF \perp AB$ and $OE \perp AC$
5.	$2AF = 2AE$ or, $AB = AC$	5.	From statements 3 and 4

Proved

Example 3

A straight line cuts at four points A, B, C and D of two concentric circles having common centre O as shown in the given figure. Then prove that AB = CD.

Given: O is a centre of concentric circles. The line AD cuts the concentric circles at points A, B, C and D.



To prove: AB = CD

Construction: OM $\perp$ AD

Proof:

S. No.	Statements	S. No.	Reasons
1.	AM = DM	1.	Perpendicular drawn from centre to the chord.
2.	BM = CM	2.	From statement 1
3.	AB = CD	3.	Subtract statement 2 from 1

Proved

Exercise 16

1. (a) Find the length of a chord at a distance 4 cm from the centre of a circle having radius 5 cm.

   (b) Find the length of a chord at a distance 8 cm from the centre of a circle having diameter 34 cm.

   (c) Find the distance between centre and the chord of length 20 cm of a circle having radius 26 cm.

   (d) Find the distance between centre and the chord of length 48 cm of a circle having radius 26 cm.

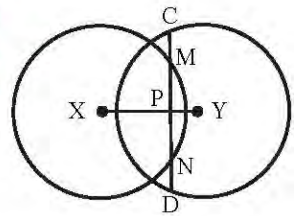
   (e) If the distance between centre and the chord having length of 24 cm is 10 cm, then find the diameter of the circle.

   (f) There are two parallel chords AB = 6 cm and CD = 12 cm in the same side of a circle having centre O. Find the radius of the circle, if the distance between two radius is 3 cm.

   (g) There are two parallel chords PQ = 8 cm and XY = 6 cm in the opposite side of a circle having centre O. Find the distance between PQ and XY, if the radius of the circle is 5 cm.

2. Draw a circle with centre O and having radius 4 cm. Again, draw a chord AB of length 5 cm. Would you be able to draw a circle having 2 cm radius passing the points A and B? Give reason.

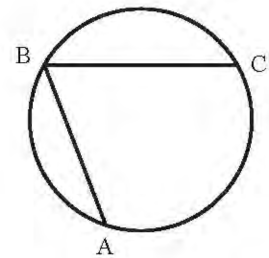
3. In the given figure, X and Y are the centre of two circles. The line CD cut at point M and N of circle with centre X; and XY cut at point P, then prove that



(i) CM = DN

(ii) CN = DM

4. In the given figure, AB and BC are chords of a circle ABC. Trace the figure in to your copy and find the centre of the circle.



Answer

1.	(a) 6 cm	(b) 30 cm	(c) 24 cm
	(d) 10 cm	(e) 52 cm	(f) $3\sqrt{5}$ cm
	(g) 7 cm		

2. No, we can't, because the length of radius is half of AB.

3 - 4. Show to your teacher.

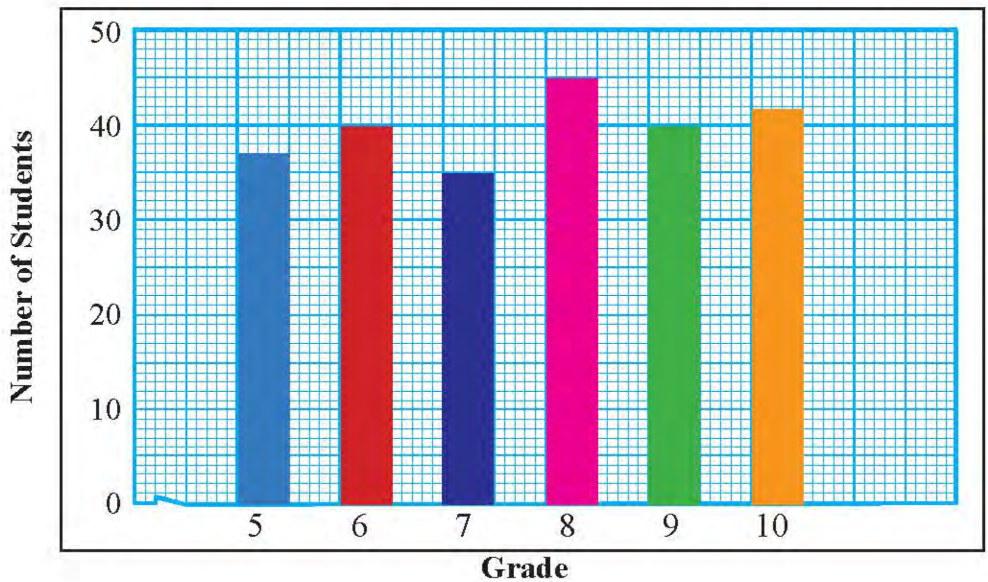
Lesson 17

Classification and Presentation of Data

17.0 Review

The bar diagram of students of Shree Janabikash Secondary School from class 5 to 10 is given as follows. Study the bar diagram and answer the following questions:

The data of Students of
Shree Janabikash Secondary School from Grade 5 to 10



(a) How many total students are there from grade 5 to grade 10?

(b) In which grade has the highest number of students and how many are there?

(c) In which grade has the lowest number of students and how many are there?

(d) In which grade has the equal number of students?

17.1 Classification of data

Activity 1

The marks obtained in mathematics by 40 students of grade 8 are given below:

73, 40, 65, 45, 53, 49, 40, 56, 45, 53,

75, 49, 63, 75, 45, 83, 73, 92, 48, 89,

65, 73, 73, 94, 75, 92, 82, 89, 45, 90,

48, 40, 82, 49, 73, 56, 63, 65, 60, 60

Complete the following frequency distribution table of from the given data.

I. A table of marks obtained in mathematics of grade 8

Marks Obtained	Tally bar	Frequency
40	|||	3
45	||||	4
48		
49		
53		
56		
60		
63		
65		
73		
75		
82		
83		
89		
90		
92		
94		
		Total number (N) =

II. Again, complete the following continuous frequency table having class interval 10/10 that starts from the class 40 – 50.

Marks Obtained	Tally bar	Frequency
40 -50	ⅢⅠ ⅢⅠ ‖	12
50 - 60		
60 - 70		
70 - 80		
80 - 90		
90 - 100		
		Total number (N) =

Answer the following questions based on the above two table:

(a) Are there any changes in total number of students from the above two table?

(b) How many ways were the above data presented through?

(c) Could we prepare the table (II) with class interval 5/5?

(d) How is the frequency 12 obtained the class interval 40 – 50?

The frequency table prepared by calculating the frequency of each data that are given or collected is called the discrete series. The table prepared from the given or collected data with a certain class interval and frequency of class interval is called the continuous series.

Example 1

The marks obtained by 30 students are given as:

Ages (Year): 45, 25, 51, 35, 42, 37, 40, 35, 51, 42, 42, 42, 37, 40, 35, 37, 42, 40, 37, 35, 37, 42, 51, 25, 42, 40, 45, 37, 40, 42

Prepare a discrete distribution from the above data.

Solution

Obtained Marks	Tally bar	Frequency
25	||	2
35	||||	4
37	$\cancel{||||}$ |	6
40	$\cancel{||||}$	5
42	$\cancel{||||}$ |||	8
45	||	2
51	|||	3
		Total number (N) = 30

Activity 2

The ages of grade 5 to 9 students studying in Setidevi Secondary School are as follows:

Ages (years)	10	11	12	13	14	15
No. of Students	40	30	33	48	37	42

Study the given table and answer the following questions:

(a) Which type of distribution is called the above distribution?

(b) Prepare a distribution having 10 years, 11 or less year, 12 or less year, 13 or less year, 14 or less year, 15 or less year of the above data.

(c) What type of distribution in (b) is called?

From the above discussion

(a) The distribution is called as discrete distribution.

(b)

Ages (years)	Frequency	Cumulative frequency
10	40	40
11	30	$40 + 30 = 70$
12	33	$70 + 33 = 103$
13	48	$103 + 48 = 151$
14	37	$151 + 37 = 188$
15	42	$188 + 42 = 230$
	Total number (N) = 230	

(c) Cumulative frequency table

Example 2

Prepare a cumulative frequency distribution of the following table

Wages (Rs.)	50	55	60	65	70	75
No. of workers	4	8	7	6	9	6

Solution

Wages (Rs.)	No. of workers	Cumulative frequency
50	4	4
55	8	$4 + 8 = 12$
60	7	$12 + 7 = 19$
65	6	$19 + 6 = 25$
70	9	$25 + 9 = 34$
75	6	$34 + 6 = 40$
	Total number (N) = 40	

Activity 3

Study the following data and answer the following:

Wages (Rs.)	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
No. of workers	6	13	22	17	7	5

(a) Which type of distribution is called the above distribution?
(b) How many students are there who obtained less than 10 marks?
(c) How many students are there who obtained greater than 0 or more marks?
(d) How can we construct a less than cumulative frequency table?
(e) How can we construct a more than cumulative frequency table?

From the above discussion,

(a) The given data is in the form of continuous distribution.
(b) There are 6 students who obtained less than 10 marks.
(c) There are 70 students who obtained greater than 0 or more marks, i.e all students?
(d) We can construct a less than cumulative frequency table as follows:

Obtained Marks	Cumulative frequency
Less than 10 (<10)	6
Less than 20 (<20)	$6 + 13 = 19$
Less than 30 (<30)	$19 + 22 = 41$
Less than 40 (<40)	$41 + 17 = 58$
Less than 50 (<50)	$58 + 7 = 65$
Less than 60 (<60)	$65 + 5 = 70$

(e) We can construct a more than cumulative frequency table as follows:

Obtained Marks	Cumulative frequency
More than 0 (≥ 0)	70
More than 10 (≥ 10)	$70 - 6 = 64$
More than 20 (≥ 20)	$64 - 13 = 51$
More than 30 (≥ 30)	$51 - 22 = 29$
More than 40 (≥ 40)	$29 - 17 = 12$
More than 50 (>50)	$12 - 7 = 5$

The table in which the frequencies are added one by one is called cumulative frequency table. To prepare the cumulative frequency table, we should arrange the frequency in increasing or decreasing order. Cumulative frequency table of continuous data can be presented into two methods: less than cumulative frequency table and more than cumulative frequency table.

Example 3

The marks obtained in mathematics by 35 students of grade 9 of full marks 60 are given as follows:

39, 50, 42, 34, 25, 35, 36, 46, 34, 32, 44, 43, 24, 43, 40, 36, 45, 34, 42, 37, 35, 43, 58, 34, 35, 33, 24, 40, 43, 52, 57, 33, 50, 38, 24

(a) Present the given marks in continuous data.

(b) Present the given data in less than cumulative frequency table.

(c) Present the given data in more than cumulative frequency table.

Solution

(a) Here, minimum marks and maximum marks are 20 and 58 respectively. Thus, taking the interval of 10/10 we can find the first class interval as 20 – 30 in which lower limit is 20 and upper limit is 30. Lower limit is included and upper limit is not included in each and every class interval. e.g 30 is not counted in class interval 20 – 30 but it is count in class 30 – 40.

Class interval	frequency
20 - 30	4
30 - 40	15
40 -50	11
50 - 60	5
	Total number (N) = 35

(b) Construction of less than cumulative frequency curve

Class interval	Frequency
Less than 30 (<30)	4
Less than 40 (<40)	4 + 15 = 19
Less than 50 (<50)	19 + 11 = 30
Less than 60 (<60)	30 + 5 = 35

(c) Construction of less than cumulative frequency curve

Class interval	Frequency
20 or more than (≥ 20)	35
30 or more than (≥ 30)	$35 - 4 = 31$
40 or more than (≥ 40)	$31 - 15 = 16$
50 or more than (≥ 50)	$16 - 11 = 5$

Exercise 17.1

1. **The marks with full marks 10 in Science of 30 students of grade 9 are as follows:**

   6, 8, 10, 6, 2, 8, 4, 6, 8, 2, 4, 6, 8, 6, 8,
   6, 10, 2, 4, 6, 8, 4, 2, 4, 8, 6, 4, 6, 10, 6

   (a) Convert the given discrete data into continuous data.

   (b) Construct the data in cumulative frequency table.

2. **Construct the given data into continuous data having the first class interval 5 – 10.**

   19, 5, 14, 17, 20, 21, 35, 39, 30, 31, 6, 8, 14, 28, 27, 39,
   30, 31, 32, 25, 26, 10, 11, 12, 15, 28, 30, 31, 24, 22

3. **Construct the following table based on the continuous data of question number of 2.**

   (a) Less than cumulative frequency table

   (b) More than cumulative frequency table

4. **Construct a cumulative frequency table from the following discrete data.**

Wages (Rs. 100)	10	20	30	40	50
No. of workers	3	7	10	8	7

5. **The following table shows that the study time given by the students in mathematics per week**

Hour	2 - 6	6 - 10	10 - 14	14 - 18	18 - 22
No. of students	100	250	325	200	150

Construct the following table into cumulative frequency table based on the given data.

(a) Less than cumulative frequency table

(b) More than cumulative frequency table

Answer

Show the answer to your teacher

17.2 Presentation of data

17.2.1 Histogram

Activity 1

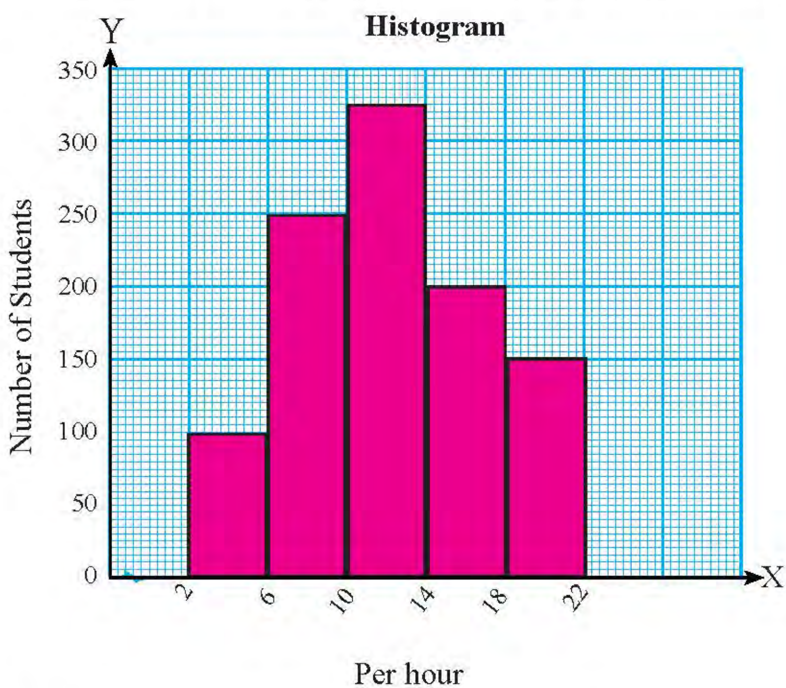
(a) Draw a bar diagram from the following data.

Grade	5	6	7	8	9	10
No. of students	37	40	35	45	40	43

(b) Answer the following questions based on the following data and histogram.

The continuous data related to the study hour in mathematics of 1025 students are presented below.

Hour (Per week)	2 - 6	6 - 10	10 - 14	14 - 18	18 - 22
No. of students	100	250	325	200	150



What is the difference between the bar diagram you made in (a) and histogram in (b)? Discuss in group and draw the conclusion.

A diagram consisting of rectangles whose area is proportional to the frequency of a variable and whose width is equal to the class interval is called histogram . Graphical representation of discrete data is bar diagram and histogram is for continuous data.

Steps of constructing histogram

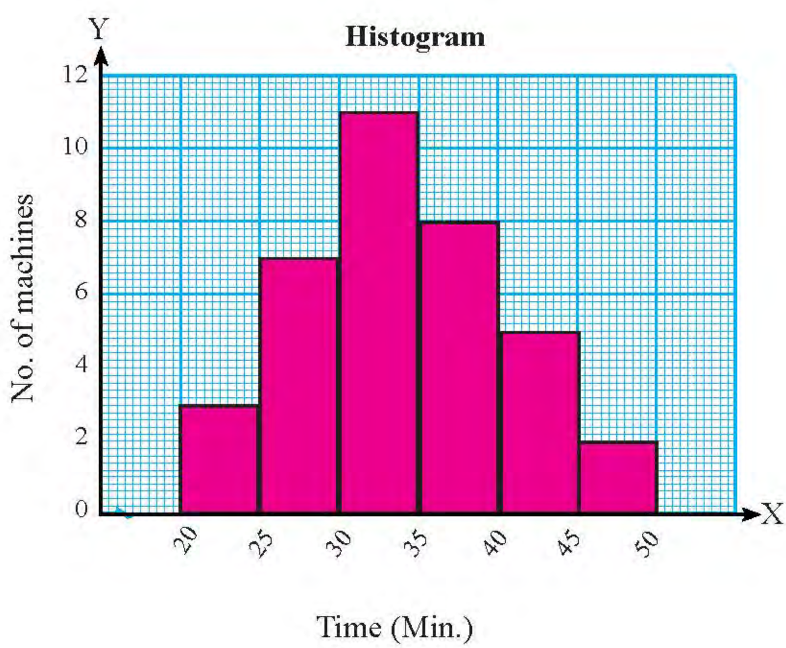
(a) Draw X- and Y- axis in the graph paper.
(b) Put the class interval in X-axis and frequency in Y-axis with a suitable measurement.
(c) Draw the joined rectangles based on the specified class interval in X-axis and frequency in Y-axis as selected scale.

Example 1

Draw a histogram from the following data.

Time (Min.)	20 - 25	25 - 30	30 - 35	35 - 40	40 - 45	45 - 50
No. of machines	3	7	11	8	5	2

Solution

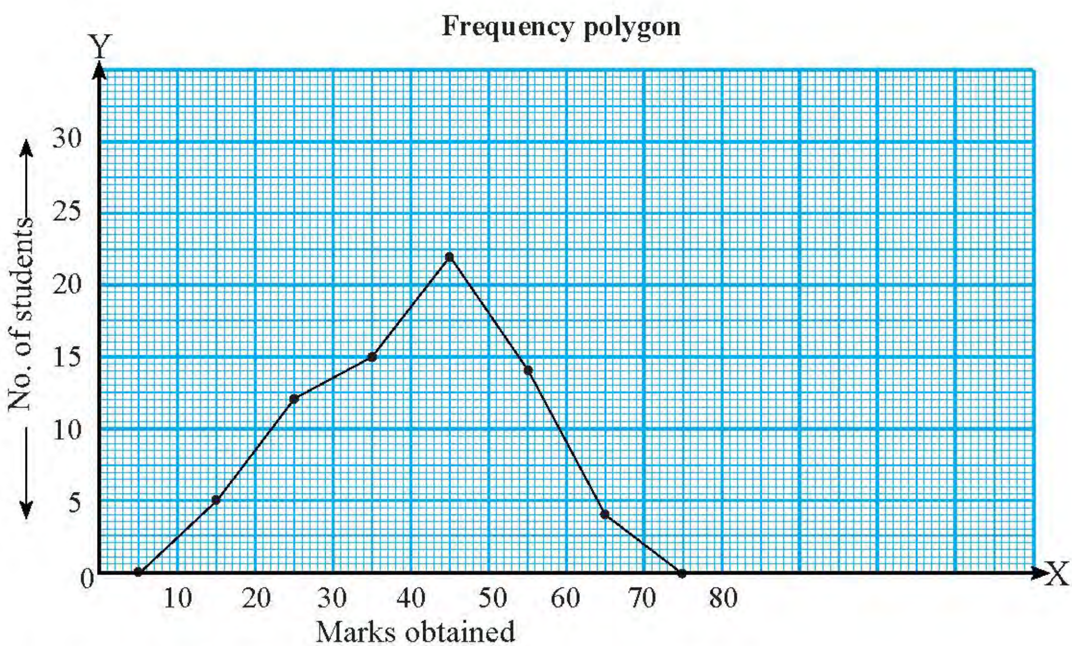


17.2.2 Frequency polygon

Activity 2

Answer following questions based on the given data and the frequency polygon constructed from the given data.

Marks obtained	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70
No. of students	5	12	15	22	14	4



(a) What is the mid-value of the first class interval?

(b) What are the mid-values of each of these class intervals?

(c) Which class interval has the highest frequency?

(d) Which class interval has the lowest frequency?

(e) How can we construct the frequency polygon from continuous data drawn as above?

A line graph which joins the mid-point of the class interval containing in X-axis and the corresponding frequencies containing in Y-axis is called frequency polygon. In frequency polygon, the line is connected with the help of scale.

Steps of constructing frequency polygon

(a) Draw X- and Y- axis in the graph paper.

(b) Put the mid-value of class interval in X-axis and corresponding frequency in Y-axis with a suitable measurement.

(c) Now, draw the mid-value and the corresponding frequency in the graph and join them respectively.

(d) At last, take a lowest class-interval in left side and a highest class-interval in right side with the same scale. And then, take the mid-value of these extreme class-intervals and join them. Then, we can get the required frequency polygon.

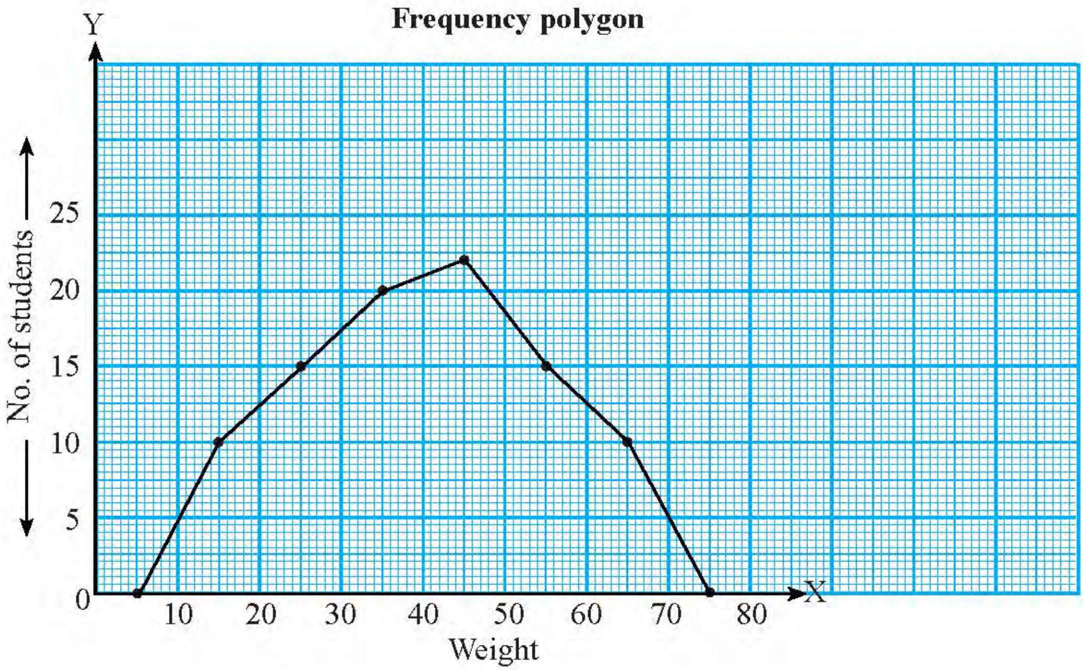
Example 2

Draw the frequency polygon from the following data.

Weight (Kg)	10 - 20	20 - 30	30 - 40	40 -50	50 - 60	60 - 70
No. of students	10	15	20	22	15	10

Solution

Weight	Mid-value	Frequency	Points (Mid-value, corresponding frequency)
0 - 10	$\frac{0+10}{2} = 5$	0	(5, 0)
10 - 20	$\frac{10+20}{2} = 15$	10	(15, 10)
20 - 30	15+10 = 25	15	(25, 15)
30 - 40	25 +10 = 35	20	(35, 20)
40 - 50	35 +10 = 45	22	(45, 22)
50 - 60	45 +10 = 55	15	(55, 15)
60 - 70	55+10 = 65	10	(65, 10)
70 - 80	65+10 = 75	0	(75, 0)
		N = 92	



17.2.3 Cumulative frequency curve or ogive

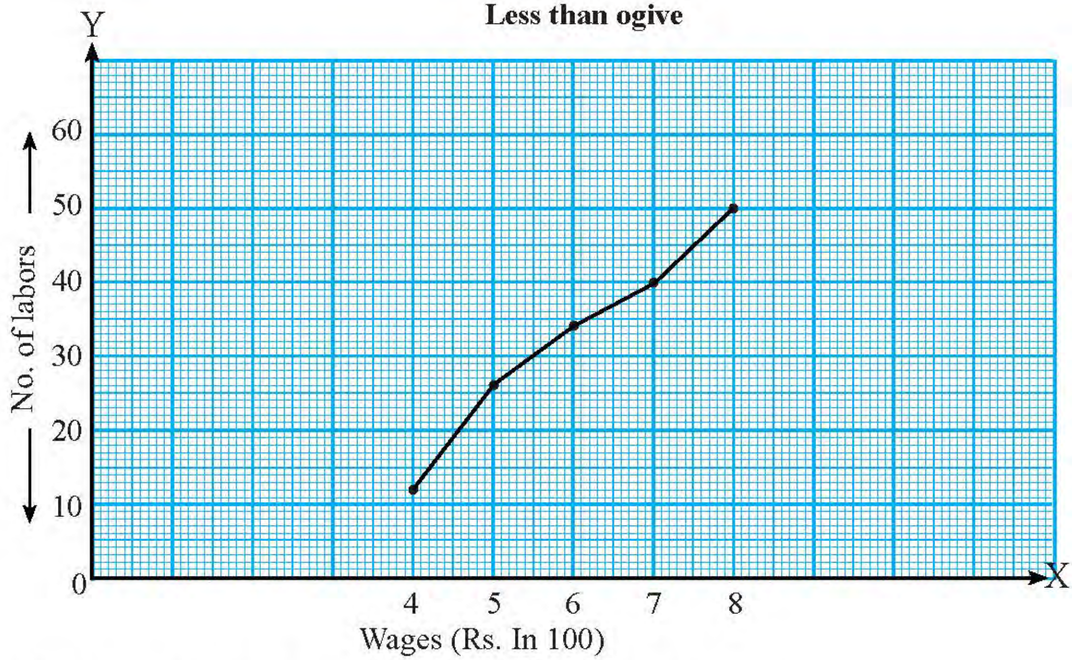
Activity 3

The wages of labors of a company in a day is given in the following table. Study the given cumulative frequency table and cumulative frequency curve:

Wages (Rs. In 100)	3 - 4	4 - 5	5 - 6	6 - 7	7 - 8
No. of labors	12	14	8	6	10

Wages (Rs. In 100)	Cumulative frequency
Less than 4 (<4)	12
Less than 5 (<5)	$12 + 14 = 26$
Less than 6 (<6)	$26 + 8 = 34$
Less than 7 (<7)	$34 + 6 = 40$
Less than 8 (<8)	$40 + 10 = 50$

Less than ogive



Discuss the following questions based on the above figure.

(a) What is the above curve line called?

(b) Which variable is allocated in X-axis?

(c) Which variable is allocated in Y-axis?

(d) How many labors are there in the company in total?

(e) How many labors are there in the company who earn less than Rs. 500 per day?

(f) How many labors are there in the company who earn less than Rs. 700 per day?

(g) How can you draw the more than ogive of the above table as like the given less than ogive?

The curve obtained by freehand drawing of the upper or lower limit of the class interval on the X-axis and the cumulative frequency of the class interval on the Y-axis is called the cumulative frequency curve or ogive.

The freehand drawing of the curved line by marking the points of upper limit of the class interval on the X-axis and the cumulative frequency of the class interval on the Y-axis is called less than cumulative frequency curve or ogive.

The freehand drawing of the curved line by marking the points of lower limit of the class interval on the X-axis and the cumulative frequency of the class interval on the Y-axis is called more than cumulative frequency curve or ogive.

Steps of constructing less than cumulative frequency curve or ogive

(a) Draw X- and Y- axis in the graph paper.
(b) Mark on X-axis in a suitable scale of upper limit of each class.
(c) Label the corresponding frequency in Y-axis with a suitable measurement.
(d) Now, locate the points that intersect the upper limit value of the class and the corresponding frequency in the graph
(e) Then, join the points respectively, you can get the less than frequency curve.

Steps of constructing more than cumulative frequency curve or ogive

(a) Draw X- and Y- axis in the graph paper.
(b) Mark on X-axis in a suitable scale of lower limit of each class.
(c) Label the corresponding frequency in Y-axis with a suitable measurement.
(d) Now, locate the points that intersect the lower limit value of the class and the corresponding frequency in the graph
(e) Then, join the points respectively, you can get the more than frequency curve.

Example 3

Draw the less than and more than curve of the following data in separate graph.

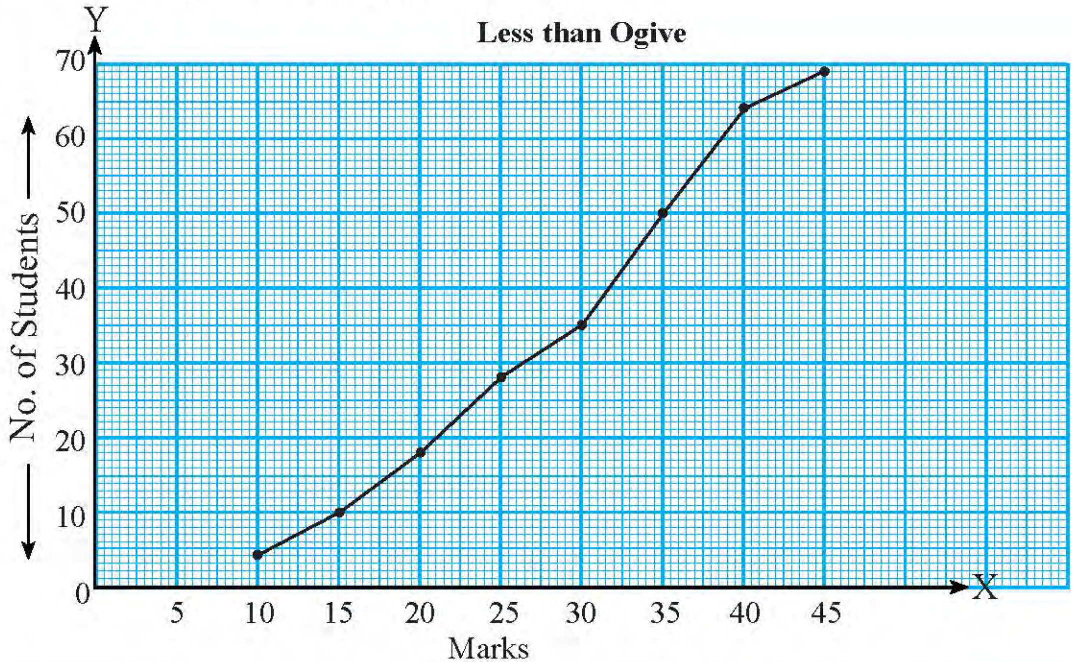
Marks	5 - 10	10 - 15	15 - 20	20 - 25	25 - 30	30 -35	35 - 40	40 - 45
No. of Students	4	6	8	10	7	15	14	5

Solution

Now, present the data into the less than cumulative frequency of the given data.

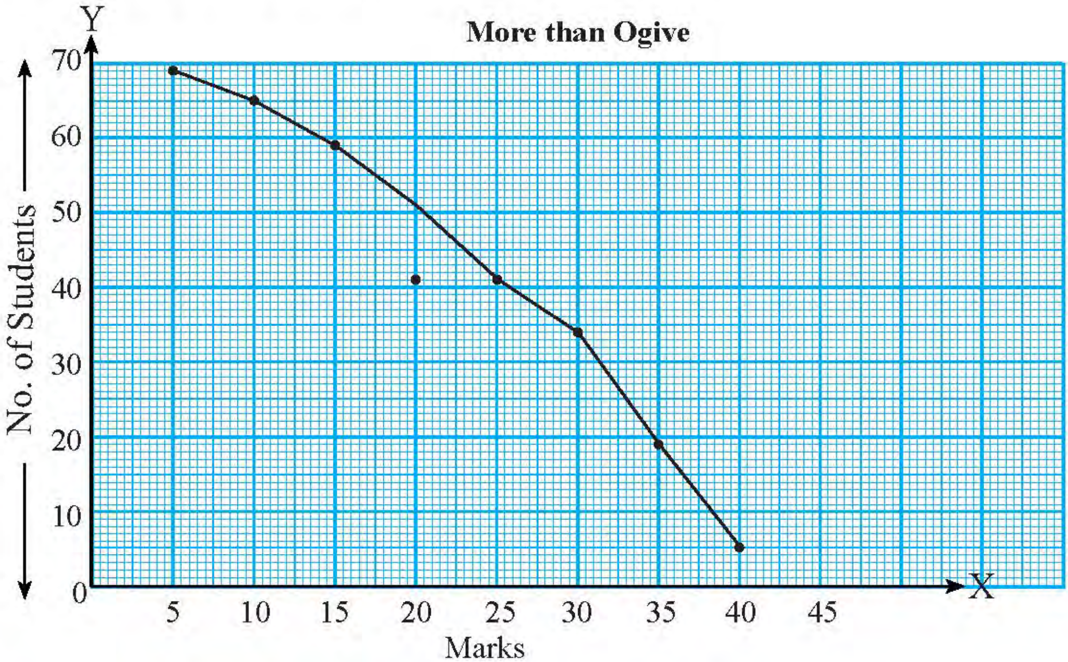
Upper limit	Cumulative frequency	Points (Upper limit, cumulative frequency)
Less than 10 (<10)	4	(10, 4)
Less than 15 (<15)	4 + 6 = 10	(15, 10)
Less than 20 (< 20)	10 + 8 = 18	(20, 18)
Less than 25 (<25)	18 + 10 = 28	(25, 28)
Less than 30 (<30)	28 + 7 = 35	(30, 35)
Less than 35 (<35)	35 + 15 = 50	(35, 50)
Less than 40 (<40)	50 + 14 = 64	(40, 64)
Less than 45 (<45)	64 + 5 = 69	(45, 69)

Now, plot the points on the graph.



Upper limit	Cumulative frequency	Points (Upper limit, cumulative frequency)
More than 5 (≥5)	69	(5, 69)
More than 10 (≥10)	69 – 4 = 65	(10, 65)
More than 15 (≥15)	65 – 6 = 59	(15, 59)
More than 20 (≥20)	59 – 8 = 51	(20, 51)
More than 25 (≥25)	51 – 10 = 41	(25, 41)
More than 30 (≥30)	41 – 7 = 34	(30, 34)
More than 35 (≥35)	34 – 15 = 19	(35, 23)
More than 40 (≥40)	19 – 14 = 5	(40, 5)

Now, plot the points on the graph.



Exercise 17.2

1. Draw the histogram of the following data:

(a)

Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90
No. of Students	4	6	8	10	8	4

(b)

Age (Year)	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
No. of patients	15	12	8	20	27

(c)

Wages (Rs. In 100	50 – 60	60 – 70	70 – 80	80 – 90	90 – 100
No. of Students	20	40	30	60	7

(d)

Class interval	0 – 5	5 – 10	10 – 15	15 – 20	20 – 25
Frequency	3	8	12	7	2

2. Draw the frequency polygon from the following data:

(a)

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
No. of Students	7	3	8	10	2

(b)

Cost of pen	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
No. of pen	15	20	30	25	5

(c)

Incomes (Rs. In 100)	20 – 35	35 – 50	50 – 65	65 – 80	80 – 95
Frequency	10	7	6	5	2

(d)

Class interval (C.I.)	11.5 – 19.5	19.5 – 27.5	27.5 – 35.5	35.5 – 43.5	43.5 – 51.5	51.5 – 59.5
Frequency(f)	7	17	10	4	1	1

3. Draw the less than and more than ogive from the following data:

(a)

Class interval (C.I.)	0 – 5	5 – 10	10 – 15	15 – 20	20 – 25	25 – 30	30 – 35
Frequency (f)	7	10	20	13	17	10	14

(b)

Marks	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90	90 – 100
Frequency	4	6	16	20	30	24

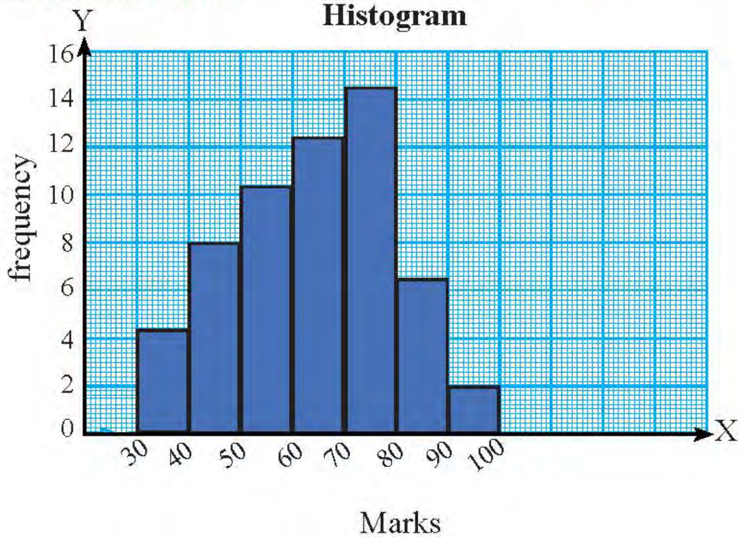
(c)

Periods	0 – 6	6 – 12	12 – 18	18 – 24	24 – 30	30 – 36
No. of teachers	3	10	20	10	5	2

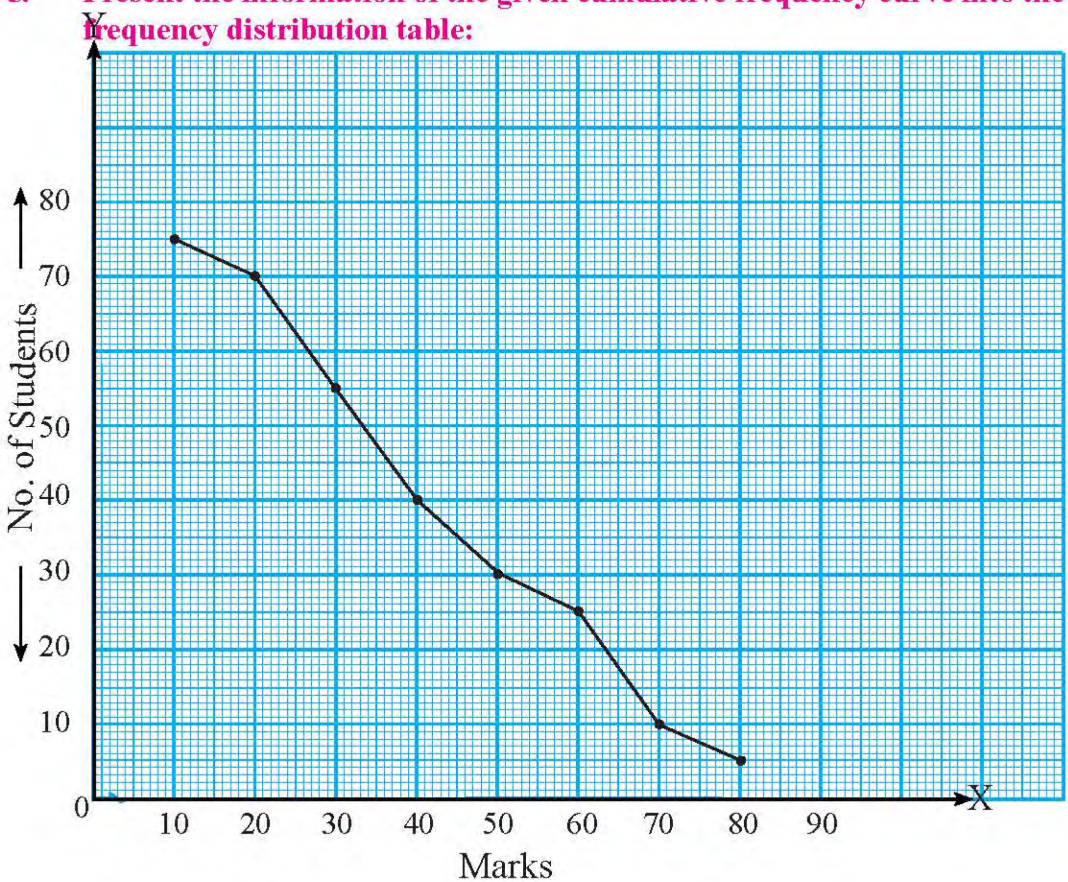
(d)

Time (Periods per week)	0 – 4	4 – 8	8 – 12	12 – 16	16 – 20
Frequency	28	35	66	40	31

4. Present the information of the following histogram into the frequency distribution table:



5. Present the information of the given cumulative frequency curve into the frequency distribution table:



6. **The monthly water consumption of 50 families in a certain locality are given as follows:**

Water consumption (In 1000 liter)	0 – 4	4 – 8	8 – 12	12 – 16	16 – 20
No. of family	28	35	66	40	31

(a) Prepare a histogram from the given information.

7. **In a survey of 40 different types of foods containing the protein quantity, the data is found:**

Protein (In gm): 23, 30, 20, 27, 44, 26, 35, 20, 29, 29, 25, 15, 18, 27, 19, 22, 12, 26, 34, 15, 27, 35, 26, 43, 35, 14, 24, 12, 23, 31, 40, 35, 38, 57, 22, 42, 24, 21, 27, 33

(a) Convert the above discrete data into continuous data with class interval 10 – 15, 15 – 20, and so on.

(b) Prepare a histogram of the given data.

(c) Draw both the less than and more than ogive in the same graph of the data.

Project work

Divide the class in suitable group and go to one of the class from grade 6 – 10, and collect the data of number of family members the students in that class have. Then

(a) Prepare a continuous data having 10 class interval from the given information.

(b) Draw a histogram of the data.

(c) Sketch the frequency polygon of the data.

(d) Draw a less than cumulative frequency ogive.

(e) Draw a more than cumulative frequency ogive.

Answer

Show the answers to your teacher.

Lesson 18

Measures of Central Tendency

18.0 Review

Discuss and find the solution of the following questions based on your class 8 gradesheet.

(a) What is the mean (average) grade point of all students?

(b) What is be the median grade point?

(c) What is the grade point secured by most of the students?

18.1 Arithmetic mean

Activity 1

The secured marks in mathematics by 32 students of grade 9 with full marks 20 are as given below:

12, 9, 14, 16, 14, 9, 12, 12, 10, 18, 14, 10, 12, 16, 18, 14, 12, 16, 16, 14, 16, 14, 14, 12, 18, 16, 16, 18, 14, 12, 14, 16

Based on the above data,

(a) How can we find the average? What is the average in another word colled? Find the average of the data.

(b) Convert the data into discrete distribution.

(c) What are the sum and average of the discrete data?

Here,

(a) The individual obtained marks are in the form of individual data of each student. Thus,

$\sum X = 9 + 9 + 10 + 10 + 12 + 12 + 12 + 12 + 12 + 12 + 12 + 14 + 14 + 14 + 14 + 14 + 14 + 14 + 14 + 14 + 16 + 16 + 16 + 16 + 16 + 16 + 16 + 16 + 18 + 18 + 18 + 18 = 448$

Total number (N) = 32

$$\overline{X} = \frac{\sum X}{N}$$

$$\overline{X} = \frac{\sum X}{N} = \frac{448}{32} = 14$$

(b) The amended data in the form of discrete data

Marks	9	10	12	14	16	18
No. of Students	2	2	7	9	8	4

(c)

Marks (X)	No. of Students (f)	f × X
9	2	18
10	2	20
12	7	84
14	9	126
16	8	128
18	4	72
	$\sum f = N = 32$	$\sum fX = 448$

Now,

Sum of marks ($\sum fX$) = 448

And total number of students ($\sum f$) = N = 32

$$\therefore \overline{X} = \frac{\sum fX}{\sum f} = \frac{\sum fX}{N} = \frac{448}{32} = 14$$

The value which is obtained from the total sum of the given data is divided by the total number of observation is called arithmetic mean or mean or average. If the data is denoted by X, then the arithmetic mean is denoted by $\overline{X}$. If there are large number of data, it should be converted into discrete data; and then we can easily find the average of the data. The average represents the whole data. Arithmetic mean of any data is an important tools of measures of central tendency.

Example 1

Find the arithmetic mean from the following data:

Marks	10	20	30	40	50	60
No. of Students	3	4	7	15	12	1

Solution

Here,

Marks (X)	No. of Students (f)	$f \times X$
10	3	$3 \times 10 = 30$
20	4	$4 \times 20 = 80$
30	7	$7 \times 30 = 210$
40	15	$15 \times 40 = 600$
50	12	$12 \times 50 = 600$
60	1	$1 \times 60 = 60$
	$\sum f = N = 42$	$\sum fX = 1580$

Now,

$$\text{Arithmetic mean } (\overline{X}) = \frac{\sum fX}{N}$$

$$= \frac{1580}{42}$$

$$= 37.62$$

$\therefore$ Therefore, the average marks is 37.62.

Example 2

If the average wages of the given data is Rs. 4100, find the value of y.

Wages (R)	2,000	3,000	4,000	5,000	6,000	7,000
No. of labor	8	12	20	y	6	4

Solution

Here,

Wage (X)	No. of labor (f)	$f \times X$
2000	8	$8 \times 2000 = 16000$
3000	12	$12 \times 3000 = 36000$
4000	20	$20 \times 4000 = 80000$
5000	y	$y \times 5000 = 5000y$
6000	6	$6 \times 6000 = 36000$
7000	4	$4 \times 7000 = 28000$
	$N = \sum f = 50 + y$	$\sum fX = 196000 + 5000y$

Now,

Average wage $(\overline{X}) = \dfrac{\sum fX}{N}$

or, $4100 = \dfrac{196000 + 5000y}{50 + y}$

or, $\dfrac{4100}{1} = \dfrac{196000 + 5000y}{50 + y}$

or, $196000 + 5000y = 4100(50 + y)$

or, $196000 + 5000y = 205000 + 4100y$

or, $5000y - 4100y = 205000 - 196000$

or, $900y = 9000$

or, $y = \dfrac{9000}{900}$

$\therefore y = 10$

18.2 Median

Activity 2

a) Stand the odd number students according to the height in the ascending order and find the height of the middle student.

b) Stand the even number students according to the height in the ascending order and find the height of the middle student.

What is the height of the just middle student called? Discuss with yor friends.

Activity 3

The age of awarded players playing in sports week of class 5 to 10 students are as follows:

Ages (Years): 10, 8, 12, 10, 14, 8, 10, 12, 18, 12, 14, 10, 12, 14,

16, 14, 8, 12, 14, 16, 14, 12, 10, 12, 12, 14, 16, 14, 12

Now, study the data and discuss the about the following questions.

(a) In which series is the data given?

(b) Arrange the data in ascending order.

(c) After arranging the data in ascending order, find the middle value of the data.

(d) What is called the middle value?

(e) Present the data in discrete series and convert the data into cumulative frequency table.

(f) Find the position of median.

Here,

(a) The given data is in individual series.

(b) The data is in ascending order as:

8, 8, 8, 10, 10, 10, 10, 10, 12, 12, 12, 12, 12, 12, 12, 12, 12, 14, 14, 14, 14, 14, 14, 14, 14, 16, 16, 16, 18

(c) Here, 12 is the value which contains in the 15th position from both left and right sides. The middle value i. e. 50% of 29 data is 12. We can also find the middle value using the following formula:

Median = $\left(\frac{N+1}{2}\right)$ item

$= \left(\frac{29+1}{2}\right)$

$= \left(\frac{30}{2}\right)$

$= 15^{th}$ item

8, 8, 8, 10, 10, 10, 10, 10, 12, 12, 12, 12, 12, 12, 12, 12, 12, 14, 14, 14, 14, 14, 14, 14, 14, 16, 16, 16, 18

$\therefore$ Therefore, median (M_d) = 12

(e) The data can be prepared in discrete series due to their repeated frequency as follows:

Age (Years)	8	10	12	14	16	18
No. of Students	3	5	9	8	3	1

Now, construct the cumulative frequency table as follows

Age (Years)	Frequency (f)	Cumulative Frequency (cf)
8	3	3
10	5	3 + 5 = 8
12	9	8 + 9 = 17
14	8	17 + 8 = 25
16	3	25 + 3 = 28
18	1	28 + 1 = 29
	Total (N) = 29	

(f) Median $= \left(\frac{N+1}{2}\right)^{th}$ item

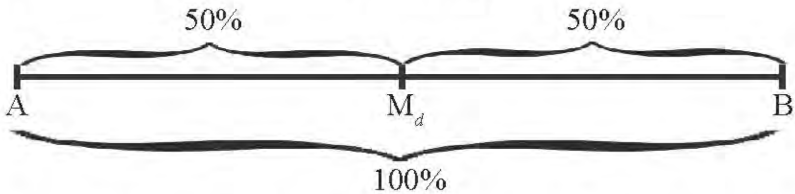
$= \left(\frac{29+1}{2}\right)^{th}$ item

$= \left(\frac{30}{2}\right)^{th}$ item

$= 15^{th}$ item

According to the table 12 is 15th position value of the data. Thus, median (M_d) = 12

Further, we can justify more from the following figure:



From the figure, the total distribution is represented by a line AB. The median M_d divides the line into two equal parts in left and right i. e, median divides the whole data into lower 50% and upper 50%.

The value which divides the whole distribution into two equal parts is called median. Median is denoted by Md. Median divides the distribution into 50/50 percent. Thus, we should divide the total number of observation by 2.

To find the median of discrete distribution, firstly we should rearrange the data in ascending or descending order. In the discrete distribution, the position of the median contains in the corresponding cumulative frequency in which we operate the cumulative frequency.

The following formula is used to calculate the median:

Total number = N, Median = $\left(\frac{N+1}{2}\right)^{\text{th}}$ item

Example 3

Find the median from the following data:

Rainfall (mm)	40	43	54	55	60	62
Places	3	6	9	5	4	2

Solution

First of all, we should confirm that the data is in certain order or not. If not, data should be converted into ascending or descending order. Here, the data is in ascending order. So, we need not rearrange them.

Rainfall (mm)	Places (f)	Cumulative Frequency (cf)

40	3	3
43	6	9
54	9	18
55	5	23
60	4	27
62	2	29
	Total (N) = $\sum f$ = 29	

Here, N = 29

We know that,

$$\text{Median} = \left(\frac{N+1}{2}\right)^{\text{th}} \text{item}$$

$$= \left(\frac{29+1}{2}\right)^{\text{th}} \text{item}$$

$$= \left(\frac{30}{2}\right)^{\text{th}} \text{item}$$

$= 15^{\text{th}}$ item

From cf column,

$\therefore$ Median (M_d) = 54 mm

Example 4

Find the median from the following data:

Marks (X)	10	15	20	25	30
Frequency (f)	4	6	8	3	5

Solution

Now, present the given data into cumulative frequency table as follows:

Marks (X)	Frequency (f)	Cumulative Frequency (cf)
10	4	4
15	6	10
20	8	18
25	3	21
30	5	26
	Total (N) = 26	

Here, N = 26

We know that,

$$= \left(\frac{N+1}{2}\right)$$

$$= \left(\frac{26+1}{2}\right)^{th} \text{ item}$$

$$= \frac{27}{2}^{th} \text{ item}$$

$$= 13.5^{th} \text{ item}$$

$$= \frac{13^{th} \text{ item} + 14^{th} \text{ item}}{2}$$

$$= \frac{20+20}{2}$$

$$= \frac{40}{2}$$

Thus, median (Md) = 20

[∵ from cf table, 13^{th} and 14^{th} item are 20 and 20]

Example 5

Find the median from the following data:

Marks (X)	10	12	14	16	18	20
Frequency (f)	3	4	2	3	5	7

Solution

Now, present the given data into cumulative frequency table as follows:

Marks (X)	Frequency (f)	Cumulative Frequency (cf)
10	3	3
12	4	7
14	2	9
16	3	12
18	5	17
20	7	24
	Total (N) = 24	

Here, N = 24

We know that,

$$\text{Median} = \left(\frac{N+1}{2}\right)^{th} \text{item}$$

$$= \left(\frac{24+1}{2}\right)^{th} \text{item}$$

$$= \frac{25}{2}^{th} \text{item}$$

$$= 12.5^{th} \text{item}$$

$$= \frac{12t^{h} \text{ item} + 13^{th} \text{ item}}{2}$$

$$= \frac{16+18}{2}$$

$$= \frac{34}{2}$$

$$= 17$$

Thus, median (Md) = 17.

18.3 Mode

Activity 4

Time (Hours): 13, 13, 15, 8, 17, 8, 10, 15, 8, 13, 17, 13, 10, 13, 12, 10, 12, 18, 15, 8, 17

(a) Rewrite the given data in discrete series.

(b) Find which one variable is the most repeated.

(c) What is the mode of the given data?

Here, (a)

Time (Hours)	8	10	12	13	15	17	18
Frequency (f)	4	3	2	5	3	3	1

(b) Here, the most repeated value is 13.

(c) Thus, mood (M_o) = 13

The most repeated value or having highest frequency of the given data is called mode.
Mode is denoted by M_o.
Therefore, mode (M_o) = Having highest frequency value

Example 6

Find the mode from the given data.

Marks	10	20	30	40	50	60	70
No. of Students	12	32	50	85	45	30	5

Solution

Here, the most number of students (85) secured the marks 40. That is, the marks 40 has the highest frequency 85.

Thus, mode (M_o) = 40

Exercise 18.1

1. Find the arithmetic mean of the following data:

(a)

Marks	10	20	30	40	50
Frequency	2	5	12	7	1

(b)

Age (Years)	15	25	35	45	55
No. of patients	40	30	50	20	10

(c)

X	40	50	55	62	75	80
f	4	6	10	8	5	2

(d)

Quantity of milk (ml)	500	700	1000	1500	2000
Number of family	8	5	9	5	3

2. Find the arithmetic mean of the following data:

(a)

Marks	10	20	30	40	50	60
Frequency	5	9	15	12	6	3

(b)

Weight (kg)	300	350	400	450	500	550	600
No. of people	6	12	18	14	7	4	1

(c)

Wages (Rs.)	20	25	30	35	40
No. of workers	5	7	10	5	3

(d)

Dividents (Rs.)	300	400	500	600	700
No. of workers	10	14	20	12	4

3. Find the median of the data from the question number 1.

4. Find the arithmetic mean of the data from the question number 2.

5. Find the value in each of the following:

(a) Find the value of k, if the mean $\overline{X} = 38$

X	30	25	35	45	55
f	2	6	k	10	3

(b) Find the value of m, if the mean $\overline{X}$ of the data is 175.

Marks	155	165	175	185	195
Frequency	3	m	5	4	2

(c) Find the value of r, if the mean of the data is 24.55 kg.

Weight (Kg)	20	22	25	26	r	34
No. of persons	4	4	5	4	2	1

6. **Find the mode of the following data:**

(a)	X	10	15	20	25	30
	f	2	4	6	4	2

(b)	Wages (Rs.)	120	150	175	200	225	300
	No. of workers	10	12	17	13	4	3

(c)	Marks	15	25	45	55	65	70
	Frequency	7	8	12	10	3	1

(d)	Weight (Kg)	10	20	30	40	50
	No. of persons	5	9	15	12	6

7. **The heights of 40 garden trees are as follows. Convert the raw data into discrete data and find the arithmetic mean and median.**

Height (cm): 20, 25, 20, 10, 30, 35, 30, 25, 20, 40,

40, 35, 30, 30, 25, 20, 25, 30, 35, 30,
20, 25, 30, 25, 35, 25, 40, 25, 40, 30
25, 35, 30, 35, 35, 25, 30, 35, 30, 30

Answer

1. (a) 30 (b) 30.33 (c) 58.31 (d) 1000 ml
2. (a) 30 (b) Rs. 400 (c) 30 kg (d) Rs. 500
3. (a) 30 (b) 35 (c) 55 (d) 1000 ml
4. (a) 32.8 (b) Rs. 416.13 (c) 29 (d) 476.67
5. (a) k = 9 (b) m = 2 (c) r = 30
6. (a) 20 (b)?= 175 (c) 45 (d) 30 kg
7. 27.5 cm and 30 cm, Mean = 29 cm, Madian = 30 cm

18.4 Quartiles

Activity 1

Study the given figures and answer these questions.

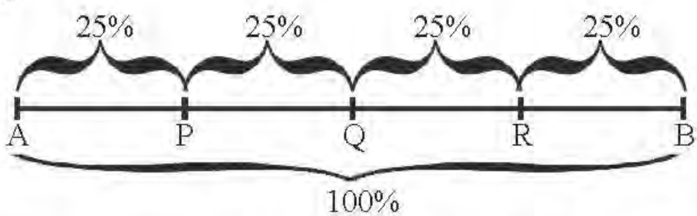
(a) In how many points should the line AB be cut to divide it into 4 equal parts?

(b) If the line AB is 100%, then how much what percentage do the line segments AP, PQ, QR and RB represent?

(c) What percentage is the line segments from A to P, from A to Q and from A to R?

(d) What is the point P that represents 25%, Q that represents 50% and R that represents 75% of the line segment AB called?

By discussion,



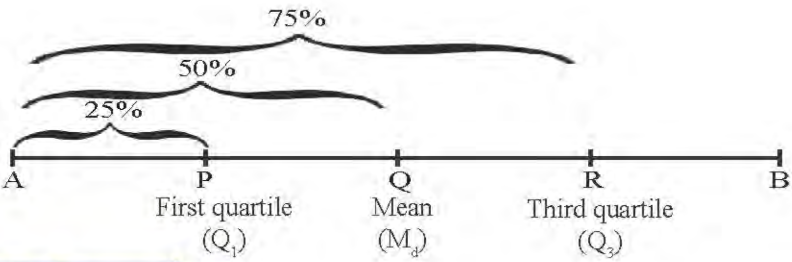
(a) To divide the line AB into 4 equal parts, we should break in 3 points. The three points are P, Q and R respectively.

(b) If AB is 100%, then

AP = 25%, PQ = 25%, QR = 25%, RB = 25%

(c) Out of 100% of AB, the maximum 25% represents the point P, the maximum 50% represents the point Q and the maximum 75% represents the point R.

(d) Thus in line AB, the maximum 25% represents the point P, the maximum 50% represents the point Q and the maximum 75% represents the point R. This concept is clearly shown in the following figure:



Activity 2

Study the given data and find the solution of the following questions:

40, 48, 37, 35, 46, 45, 47

(a) Arrange the given data in ascending order.

(b) Which value is contained in 25% and 75%?

(c) If we convert 25% and 75%, what may be the fraction?

(d) Add 1 to the total number of items and then write the fraction of 25% and 75%; and multiply by it. Then you can find the position of Q1 and Q3. Write the value of Q1 and Q3.

By discussion

(a) The least value of the data is 35, so arrange the data in ascending order from the 35.

35, 37, 40, 45, 46, 47, 48 x'G5 .

(b) 35, 37, 40, 45, 46, 37 48

Q_1 M_d Q_3

(c) $25\% = \dfrac{25}{100} = \dfrac{1 \times 25}{4 \times 25} = \dfrac{1}{4}$ = 1 part out of 4 parts and

$75\% = \dfrac{75}{100} = \dfrac{3 \times 25}{4 \times 25} = \dfrac{3}{4}$ = 3 parts out of 4 parts

(d) There are 7 items in the given data.

Q_1 lies in the position $= \dfrac{1}{4}(N + 1)^{th}$ item $= \dfrac{1}{4}(7 + 1)^{th}$ item $= \dfrac{1}{4}\ 8^{th}$ item $= 2^{nd}$ item

$Q_1 = 2^{nd}$ item $= 37$

Q_3 lies in the position $= \dfrac{3}{4}\ (N + 1)^{th}$ item $= \dfrac{3}{4}\ (7 + 1)^{th}$ item $= \dfrac{3}{4}\ 8^{th}$ item $= 6^{th}$ item

$Q_3 = 6^{th}$ item $= 47$

$\therefore Q_1 = 37$

$Q_3 = 47$

Activity 3

Study the given data and answer the following questions:

Weight (Kg)	20	22	24	27	28	30	31
Frequency	6	8	9	7	5	5	3

Rearrange the data in ascending order if necessary.

(a) Represent the data in cumulative frequency.

(b) Find the position that lie the first and third quartiles.

(c) Find the first and third quartiles from the cumulative frequency data.

From discussion,

(a) Construction of cumulative frequency table:

Weight (Kg)	Frequency (f)	Cumulative frequency (cf)
20	6	6
22	8	14
24	9	23
27	7	30
28	5	35
30	5	40
31	3	43
	Total number (N) = 43	

(b) From the cumulative frequency table:

Q_1 lies in the position = $\frac{1}{4}$ $(N + 1)^{th}$ item

$= \frac{1}{4} \times (43 + 1)^{th}$ item

$= 11^{th}$ item

$\therefore Q_1 = 11^{nd}$ item $= 22$

Q_3 lies in the position = $\frac{3}{4}$ (N + 1)th item

$= \frac{3}{4}$ (43 + 1)th item

= 33rd item

$\therefore Q_3$ = 33rd item = 28

Thus, $Q_1 = 22$

$Q_3 = 28$

The three values which divide the whole data into four equal parts is called quartiles. The first quartile, second quartile and third quartile are denoted by Q_1, Q_2 and Q_3 respectively. The points Q_1 and Q_3 divide the whole data into 25% and 75% respectively.

Here, Q_2 divides the whole data into 50%, so it is called the median.

Example 1

Find Q_1 and Q_3 from the following data.

57, 59, 52, 54, 51, 53, 55

Solution

Rearrange the data in ascending order as:

51, 52, 53, 54, 55, 57, 59

Here, total number of items (N) = 7

Q_1 lies in the position $= \frac{1}{4}$(N + 1)th item $= \frac{1}{4}$(7 + 1)th item $= \frac{1}{4}$ 8th item = 2nd item

Q_1 lies in the position = 2nd position = 52

Q_3 lies in the position $= \frac{3}{4}$ (N + 1)th item $= \frac{3}{4}$ (7 + 1)th item $= \frac{3}{4}$ 8th item = 6th item

Q_3 lies in the position = 6th item = 57

$\therefore Q_1 = 52$

$Q_3 = 57$

Example 2

Find first quartile Q_1 and third quartile Q_3 from the following data.

Marks	10	20	30	40	50	60	70
No. of Students	12	32	50	85	45	30	5

Solution

Now, construct the cumulative frequency table of the given data.

Marks	Frequency (f)	Cumulative frequency (cf)
10	6	6
20	4	10
30	2	12
40	1	13
50	5	18
60	8	26
70	9	35
	Total number (N) = 35	

There are 35 items in the given data, i. e. N = 35

Q_1 lies in the position = $\frac{1}{4}(N+1)^{th}$ item = $\frac{1}{4}(35+1)$th item = $\frac{1}{4}$ 36^{th} item = 9^{th} item

Q_1 lies in the position = Value of 9^{th} term = 20

Q_3 lies in the position = $\frac{3}{4}$ $(N+1)^{th}$ item = $\frac{3}{4}$ $(35+1)^{th}$ item = $\frac{3}{4}$ 36^{th} item = 27^{th} item

$\therefore Q_3 = 27^{th}$ item = 70

Thus, $Q_1 = 20$

$Q_3 = 70$

Example 3

Find the first quartile Q_1 and third quartile Q_3 from the following data.

Marks	12	13	14	15	16	17	18
No. of Students	6	8	9	3	3	5	4

Solution

Now, construct the cumulative frequency table of the given data.

Marks	Number of students (f)	Cumulative frequency (cf)
12	6	6
13	8	14
14	9	23
15	3	26
16	3	29
17	5	34
18	4	38
	Total number (N) = 38	

There are 38 items in the given data, i. e. N = 38

Q_1 lies in the position = $\frac{1}{4}(N + 1)^{th}$ item = $\frac{1}{4}(38 + 1)^{th}$ item = $\frac{1}{4}$ 39^{th} item = 9.75^{th} item

Q_1 = lies in the position + 0.75 (10^{th} item – 9^{th} item)

= 13 + 0.75 (13 – 13)

= 13

Q_3 lies in the position = $\frac{3}{4}$ $(38+ 1)^{th}$ item = $\frac{3}{4}$ $(38 + 1)^{th}$ item = $\frac{3}{4}$ 39^{th} item = 29.25 th item

Q_3 = 29^{th} item + 0.25 (30 th item – 29^{th} item

= 16 + 0.25 (17 – 16)

= 16 + 0.25

= 16.25

Exercise 18.2

1. Find the first quartile and third quartile from the following data:

(a) 20, 40, 30, 15, 60, 90, 80

(b) Rs. 400, Rs. 600, Rs. 350, Rs. 200, Rs. 550, Rs. 700, Rs. 320, Rs. 625, Rs. 370, Rs. 650, Rs. 275

(c) 20, 15, 5, 10, 25

(d) 15 kg, 9 kg, 12 kg, 24 kg, 18 kg, 21 kg

(e) 18 °C, 28 °C, 26 °C, 16°C, 21°C, 29 °C, 25 °C, 12 °C, 23 °C

2. Find the first quartile (Q_1) and third quartile (Q_3) from the following data:

(a)

Marks	5	10	15	20	25	30
No. of Students	3	5	8	7	4	3

(b)

Wages	300	250	350	450	550
Number of labors	2	6	9	10	1

(c)

Weight (Kg)	105	115	125	135	145	155
Frequency	8	10	7	12	7	6

(d)

X	16	12	20	14	18
f	7	3	6	8	4

3. The mathematics marks with full marks 30 of 19 students studying in 9 class are as follows:

15, 10, 25, 10, 20, 15, 20, 25, 15, 20, 10, 20, 25, 15, 30, 20, 20, 30, 25

Convert the given data into discrete data and find the first quartile and third quartile

4. In the survey of class 10 students, the weight (Kg) of 45 students are as follows:

25, 40, 35, 35, 29, 40, 38, 25, 29, 27, 29, 27, 35, 40, 35, 29, 35, 38, 27, 29, 29, 40, 35, 35, 38, 29, 40, 27, 29, 35, 38, 27, 27, 29, 29, 25, 40, 27, 38, 25, 35, 25, 35, 29, 29

Find the lower and upper quartile of the given data.

5. Find the first quartile and third quartile from the following data:

(a) 47, 49, 52, 54, 51, 53, 55

(b) 7, 9, 6, 5, 7, 6, 4, 10, 9

(c) 5, 10, 15, 20, 25

(d) 64, 60, 70, 72, 68, 80, 85, 56

6. Find the first quartile (Q_1) and third quartile (Q_3) from the following data:

(a)	Marks	60	70	80	85	90	95	98
	No. of Students	5	7	12	15	18	3	2

(b)	Weekly income (Rs.)	300	400	500	600	700	800	900
	Number of labors	40	20	15	17	25	10	5

(c)	Height (cm)	120	121	122	123	124	125	126
	Number of students	5	10	12	17	16	3	4

(d)	Average temperature (°C)	20	22	25	26	27	29
	Day	4	6	8	5	3	4

Project work

Divide the students in a suitable group and collect the marks of own class and preceding class. Then find the mean, median, mode, first quartile and third quartile of the collected data subject wise.

Answer

1. (a) $Q_1 = 20, Q_3 = 80$ (b) $Q_1 = 320, Q_3 = 625$
   (c) $Q_1 = 7.5, Q_3 = 22.5$ (d) $Q_1 = 11.25, Q_3 = 21.75$
   (e) $Q_1 = 17, Q_3 = 27$
2. (a) $Q_1 = 10, Q_3 = 25$ (b) $Q_1 = 300, Q_3 = 450$
   (c) $Q_1 = 115, Q_3 = 137.5$ (d) $Q_1 = 14, Q_3 = 18$
3. $Q_1 = 15, Q_3 = 25$
4. $Q_1 = 27, Q_3 = 38$
5. (a) $Q_1 = 49, Q_3 = 54$ (b) $Q_1 = 5.25, Q_3 = 8.5$
   (c) $Q_1 = 7.5, Q_3 = 22.5$ (d) $Q_1 = 61, Q_3 = 78$
6. (a) $Q_1 = 80, Q_3 = 90$ (b) $Q_1 = 300, Q_3 = 700$
   (c) $Q_1 = 122, Q_3 = 124$ (d) $Q_1 = 22, Q_3 = 26$

Lesson 19

Probability

19.0 Review

Study the following statements:

(a) Today is cloudy, perhaps it rains.

(b) I am in confusion of passing the exam.

(c) The political situation may drastically change.

(d) There is a equal chance of winning the election by two candidates.

(e) What is the probability of winning the football 2022 by the Spain?

There are some words in the above statements like: perhaps, confusion, chance, probability that indicate uncertainty. That is, the result may happen or not. We can predict the result based on the past experiences and events. From the ancient period, people have been predicting such type of daily life events.

19.1 Probability

Estimation is the unpredictable fortune and probability. Thus, the measurement of uncertainty is called the probability. From the above statements: today is cloudy means it may rain. In this situation, it is better to take an umbrella. Here, raining is only the estimation i.e, there is no certainty of raining. Thus, the cloudy day is only the sign of raining.

Some technical terms of probability

Activity 1

What may occur in tossing a coin and throwing a dice?

In tossing a coin, either a head (H) or tail (T) can occur.

The total number of possible occurrence in the case of tossing a coin can be given in the form of set is as: S = {H, T}.

Similarly in throwing a dice, the possible occurrence is: 1, 2, 3, 4, 5 or 6. That is, it can be written in the form of set as: S = {1, 2, 3, 4, 5, 6}.

Study the following technical terms based on the above examples.

(a) Experiment

The result or outcomes that cannot be predicted is called the experiment. In above example, we cannot predict the turning of H or T. Thus, the tossing a coin is an experiment.

(b) Random experiment

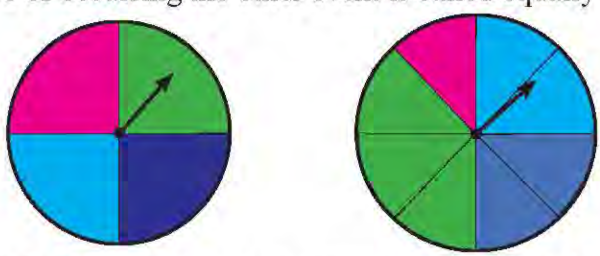
An experiment whose outcomes cannot be predicted with certainty is called random experiment. For example: we cannot say who can win the prize from a lottery. That is, nobody can predict the result of the lottery. Thus, drawing a lottery is a random experiment. Similarly from the above example: tossing a coin, rolling a die, etc are the example of random experiment.

(c) Outcomes

The results of random experiment are called outcomes or events. For example: in tossing a coin, any one out of H or T will turn. Then, it is a outcome of the experiment.

(d) Equally likely outcomes

An experiment in which the chance of occurring any one of the event is equal to the chance of occurring the other event is called equally likely events.



In the above first figure, there is an equal chance of indicating the needle to the four color on rotating the spinner because all four color covers the equal area. Thus, the result of indicating the four color: red, green, yellow and blue are the cases of equally likely outcomes.

Again in second figure, if we rotate the spinne, there is no equal chance of indicating the four colors because all four colors do not cover the equal area. There is not equal chance of indicating the four colors: red, green, yellow and blue and thus the second figure does not show the equally likely outcomes.

(e) Sample space

The set of all possible outcomes of a random experiment is called as sample space. Generally, it is denoted by S.

The sample space in the case of tossing a coin (S) = {H, T} and n(S) = 2.

The sample space in the case of rolling a dice (S) = {1, 2, 3, 4, 5, 6} and n(S) = 6

(f) Event

The set of outcomes of any experiment is called an event. It is denoted by E.

In the case of tossing a coin, the result may be S = {H, T} then the possible cases: not turning both, turning head, turning tail, etc. are the events. Thus, { }, {H}, {T}, {H, T} are all the events in tossing a coin.

(g) Mutually exclusive events

In tossing a coin, record the turning face. In the first tossing, the turn face may be a T. Could we see the face H in this case? If we toss the coin again, then we found the face H. Could we see the face T in this case?

In this way, the experiment in which any one of the event can occur but another event cannot occur simultaneously, then such types of event is called mutually exclusive event. For example: in tossing a coin, out of H or T, only one can occur, but not both the H and T can occur simultaneously. Thus, turning H and T are the mutually exclusive events in the case of tossing coin.

Now, discuss in your class whether the turning faces from 1 to 6 are mutually exclusive events or not in the case of rolling a dice.

(h) Number of favourable outcomes

The outcomes of random experiment, which entails the happening of an event, are known as the number of favorable outcomes.

(i) Elementary event

The set of all possible outcomes in an experiment is called elementary event. For example: if we roll a dice, then the elementary events are: {1, 2, 3, 4, 5, 6}.

Activity 2

In tossing a coin,

What is the probability of getting a head (H)?

What is the probability of getting a tail (T)?

What are the total possible outcomes in tossing a coin?

Here, in tossing a coin the total possible outcomes in tossing a coin are: S = {H, T}

Thus, total number of sample space: n(S) = 2

Occurring the event of head: (E) = {H}

Thus, number of favorable outcomes: n(E) = 1

The probability of any event: P(E) = $\frac{n(E)}{n(S)}$ = $\frac{\text{The probability of}}{\text{Total number of sample space}}$

Thus, the probability of getting head = $\frac{1}{2}$

Example 1

What is the probability of getting even numbers in throwing a dice?

Solution

In throwing a dice,

The sample space (S) = {1, 2, 3, 4, 5, 6}

Thus, the total number of sample space: n(S) = 6

Favorable cases (E) = Even numbers = {2, 4, 6}

Thus, the number of favorable cases n(E) = 3

Probability of occurring the even number P(E) =?

Now P(E) = $\frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2} = 0.5$

Example 2

The result of 80 students in a exam is given below:

Grade	A^+	A	B^+	B
Number of Students	12	20	18	30

Find the probability of the following if a student is selected randomly.

(a) Securing A grade

(b) Securing B grade

Solution

Here, total number of students: n(S) = 80

Number of students securing A grade: n(A) = 12

Number of students securing B grade: n(B) = 30

(a) Probability of securing A grade: $p(\text{A})$ =?

Now, $p(\text{A}) = \frac{n(A)}{n(S)} = \frac{12}{80} = \frac{3}{20}$

Thus, the probability of securing A grade = $\frac{3}{20}$

(b) Probability of securing B grade: $p(\text{B})$ =?

Now, $p(\text{B}) = \frac{n(B)}{n(S)} = \frac{30}{80} = \frac{3 \times 10}{8 \times 10} = \frac{3}{8}$

Thus, the probability of securing B grade: $\text{P(B)} = \frac{3}{8}$

Example 3

What is the probability of not selecting a square number in selecting number cards labeled from 1 to 10?

Solution

In selecting a number cards,

The sample space (S) = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

Thus, the total number of sample space: $n(\text{S}) = 10$

Favorable cases (E) = Square numbers = {1, 4, 9}

Thus, the number of favorable cases: $n(\text{E}) = 3$

Probability of occurring the square number: P(E) =?

Now, $\text{P(E)} = \frac{n(E)}{n(S)} = \frac{3}{10}$

Then, the probability of not occurring square number: $\text{P}(\overline{\text{E}}) = 1 - \text{P(E)}$

$= 1 - \frac{3}{10} = \frac{7}{10}$

Thus, the probability of not occurring square number: $= \frac{7}{10}$

Alternate method

Number of sample space $n(s) = 10$

Non squared number $n(\overline{E}) = \{2, 3, 5, 6, 7, 8, 10\} = 7$

The number of non squared number $n(\overline{E}) = 7$

The probability of non squared number $P(\overline{E}) = ?$

Now, $P(\overline{E}) = \dfrac{n(\overline{E})}{n(S)}$

Therefore, the probability of non-squared number is $P(\overline{E}) = \dfrac{7}{10}$

Exercise 19.1

1. Find the sample space of the following experiment:

(a) Roll a dice in one time

(b) Tossing a coin in two times

(c) Selecting a day from in a week

(d) Indicating a color in spinning a spinner

2. Write down the events/favorable cases from the following condition:

(a) Getting even numbers in rolling a dice in one time

(b) Getting two head in tossing a coins in two times

(c) Turning odd day in selecting a day from a week numbered from 1 to 7.

(d) Turning a head and a tail in tossing two coins in one times

3. Find the probability of getting prime number in rolling a dice.

4. Find the probability of one head and one tail in one time tossing two coins.

5. Find the probability of the following based on the given table.

Number of children	0	1	2	3
Total number of family	9	15	36	30

(a) Probability of getting 0 children family

(b) Probability of getting 2 children family

(c) Probability of getting at least 2 children family

6. Divide the class into four groups. Take a pack of well-shuffled deck of 52 cards by each group. Fill the number of Red, Black, Heart, Diamond, Spade, Club, Face cards, Jack, Queen, King, Ace cards in the following table.

(a)

♣	♦	♥	♠

(b)

A	J	Q	K

(c)

Red	Black	Face card

(d) Find the probability of the following based on the above table:

(i) Spade (ii) Ace (A) (iii) Red card

(iv) Black king (v) Jack and Queen (vi) Not the king

Answer

1. (a) {1, 2, 3, 4, 5, 6} (b) {HH, HT, TH, TT}

   (c) {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}

   (d) {Red, Green, Blue, Yello}

2. (a) {2, 4, 6} (b) {HH} (c) {1, 3, 5, 7} (d) {HT, TH}

3. $\frac{1}{2}$ 4. $\frac{1}{2}$ 5. (a) $\frac{1}{10}$ (b) $\frac{2}{5}$ (c) $\frac{11}{15}$

6. Show the answers to the teacher

19.2 Probability scale

Activity 1

Write the sample space in the case of tossing a coin. Similarly, write the possible outcomes in tossing a coin.

S = {H, T},{H}, {T}

Find the probability of each of the above cases.

P(A) = ..., P(B) = ...,

$\frac{1}{2}$

$\frac{1}{2}$

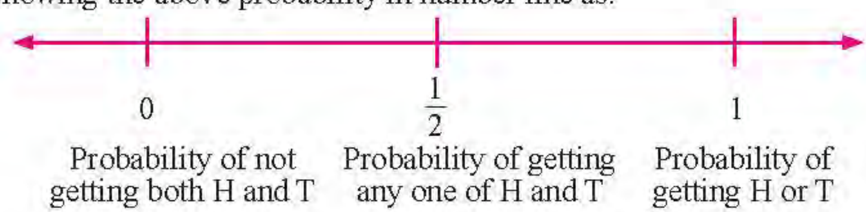
Add all the probability after finding the probability.

P(H) = ... or, P(T) = ...

P(H) + P(T) = ...

Similarly, what is the probability of not occurring H and T?

Showing the above probability in number line as:



We can conclude as following based on the above number line

Probability of getting H or T: $P(H) = P(T) = \frac{1}{2}$

Probability of getting in tossing a coin H or T: $P(H \text{ or } T) = \frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1$

$P(S) = 1$ where $S = \{H, T\}$

The least value of probability is 0, it means that the probability of occurring the event is completely impossible. And the highest value of probability is 1 that means the probability of occurring the event is certain. In this way, the probability of getting any event E lies between 0 and 1, i. e., $0 \leq P(E) \leq 1$. It is called a probability scale.

Activity 2

Put a red, two green and three white same size tennis ball in a box. Draw a ball randomly by a student. Then, write down the possible probability in the blackboard.

Let us assume that: Red = R, Green = G and White = W.

(a) Find the separate probability of getting these three different color.

P(R) = ……., P(G) = ……., P(W) = ………

(b) Adding all the probability.

P(R) + P(G) + P(W) = ……………………= ………..

Example 1

Find the probability of indicating the blue color and not indicating the blue color in the case of turning a spinner having three different color: blue, yellow and red.

Solution

Here, there are three different colors: blue, yellow and red in a spinner.

∴ Total number of sample space cases: n(S) = 3

Number of blue color part = Number of favorable cases $n(E) = 1$

∴ The probability of indicating the blue color part: (E) = $\frac{n(A)}{n(S)} = \frac{1}{3}$

Again, the probability of not indicating the blue color part: $P(\overline{E}) = 1 - P(E)$

$$= 1 - \frac{1}{3}$$

$$= \frac{3-1}{3}$$

$$= \frac{2}{3}$$

Example 2

What is the probability of drawing a king from a well shuffled deck of 52 cards?

Solution

Here in the case of drawing a card from a well shuffled deck of 52 cards,

Total number of sample space cases: $n(S) = 52$

The number of favorable cases = Number of king: $n(K) = 4$

Now, the probability of king P(K) =?

The probability of indicating the blue color part: $P(K) = \frac{n(K)}{n(S)} = \frac{4}{5^2}$

$$\therefore P(K) = \frac{1}{13}$$

Example 3

What is the probability of not getting 5 in throwing a dice?

Solution

Here in the case of throwing a dice $(S) = \{1, 2, 3, 4, 5, 6\}$

Total number of sample space cases: $n(S) = 6$

The number of favorable cases = Number of 5: $n(5) = 1$

Now, the probability of king P(5) =?

$\therefore$ The probability of getting 5: $P(E) = \frac{n(E)}{n(S)}$

$$= \frac{1}{6}$$

Again, the probability of not getting 5: $P(\overline{E}) = 1 - P(E)$

$$= 1 - \frac{1}{6}$$

$$= \frac{6-1}{6}$$

$$= \frac{5}{6}$$

$\therefore$ The probability of not getting 5 in throwing a dice $= \frac{5}{6}$

Example 4

There are 2 red, 3 black and 4 green marbles in a bag. If 1 marble is drawn randomly, then find the probability of the following:

(i) getting red marble

(ii) getting green marble

(iii) not getting red marble

Solution

There are 2 red, 3 black and 4 green marbles in a bag.

Total number of sample space cases: $n(S) = 2 + 3 + 4 = 9$

(i) The number of red balls: $n(R) = 2$

$\therefore$ The probability of getting red ball: $P(R) = \dfrac{n(R)}{n(S)} = \dfrac{2}{9}$

(ii) The number of green balls: $n(G) = 4$

$\therefore$ The probability of getting green ball: $P(R) = \dfrac{n(G)}{n(S)} = \dfrac{4}{9}$

(iii) The number of black balls: $n(B) = 3$

Now, the probability of getting black ball: $P(B) = \dfrac{n(B)}{n(S)} = \dfrac{3}{9} = \dfrac{1}{3}$

$\therefore$ The probability of not getting black ball: $P(\overline{B}) = 1 - \dfrac{1}{3} = \dfrac{3-1}{3} = \dfrac{2}{3}$

Exercise 19.2

1. Show the result in a probability scale (0 – 1) in the case of tossing a coin.

2. **There are 2 blue, 3 black and 5 red marbles in a bag. If one marble is drawn randomly, then find the probability of the following:**

   (a) getting black marble

   (b) not getting black marble

   (c) Show the result in a probability scale (0 – 1) in the case of (a) and (b).

3. What is the probability of getting public holiday in Sunday throughout of the year? Similarly, find the probability of not getting public holiday in Sunday.

4. **In the case of drawing a card from a well shuffled deck of 52 cards,**

   (a) What is the probability of getting red card?

   (b) What is the probability of getting red or black card?

   (c) Show the result in a probability scale (0 – 1) of the above two probability.

5. **Find the probability of indicating the following colours in the case of turning the spinner, and show the result in a probability scale (0 – 1).**

(a) indicating the white color

(b) indicating the yellow color

(c) indicating the yellow or red color

(d) indicating the yellow or red or green color

(e) indicating the yellow or red or green or blue color

(f) show the result in a probability scale (0 – 1) of all the above

6. Show the result in a probability scale (0 – 1) in the case of throwing a dice having 6 faces.

7. **If a number card numbered from 2 to 25 is drawn randomly, then**

(a) What is the probability of getting a number card, which is exactly divisible by 3?

(b) What is the probability of getting even number cards?

Answer

1. Show the answer to your teacher. 2. (a) $\frac{3}{10}$ (b) $\frac{7}{10}$

(c) Show the answer to your teacher. 3. $\frac{52}{365}, \frac{313}{365}$

4. (a) $\frac{1}{2}$ (b) 1 (c) Show the answer to your teacher.

5. (a) 0 (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$ (e) 1

6. Show the answer to your teacher. 7. (a) $\frac{1}{3}$ (b) $\frac{1}{2}$

19.3 Empirical probability

Activity 1

(a) If a dice is thrown 10 times, 15 times, 20 times respectively, observe the fraction of turning 1, 2, 3, 4, 5 and 6 to the total number of throwing the ball.

We know that, if a coin is tossed, then the probability of turning head is $\frac{1}{2}$.

That is, $P(H) = \frac{1}{2}$ and $P(\overline{H}) = P(T) = 1 - \frac{1}{2} = \frac{1}{2}$ Based on this theory, if we turn a coin 10 or 20, then the probability of head is $10 \times \frac{1}{2} = 5$ or, $20 \times \frac{1}{2} = 10$. But in practice, the result of the experiment may differ. In this way, the probability obtained from a practice or experiment is called empirical probability. Similarly, the process of obtaining the probability through practical base is called empirical probability. The following formula is used for finding the empirical probability.

$$\text{Empirical Probability} = \frac{\text{Number of outcomes through experimen}}{\text{Total number of experiment}}$$

Example 1

The following result is obtained in throwing a coin 50 times:

Experiment (E)	Head (H)	Tail (T)
Frequency (f)	23	27

What is the empirical probability of getting head?

Solution

Here, the empirical probability of head: $P(H) = \frac{23}{50}$ and $P(T) = \frac{27}{50}$

Example 2

If 2000 bricks were unloaded from a mini truck, the probability of breaking the bricks is 0.1, and then what is the number of unbroken bricks?

Solution:

Here, the number of bricks: N(S) = 2000

Probability of breaking the bricks: P(E) = 0.1

Probability of unbroken bricks: $p(\overline{E}) = 1 - 0.1 = 0.9$

$\therefore$ The number of unbroken bricks: $= n(S) \times p(\overline{E})$

$= 2{,}000 \times 0.9$

$= 1{,}800$

Exercise 19.3

1. There are 50 students in a class out of which 28 are girls. If a student is selected randomly, then what is the probability of girl student to be selected in essay competition?

2. Among 35 students of class 9, 25 students can fluently speak and write English language. If a student is selected randomly, then what is the probability of selecting a student who can fluently speak and write English language in a quiz competition?

3. **Toss three coins simultaneously, and then answer the following questions based on the result.**

   (a) What is the probability of turning all heads?

   (b) What is the probability of turning a head and two tails?

   (c) What is the probability of turning two heads and a tail?

   (d) What is the probability of turning all tails?

4. **Write the sample space in rolling a dice and find the following probability:**

   a) turning 5 face

   b) turning even number face

   c) turning odd number face

5. **The following tables shows the result of throwing a dice 60 times:**

Event (E)	1	2	3	4	5	6
Frequency (f)	8	9	13	15	11	4

Find the following empirical probability:

(a) turning 4

(b) turning more than 5

(c) turning less than 3

(d) turning the sum of 7

6. 58 baby girls out of 100 children were bornt in a hospital. Then, find the empirical probability of borning baby boy.

Project work

Work in paris and do the following task:

(a) Put number balls numbered from 1 to 9 in a pot like as a given figure.

(b) Draw a ball one by one respectively and replace it; and shake well

(c) Repeat the above task in 100 times.

(d) Complete the following table using tally bar from the above result.

Number	...	...	...	...	...	...	...	...	...
Tally bar									
Frequency									

(e) Find the probability of turning 1 to 9 and present the result in your class.

Answer

1. $\frac{14}{25}$ 2. $\frac{5}{7}$ 3 - 4. Show the answers to your teacher

5. (a) $\frac{1}{4}$ (b) $\frac{1}{15}$ (c) $\frac{17}{60}$ (d) $\frac{21}{50}$

Mixed Exercise

1. **Construct a continuous data having 10/10 class interval of the following raw data:**

   50, 20, 65, 52, 41, 42, 70, 27, 59, 50, 39, 43, 33, 63, 60, 47, 44, 64, 32, 51, 57, 18, 55, 38, 54, 22, 31, 68, 26, 24, 37, 40, 66, 28, 35, 14, 43, 36, 34, 12

Now, from the table

   (a) Draw histogram.

   (b) Draw frequency polygon.

   (c) Sketch less than and more than cumulative frequency ogive in the same graph.

2. **Construct a frequency table having 5/5 class interval of the following raw data:**

   45, 20, 23, 49, 47, 27, 29, 30, 32, 25, 34, 47, 45, 33, 33, 32, 37, 44, 42, 31, 42, 40, 35, 39, 26, 35, 27, 38, 34, 37

From the frequency table,

   (a) Draw histogram.

   (b) Draw frequency polygon.

   (c) Sketch less than and more than cumulative frequency ogive in the same graph.

3. **Answer the following questions:**

   (a) Write any 5 data having median 15.

   (b) Write any 5 data having both mean and median 15.

4. **Find the arithmetic mean, median, mode, first quartile (Q_1) and third quartile (Q_3) from the following data:**

(a)

X	35	40	45	50	55	60	65
f	2	9	10	9	6	3	1

(b)

Age (Yrs)	5	15	25	35	45	55
Number of persons	8	10	12	18	12	4

(c)

Cost (Rs.)	50	60	75	85	90	92
Number of shops	10	4	6	8	7	5

(d)

Marks	5	10	15	20	25	30
Number of students	7	6	8	4	5	10

5. A fair dice having 6 faces is rolled 56 times and found the following result:

1, 6, 5, 6, 4, 3, 4, 4, 3, 5, 1, 1, 6, 6, 5, 4, 3, 2, 2, 2, 6, 5, 1, 4, 3, 1, 5, 4, 6, 4, 3, 2, 2, 3, 1, 1, 1, 1, 6, 6, 2, 1, 3, 6, 4, 1, 5, 6, 6, 1, 3, 5, 5, 4, 6, 3

Present the above data into discrete data and find the following

(a) Find arithmetic mean.

(b) Determine the first quartile, second quartile and third quartile.

(c) Calculate mode and range.

Answer

1-3. Show the answers to your teacher.

4. (a)

$\overline{X} = 47.63$	$M_d = 45$	$M_o = 45$	$Q_1 = 40$	$Q_3 = 53.75$

(b)

$\overline{X} = 29.38$	$M_d = 35$	$M_o = 35$	$Q_1 = 15$	$Q_3 = 42.5$

(c)

$\overline{X} = 74$	$M_d = 80$	$M_o = 50$	$Q_1 = 52.5$	$Q_3 = 90$

(d)

$\overline{X} = 18$	$M_d = 15$	$M_o = 30$	$Q_1 = 10$	$Q_3 = 28.75$

5. (a) $\overline{X} = 3.55$ (b) $Q_1 = 2, M_d = 4, Q_3 = 5$ (c) $M_o = 1 / 6$

Lesson 20 Trigonometry

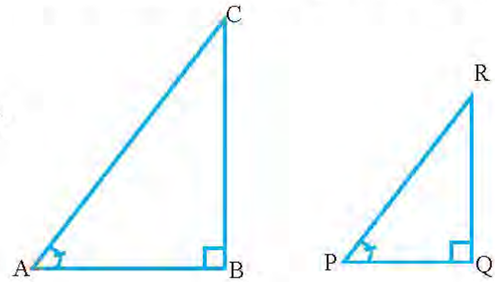
20.0 Review

Divide the students in suitable groups. Observe the given triangle ΔABC and ΔPQR; and measure all the sides and angles. Then, discuss within your group based on the following questions:

Which angles are equal?

Are the measurements of sides equal?

Are the ΔABC and ΔPQR similar?



Find the ratios of the sides of triangle ABC $\frac{AB}{BC}, \frac{BC}{AC}$ and $\frac{AB}{AC}$

Find the ratios of the sides of triangle PQR: $\frac{PQ}{QR}, \frac{QR}{PR}$, and $\frac{PQ}{PR}$

Now, write the ratios of right angled triangles ΔABC and ΔPQR based on perpendicular, base and hypotenuse.

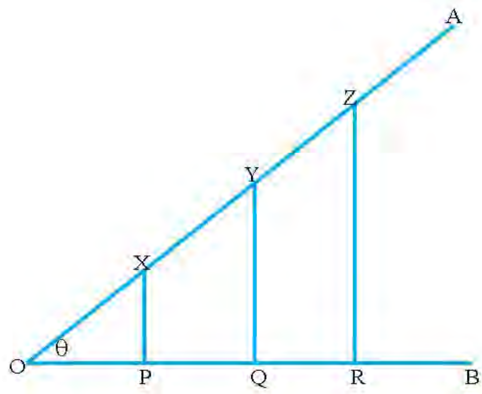
20.1 Trigonometric ratios

Activity 1

Draw a acute angle ∠AOB and name it as θ

Mark three points P, Q and R on line OB in a certain distance.

Draw perpendicular from points P, Q and R on OB using set square which meet at points X, Y and Z respectively of line OA.



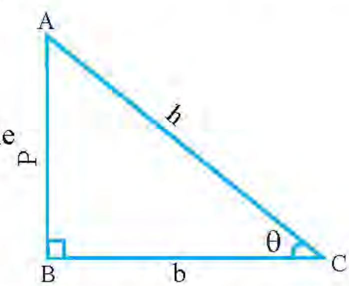
Now, measure the following parts and find the ratios of the following:

(a) $\frac{XP}{OX}, \frac{YQ}{OY}$ and $\frac{ZR}{OZ}$ (b) $\frac{OP}{OX}, \frac{OQ}{OY}$ and $\frac{OR}{OZ}$ (c) $\frac{XP}{OP}, \frac{YQ}{OQ} / \frac{ZR}{OR}$

Discuss in your group and present the conclusion in the class.

The above ratios are the trigonometric ratios. Trigonometry studies the relationship between sides and angles. Trigonometry is used to find the height, length and angles of different unusual situations. It is impossible to develop and expand the mathematics, physics and engineering without trigonometrical ratios. Thus, trigonometry is the most important and indispensible contents of mathematics and science.

In the given right angled triangle ABC, $\angle ACB = \theta$ be a reference angle, AB = perpendicular (p), BC = base (b), AC = hypotenuse (h)



Now, in right angled triangle ABC,

$\frac{AB}{AC} = \frac{p}{h} = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{\text{opposite side}}{\text{hypotenuse}}$ is called the sine ratio of reference angle θ.

In short, it is written as sin θ.

$\frac{AB}{BC} = \frac{b}{h} = \frac{\text{base}}{\text{hypotenuse}} = \frac{\text{adjacent side}}{\text{hypotenuse}}$ is called the cosine ratio of reference angle θ.

In short, it is written as cos θ.

$\frac{AB}{BC} = \frac{p}{b} = \frac{\text{perpendicular}}{\text{base}} = \frac{\text{opposite side}}{\text{adjacent side}}$ is called the tangent ratio of reference angle θ.

In short, it is written as tan θ.

In this way, the three ratios $\sin\theta = \frac{p}{h}$, $\cos\theta = \frac{b}{h}$, $\tan\theta = \frac{p}{b}$ are called the fundamental trigonometric ratios.

Write the opposite trigonometric ratios of the above ratios.

$$\frac{1}{\sin\theta} = \frac{\mathrm{h}}{\mathrm{p}} = \theta \text{ cosecant} = \text{cosec}\theta$$

$$\frac{1}{\cos\theta} = \frac{\mathrm{h}}{\mathrm{b}} = \theta \text{ secant} = \sec\theta$$

$$\frac{1}{\tan\theta} = \frac{\mathrm{b}}{\mathrm{p}} = \theta \text{ cotangent} = \cot\theta$$

Generally, angle is denoted by Greek letters. Some of them are as follows:

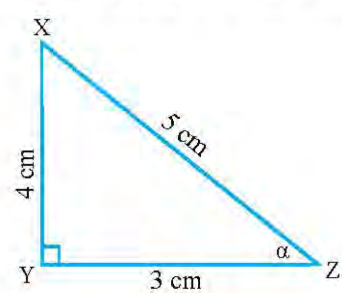
Letters	English name
α	Alpha
β	Beta
γ	Gamma
δ	Delta
θ	theta
ϕ	phi
π	pi
ψ	shi
λ	lambda

Example 1

Write the 6 trigonometric ratios from the given triangle.

Solution:

In right angled triangle XYZ, perpendicular (p) = 4 cm, base (b) = 4 cm and hypotenuse (h) = 4 cm.



Reference angle ∠xyz = α

$\sin\alpha = \frac{xy}{xz} = \frac{p}{h} = \frac{4}{5}$ $\qquad$ $\text{cosec}\,\alpha = \frac{xz}{xy} = \frac{h}{p} = \frac{5}{4}$

$\cos\alpha = \frac{yz}{xz} = \frac{b}{h} = \frac{3}{5}$ $\qquad$ $\sec\alpha = \frac{xz}{yz} = \frac{h}{b} = \frac{5}{3}$

$\tan\alpha = \frac{xy}{yz} = \frac{p}{b} = \frac{4}{3}$ $\cot\alpha = \frac{yz}{xy} = \frac{3}{4}$

Example 2

From the given right-angled triangles ΔCED and ΔABC, find the trigonometric ratios: sin, cos and tan.

Solution:

In right angled triangle ΔCED

$\sin\theta = \frac{DE}{DC} = \frac{p}{h} = \frac{3}{5}$

$\cos\theta = \frac{EC}{DC} = \frac{b}{h} = \frac{4}{5}$

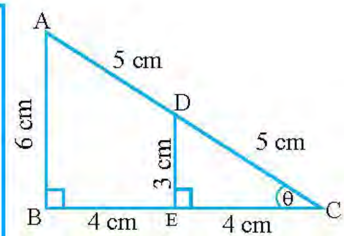
$\tan\theta = \frac{DE}{EC} = \frac{p}{b} = \frac{3}{4}$

In right angled triangle ΔCED

$\sin\theta = \frac{AB}{AC} = \frac{p}{h} = \frac{6}{10} = \frac{3}{5}$

$\cos\theta = \frac{BC}{AC} = \frac{b}{h} = \frac{8}{10} = \frac{4}{5}$

$\tan\theta = \frac{AB}{BC} = \frac{p}{b} = \frac{6}{8} = \frac{3}{4}$



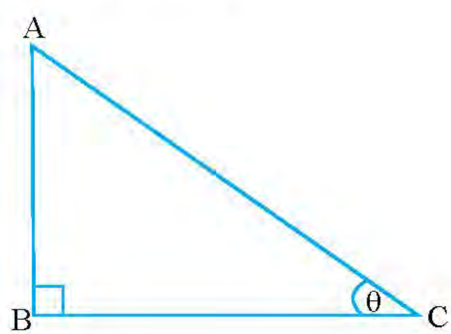
Activity 2

Draw a right angled triangle ABC.

Taking ∠BCA = θ as a reference angle, write a Pythagorian relation.

$AB^2 + BC^2 = AC^2$.............. (i)

Dividing both sides by AC^2



$\frac{AB^2}{AC^2} + \frac{BC^2}{AC^2} = \frac{AC^2}{AC^2}$

Or $\left(\frac{AB}{AC}\right)^2 + \left(\frac{BC}{AC}\right)^2 = \left(\frac{AC}{AC}\right)^2$

Or $\left(\frac{p}{h}\right)^2 + \left(\frac{b}{h}\right)^2 = 1$

or, $(\sin\theta)^2 + (\cos\theta)^2 = 1$

or, $\sin^2\theta + \cos^2\theta = 1$

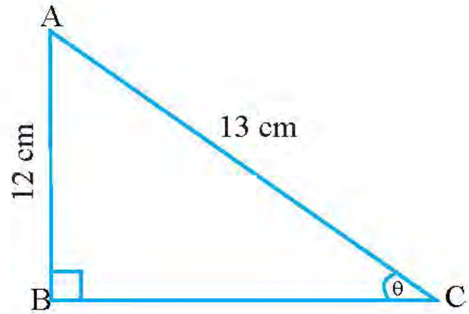
Similarly, divide the equation (i) by AB^2 and BC^2.

Discuss the result in the class.

Example 3

In right angled triangle ABC, $\angle B = 90°$, $\angle C = \theta$, AB = 12 cm, AC = 13 cm, then prove that $\sin^2\theta + \cos^2\theta = 1$

Solution



Here, perpendicular (AB) = p = 12 cm, hypotenuse (AC) = h = 13 cm, base (BC) = b =?

By Pythagoras theorem,

$h^2 = p^2 + b^2$

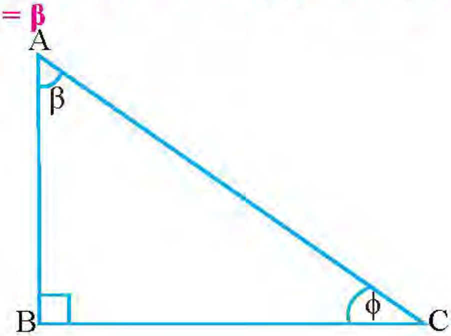
or, $13^2 = 12^2 + b^2$

or, $b^2 = 169 - 144 = 25$ Thus, b = 5

Now, $\sin^2\theta + \cos^2\theta = \frac{p^2}{h^2} + \frac{b^2}{h^2} = \frac{(12)^2}{(13)^2} + \frac{(5)^2}{(13)^2} = \frac{169}{169} = 1$

Exercise 20.1

1. In a right angled triangle ABC, answer the following questions as reference angle $\angle ACB = \phi$ and $\angle BAC = \beta$



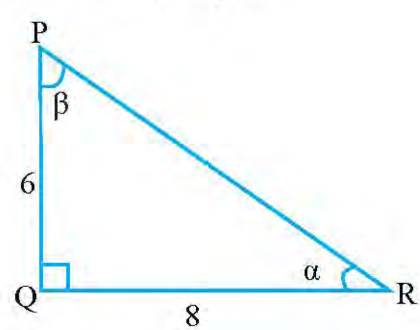
(a) Determine the perpendicular, base and hypotenuse taking ϕ as a reference angle.

(b) Find the 6 trigonometric ratios taking ϕ as a reference angle.

(c) Determine the perpendicular, base and hypotenuse taking β as a reference angle.

(d) Find the 6 trigonometric ratios taken α as a reference angle.

2. In right angled triangle PQR,



(a) Find the value of $\sin\beta$, $\cos\beta$ and $\tan\beta$.

(b) Find the value of $\operatorname{cosec}\beta$, $\sec\beta$ and $\cot\beta$.

(c) Find the value of $\sin\alpha$, $\cos\alpha$, $\tan\alpha$, $\operatorname{cosec}\alpha$, $\sec\alpha$ and $\cot\alpha$.

3. (a) If $\sin\theta = \frac{6}{10}$ and $h = 20$, then $b = ?$

(b) If $\tan\theta = \frac{5}{12}$, then find the following

(i) $h = ?$

(ii) Find the value of $\sin\theta$ and $\cos\theta$.

4. Determine the value of the following on the basis of trigonometric ratio:

(a) $\sin^2 A + \cos^2 A$ (b) $1 - \cos^2 A$ (c) $\sin\theta \div \cos\theta$

(d) $\tan^2\theta + 1$ (e) $\cos\theta \div \sin\theta$ (f) $\text{cosec}^2\theta - \cot^2\theta$

5. Prove that the following on the basis of trigonometric ratio:

(a) $\sin^2\theta + \cos^2\theta = 1$ (b) $1 - \sin^2\theta = \cos^2\theta$ (c) $\frac{\sin\theta}{\cos\theta} = \tan\theta$

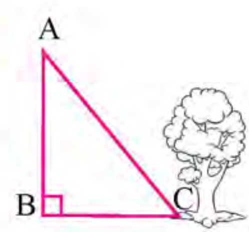
(d) $\sec^2\theta - 1 = \tan^2\theta$ (e) $\text{cosec}^2\theta - \cot^2\theta = 1$

6. A inclined plane having length 29 ft is constructed to carry out the particles to the 20 ft height building, then

(a) Find the distance between the tower and lower part of the inclined plane.

(b) Find the value of $\tan\theta$, $\sin\theta$ and $\cos\theta$, if the angle made by the inclined plane with ground is θ.

7. In the given figure, AC is a length from tree to the top of the lamp post AB, and BC is distance between the lamp post and the tree.



(a) If BC = 24 m, AC = 40 m, then find the height of the lamp post.

(b) Find the value of $\tan\theta$, $\sin\theta$ and $\cos\theta$, if the $\angle ACB$ in the reference angle.

Answer

1 - 2. Show the answer to your teacher.

3. (a) 16 (b) (i) 13 (ii) $\frac{5}{13}, \frac{12}{13}$ (c) $\frac{15}{17}, \frac{8}{17}$

4. (a) 1 (b) $\sin^2 A$ (c) $\tan\theta$ (d) $\sec^2\theta$ (e) $\cot\theta$ (f) 1

5. Show the answer to your teacher. 6. (a) 21 ft (b) $\frac{20}{21}, \frac{20}{29}, \frac{21}{29}$

7. (a) 32 m (b) $\frac{4}{3}, \frac{4}{5}, \frac{3}{5}$

20.2 Trigonometric ratio of special angles

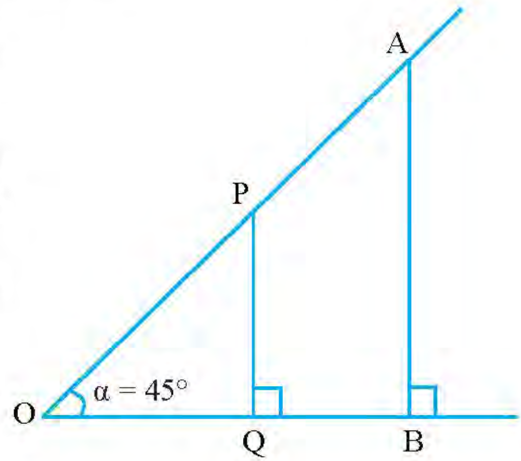
(a) 45° Trigonometric value of angle

As shown in the given figure, draw an angle $\angle AOB = \alpha = 45^\circ$ Measure the following parts of the given figure. $AB \perp OB$ and $PQ \perp OB$

OP = ……, OQ = ……, PQ = ……

OA= ……, OB = ……, AB = ……

Here, ΔOQP and ΔOBA



Now, find separate value of sin 45°, cos 45° and tan 45° from triangles ΔOQP and ΔOBA.

Comparing the different trigonometric ratios in both triangles.

Sin 45° = ...

Cos 45° = ...

And tan 45° = ...

Here $\angle POQ = \angle AOB = 45^\circ$ x'G5 .

Thus, side OQ = PQ = x

Now, from Pythagoras theorem,

$$OP^2 = PQ^2 + OQ^2 = x^2 + x^2 = 2x^2$$

$$OP = \sqrt{2x^2}$$

In ΔOQP

$$\text{Sin } \alpha = \sin 45^\circ = \frac{p}{h} = \frac{PQ}{OP} = \frac{x}{\sqrt{2x^2}} = \frac{1}{\sqrt{2}} = 0.707$$

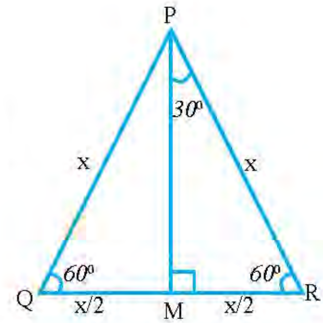
$$\text{Cos } \alpha = \text{Cos} 45^\circ = \frac{b}{h} = \frac{OQ}{OP} = \frac{x}{\sqrt{2x^2}} = \frac{1}{\sqrt{2}} = 0.707$$

$$\tan \alpha = \tan 45^\circ = \frac{p}{b} = \frac{PQ}{OQ} = \frac{x}{x} = 1 = 1.000$$

Again, find the value of cosec cosec 45°, sec 45° and cot 45° on the basis of above values.

(b) Trigonometric value of angle 30° and 60°

Draw an equilateral triangle ΔPQR to find the trigonometric values 30° and 60° where $\angle P = \angle Q = \angle R = 60°$



Thus, sides $PQ = QR = PR = x$ (say)

For finding the values of trigonometric ratios, we need a right angled triangle. Thus, draw line PM on base QR in which PM divides QR into two equal parts.

Here, $QM = \frac{x}{2} = MR$

Now in right angled triangle ΔPMR, $\angle R = 60°$; $\angle M = 90°$ then $\angle MPR = 30°$

By Pythagoras theorem in ΔPMR,

$$(PM)^2 = (PR)^2 - (MR)^2$$

$$(PM)^2 = x^2 - \left(\frac{x}{2}\right)^2 = x^2 - \frac{x^2}{4}$$

$$(PM)^2 = \frac{3x^2}{4}$$

$$PM = \sqrt{\frac{3x^2}{4}} = \frac{\sqrt{3}}{2}x$$

Now, $\angle MPR = 30°$ then $P = MR = \frac{x}{2}$, $h = PR = x$, $b = PM = \frac{\sqrt{3}x}{2}$

$$\sin 30° = \frac{p}{h} = \frac{MR}{PR} = \frac{\frac{x}{2}}{x} = \frac{x}{2} \times \frac{1}{x} = \frac{1}{2} = 0.5$$

$$\cos 30° = \frac{b}{h} = \frac{PM}{PR} = \frac{\frac{\sqrt{3}}{2}x}{x} = \frac{\sqrt{3}}{2} = 0.866$$

$$\tan 30° = \frac{p}{b} = \frac{\frac{x}{2}}{\frac{\sqrt{3}}{2}x} = \frac{x}{2} \times \frac{2}{\sqrt{3}x} = \frac{1}{\sqrt{3}} = 0.577$$

Similarly, find the value of cosec 30°, sec 30° and cot 30° based on above values.

Again in right angled triangle PMR, taking the reference angle $\angle R = 60°$

$$\sin 60° = \frac{p}{h} = \frac{PM}{PR} = \frac{\frac{\sqrt{3}}{2}x}{x} = \frac{\sqrt{3}x}{2} \times \frac{1}{x} = \frac{\sqrt{3}}{2}$$

$$\cos 60° = \frac{b}{h} = \frac{\frac{x}{2}}{x} = \frac{x}{2} \times \frac{1}{x} = \frac{1}{2} = 0.5$$

$$\tan 60° = \frac{p}{b} = \frac{\sqrt{3}x}{2} \times \frac{1}{x} = \sqrt{3} = 1.732$$

Similarly, find the value of cosec 60°, sec 60° and cot 60° on the basis of above values. cosec 60°.

Note: In this way, $\sin 30° = \frac{1}{2} = \cos 60° = \cos(90° - 30°)$

$\therefore \sin\theta = \cos(90° - \theta)$

Similarly, $\cos 30° = \frac{\sqrt{3}}{2} = \sin 60° = \sin(90° - 60°)$

$\cos\theta = \sin(90° - \theta)$

(c) Trigonometric value of angle 0° and 90°

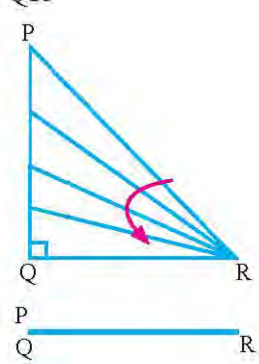
Angled triangle PQR, if we rotate the side RP, then the length of side PQ being continuously decreaseds and at last the point Poverlaps with point Q, and thus, the line PR and QR completely overlap to each other.

In this situation, $\angle PRQ$ is going to decrease at PQ = 0, PR = QR

$$\sin 0° = \frac{PQ}{PR} = \frac{0}{x} = 0$$

$$\cos 0° = \frac{QR}{PR} = \frac{x}{x} = 1$$

$$\tan 0° = \frac{PQ}{QR} = \frac{0}{x} = 0$$



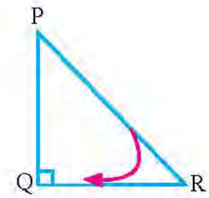
Similarly in triangle PQR, if we increase $\angle$PRQ to 90°, i. e. $\angle$PRQ = 90°, then PQ = PR.

That is, PQ = x implies PR = x and QR = 0.

$$\sin 90^\circ = \frac{PQ}{PR} = \frac{x}{x} = 1$$

$$\cos 90^\circ = \frac{QR}{PR} = \frac{0}{x} = 0$$

$$\tan 90^\circ = \frac{PQ}{PR} = \frac{x}{0} \ (\infty, \text{undefined})$$



Activity 3

Trigonometric values of standard angles in unit circle

Draw a circle with centre O using compass having radius 1 inch.

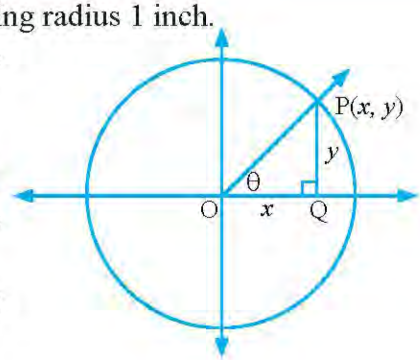
Take a point P(x, y) in the circumference of the circle.

Rotate line OP in anti-clockwise direction.

Draw perpendicular PQ on x-axis from point P.

Now in right angled triangle OPQ, write the trigonometric

ratios of sin θ, cos θ and tan θ taking <POQ = θ as a reference angle.



$$\sin\theta = \frac{y}{OP}, \ \cos\theta = \frac{x}{OP}, \text{र} \ \tan\theta = \frac{y}{x}$$

Now, find the value of trigonometric ratios making angles 30°, 45°, 60°, 90° with x-axis on rotating the line OP in anti-clockwise direction.

The values of different trigonometric ratios of standard angles: like sin 0°, sin 30°, sin 45°, sin 60°, sin 90°, ……….. are given in the following table:

Angle Ratios	0°	30°	45°	60°	90°
	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞, infinite

Note: Methods of remembering the value of standard angles of trigonometric ratios

Write numbers from 0 to 4 respectively. Divide all the numbers by 4. And find the square root of the entire fraction. Then obtained values are the values of sin 0°, sin 30°, sin 45°, sin 60° and sin 90°.

Example 1

Find the values of the following trigonometric ratios:

(a) $\sin 45^\circ + \cos 45^\circ$

(b) $\dfrac{\cos 30^\circ}{\sin 30^\circ}$

Solution:

(a) oxfF $\sin 45^\circ + \cos 45^\circ = \dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{2}} = \dfrac{2}{\sqrt{2}} = \dfrac{\sqrt{2}\times\sqrt{2}}{\sqrt{2}} = \sqrt{2}$

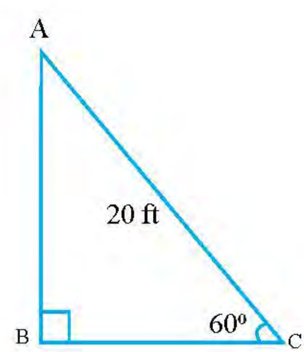
(b) $\dfrac{\cos 30^\circ}{\sin 30^\circ} = \dfrac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$

Example 2

From the given right angled triangle, find the perpendicular and base if reference angle is 60° and length of hypotenuse is 20 ft:

Solution:

Given, in right angled triangle ABC, reference angle $\angle C = \theta$, i. e. $\theta = 60^\circ$ and hypotenuse AC (h) = 20 ft, base (b) =? and perpendicular (p) =?



We know that, $\sin\theta = \dfrac{p}{h}$

or, $\sin 60^\circ = \dfrac{p}{20}$

or, $\dfrac{\sqrt{3}}{2} = \dfrac{p}{20}$

or, $p = 10\sqrt{3}$ ft

Again, $h^2 = p^2 + b^2$

$b^2 = h^2 - p^2$

$= 20^2 - \left(10\sqrt{3}\right)^2$

$= 400 - 300$

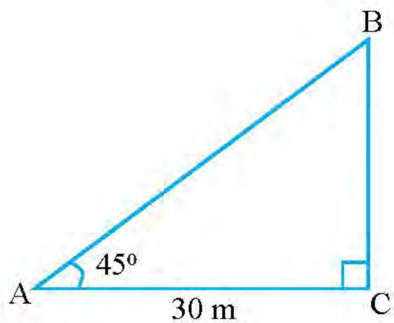
$= 100$

$b = \sqrt{100} = \sqrt{10 \times 10} = 10$ ft

Example 3

A straight tree of 30 m height is broken so that its top touches the ground and makes an angle 45°. Find the original height of the tree.

Solution:



Here, reference angle (θ) = 45°

Base (b) = 30 m

Original height of tree (H) =?

We know that, $\cos\theta = \dfrac{b}{h}$ and $\tan\theta = \dfrac{p}{b}$

or, $\cos 45° = \dfrac{30}{h}$ and $\tan 45° = \dfrac{p}{30}$

or, $\dfrac{1}{\sqrt{2}} = \dfrac{30}{h}$ and $1 = \dfrac{p}{30}$

or, $h = 30\sqrt{2}$ and $p = 30$

Thus, the original height of the tree (H) $= h + p = 30\sqrt{2} + 30 = 30(\sqrt{2} + 1)$

$= 30 \times (1.4142 + 1)$

$= 30 \times 2.4142$

$= 72.426$ m

Exercise 20.3

1. Find the value of the following trigonometric ratios:

(a) $\sin 90°$ (b) $\cos 60°$ (c) $\tan 30°$

(d) $\cot 45°$ (e) $\text{cosec } 45°$

2. Find the value of reference angles of the given right angled triangles from the following problems:

(a) $p = 4$ cm and $b = 4\sqrt{3}$ cm (b) $p = 7\sqrt{3}$ ft and $h = 14$ ft

(c) $b = 5$ cm and $h^2 = 50$ cm^2 (d) $p = x$ inch and $b = x$ inch

3. Find the remaining side of the given right angled triangles:

(a) Length of hypotenuse 10 cm and reference angle is 60°.

(b) Length of perpendicular 10 cm and reference angle is 45°

(c) Length of base 36 inch and reference angle is 30°.

4. Find the value of the following trigonometric expressions:

(a) $\sin30° + \cos60°$ (b) $\tan30° + \sin60°$

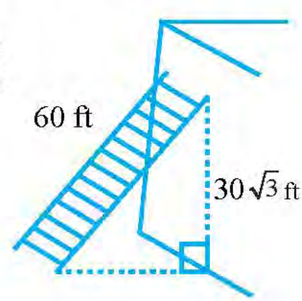
(c) $\sin30° + \cos30° \times \tan30°$ (d) $\dfrac{\sin60° + \cos30°}{\tan60°}$

5. Prove that:

(a) $\cos^2 45° + \sin^2 45° = 1$ (b) $\dfrac{2\tan30°}{1 - \tan^2 30°} = \tan60°$

(c) $\dfrac{\tan60° - \tan30°}{1 + \tan60° \tan30°} = \tan30°$ (d) $\tan 45° - \sin90° = 1 - 2\sin^2 45°$

6. (a) Find the angle formed by the ladder with the ground and the wall respectively in the given figure.

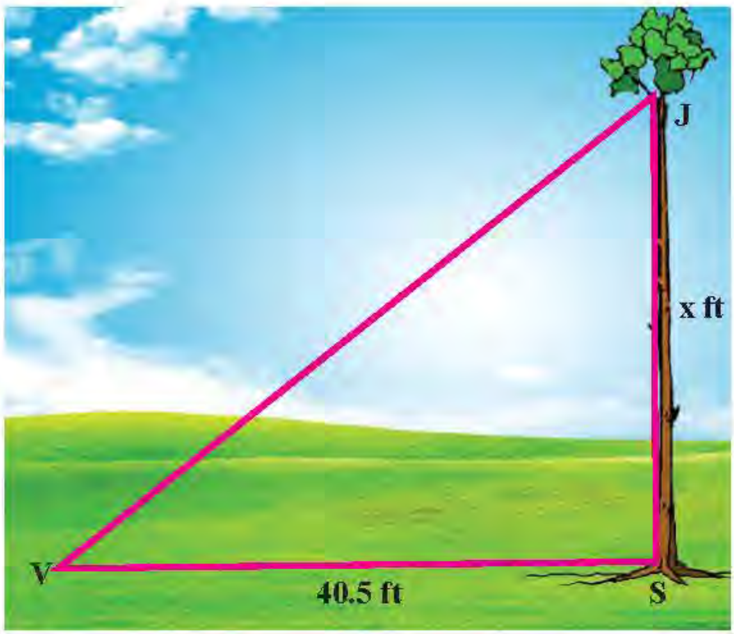


(b) Find the angle of the top of a clock tower of height 36 m observed from a point $12\sqrt{3}$ m away.

(c) In the given figure,

(i) What may be the value of x, if both angles are of acute angle 45°?

(ii) What is the measurement of VJ?



(d) A ladder is supported by a wall of 12 ft height and makes an angle of 60° with a ground. Find the length of the ladder.

Project work

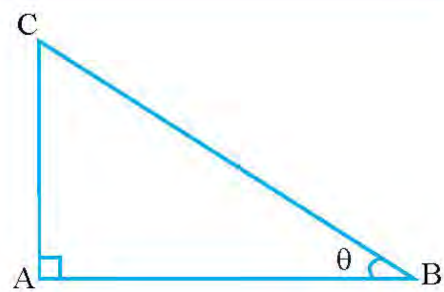
Take a long thread. Tie one end of the thread to a pole standing on the ground. Then fix another end of thread in different places of the ground that makes the angles 30°, 45°, 60° and 90°. Find different trigonometric ratios: sin, cos and tan based on the distance between pole and the thread tied on the ground as well length of the thread.

Answer

1. (a) 1 (b) $\frac{1}{2}$ (c) $1/\sqrt{3}$ (d) 1 (e) $\sqrt{2}$
2. (a) 30° (b) 60° (c) 45° (d) 90°
3. (a) $p = 5\sqrt{3}$, $b = 5$ (c) $b = 15$, $h = 15\sqrt{2}$ (d) $p = 12\sqrt{3}$, h $=24\sqrt{3}$
4. (a) 1 (b) $5/2\sqrt{3}$ (c) 1 (d) 1
5. Show the answer to your teacher.
6. (a) 60° and 30° (b) 30° (c) (i) 40.5 ft (ii) 57.28 ft (d) $8\sqrt{3}$

Mixed Exercise

1. Answer the following questions based on the given figure:



(a) Determine perpendicular, base and hypotenuse taking as reference angle ∠B.

(b) Find the 6 trigonometric ratios based on (a).

(c) If $h = 20$ and $b = 12$, then find the value of tan B.

(d) Find the value of $\sin^2$ B, $\cos^2$ B and $\tan^2$ B on the basis of (c).

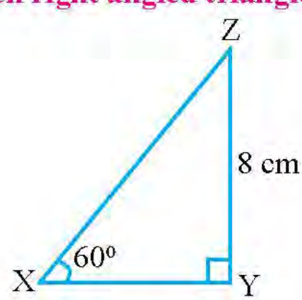
2. Answer the following questions based on the given right angled triangle:

(a) Find the value of XY and ZX.

(b) Prove that: $\sin^2 X + \cos^2 X = 1$.

(c) Find the value of sin Z, cos Z and tan Z.

(d) Find the value of $\sin^2$ B, $\cos^2$ B and $\tan^2$ B on the basis of (c).



3. (a) Top of a bamboo tree makes an angle of 30° from a distance 30 ft of ground level, and then what is the height of the bamboo tree?

(b) How long distance should be travelled from the bamboo tree to make an angle of 45°?

(c) If the bamboo tree grows up to $30\sqrt{3}$ ft after 4 days, then what angle does the top of the tree from the initial point make?

Answer

1-2. Show the answers to your teacher.	3. (a) $10\sqrt{3}$ ft
(b) 12.68 ft	(c) 60°