Lesson 7

Quadratic Equation

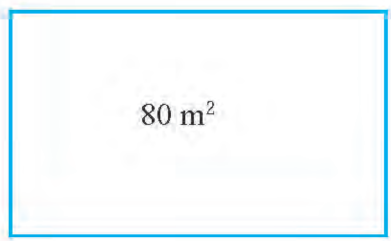
7.0 Review

A school is carpeting its office room which is rectangular in shape and the amount of the carpet used is 80 m^2 . Discuss in pair and answer the following questions.

(a) What is the length and breadth of the office room?

(b) What is the length and breadth of the room if the length is greater than its breadth by 2 meter?

If the breadth of the room is x, then its length will be = $x + 2$



Area of the room = 80 m^2

$(x+2)x = 80$

or, $x^2 + 2x - 80 = 0$

or, $x^2 + 10x - 8x - 80 = 0$

or, $x(x+10) - 8(x+10) = 0$

or, $(x+10)\ (x-8) = 0$

either $x + 10 = 0$ $\quad\therefore x = -10$, which is not possible.

or, $x - 8 = 0$ $\quad\therefore\ x = 8$

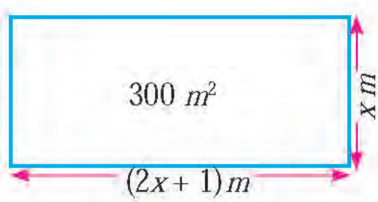
Here, the length of the room (l) = $x + 2 = 8 + 2 = 10\ m$, breath of the room (b) = x = 8m

A quadratic equation is a second degree equation of one variable. It is in the form of $ax^2 + bx + c = 0$, where a $\neq$ 0. There are two values of the variable satisfying the equation.

7.1 Solving Quadratic Equation

(a) Factorization Method

Activity 1



The area of a rectangular playground is 300 m^2. If its length is greater than breadth by 1 m, discuss in your group and find its length and breadth.

Here, area of a playground = 300 m^2

If breadth of the playground $= x$

then, length of playground $= 2x + 1$

Now, area of rectangular playground = length × breadth

$300 = (2x + 1)\, x$

or, $2x^2 + x - 300 = 0$ [∵ This is quadratic equation.]

To find the value of x from above quadratic equation,

$2x^2 + (25 - 24)x - 300 = 0$

or, $2x^2 + 25x - 24x - 300 = 0$

or, $x(2x + 25) - 12(2x + 25) = 0$

or, $(2x + 25)\ (x - 12) = 0$

If multiplication of two factors is zero, then one factor should be zero.

either $(2x + 25) = 0$ or $(x - 12) = 0)$

If $2x + 25 = 0$

then, $2x = -25$

$x = -\frac{25}{2}$ is impossible

if, $x - 12 = 0$

or, $x = 12$ $\therefore x = 12$

Breadth of the playground $(x) = 12\ m$, then length $= 2x + 1 = 2 \times 12 + 1 = 25\ m$

Example 1

Solve the equation and examine whether it is correct or not.

(a) $x^2 + 4x = 0$ (b) $x^2 + 6x + 8 = 0$ (c) $x^2 - 5x + 6 = 0$

(d) $x^2 - x - 6 = 0$ (e) $2x^2 + 7x + 6 = 0$

Solution

(a) $x^2 + 4x = 0$

or, $x(x + 4) = 0$

either $x = 0$

or, $x + 4 = 0$

$x = -4$

Thus, $x = 0, -4$

When examined,

$x = 0$ is putting in $x^2 + 4x = 0$

LHS $= 0 + 4 \times 0 = 0 =$ RHS

When placed $x = -4$

LHS $= (-4)^2 - 4 \times (-4) = 16 - 16 = 0 =$ RHS

(b) $x^2 + 6x + 8 = 0$

or, $x^2 + (4 + 2)x + 8 = 0$

or, $x^2 + 4x + 2x + 8 = 0$

or, $x(x + 4) + 2(x + 4) = 0$

or, $(x + 4)(x + 2) = 0$

either $x + 4 = 0$ $\quad \therefore x = -4$

or, $(x + 2) = 0$ $\quad \therefore x = -2$

When examined,

$x = -2$ putting $x^2 + 6x + 8 = 0$

LHS $= (-2)^2 + 6 \times (-2) + 80$

$4 - 12 + 8 = 0 =$ RHS

$x = -4$ is placed in $x^2 + 6x + 8 = 0$

LHS $= (-4)^2 + 6 \times (-4) + 8$

$= 16 - 24 + 8 = 0 =$ RHS

$\therefore$ The roots of quadratic equations are $-2, -4$

(c) $x^2 - 5x + 6 = 0$

or, $x^2 - (3 + 2)x + 6 = 0$

or, $x^2 - (3 + 2)x + 6 = 0$

or, $x^2 - 3x - 2x + 6 = 0$

or, $x(x - 3) -2(x - 3)\ 0$

or, $(x - 3)(x - 2) = 0$

either, $(x - 3) = 0$ $\quad \therefore x = 3$

or, $x - 2 = 0$ $\quad \therefore x = 2$

When examined,

Putting $x = 2$

$x^2 - 5x + 6 = (2)^2 - 5 \times 2 + 6$

$= 4 - 10 + 6 = 0$

LHS = RHS

Again, when placed $x = 3$,

$(3)^2 - 5 \times 3 + 6 = 9 - 15 + 6 = 0$

LHS = RHS

$\therefore$ The roots of quadratic equation are 2 and 3.

(d) $x^2 - x - 6 = 0$

or, $x^2 - (3 - 2)x - 6 = 0$

or, $x^2 - 3x + 2x - 6 = 0$

or, $x(x - 3) + 2(x - 3) = 0$

or, $(x - 3)(x + 2) = 0$

either, $(x - 3) = 0$ $\quad \therefore x = 3$

or, $x + 2 = 0$ $\quad \therefore x = -2$

When examined,

Putting $x = 3$,

LHS $= (3)^2 - 3 - 6$

$= 9 - 9 = 0 =$ RHS

When placed $x = -2$

LHS $= (-2)^2 - 2 - 6$

$= 4 + 2 - 6 = 0 =$ RHS

$\therefore$ The roots of quadratic equation are 3 and -2.

(e) $2x^2 + 7x + 6 = 0$

or, $2x^2 + 7x + 6 = 0$

or, $2x^2 + (4 + 3)\ x + 6 = 0$

or, $2x^2 + 4x + 3x + 6 = 0$

or, $2x\ (x + 2) + 3(x + 2) = 0$

or, $(x + 2)\ (2x + 3) = 0$

either, $(x + 2) = 0$. $\therefore$ $x = -2$

or, $2x + 3 = 0$. $\therefore$ $x = -\frac{3}{2}$

$\therefore$ The roots of $2x^2 + 7x + 6 = 0$ are -2 and $-\frac{3}{2}$.

When examined

Putting $x = -2$

LHS $= 2(-2)^2 + 7 \times (-2) + 6$

$= 8 - 14 + 6 = 0 =$ RHS

Putting $x = -\frac{3}{2}$,

LHS $= 2 \times \left(-\frac{3}{2}\right)^2 + 7 \times -\frac{3}{2} + 6$

$= \frac{9}{2} - \frac{21}{2} + 6 = \frac{21 - 21}{2}$

$= 0 =$ RHS

(b) Solving quadratic equation by completing square

Activity 2

Solve the given quadratic equation

(a) $x^2 - 9 = 0$ (b) $x^2 - 5x + 6 = 0$

Solution

(a) $x^2 - 9 = 0$

or, $x^2 - 3^2 = 0$

or, $(x + 3)\ (x - 3) = 0$

Either $x + 3 = 0$ $\therefore$ $x = -3$

or, $x - 3 = 0$ $\therefore$ $x = 3$

$\therefore$ $x = \pm 3$

We can also solve in this way.

$x^2 - 9 = 0$

or, $x^2 = 9$

or, $x^2 = 3^2$ [$\because$ Here x^2 and 9 both are square.]

or, $x = \pm 3$

The roots of $x^2 = a^2$ form of quadratic equation is $x \pm a$.

(b) $x^2 - 5x + 6 = 0$

or, $x^2 - 5x = -6$

or, $x^2 - 2\frac{5}{2}x + \left(\frac{5}{2}\right)^2 = \left(\frac{5}{2}\right)^2 - 6$ [$\because (a-b)^2 = a^2 - 2ab + b^2$]

or, $\left(x - \frac{5}{2}\right)^2 = \frac{25}{4} - 6 = \frac{25-24}{4} = \frac{1}{4}$

or, $\left(x - \frac{5}{2}\right)^2 = \frac{1}{4}$

or, $\left(x - \frac{5}{2}\right)^2 = \left(\frac{1}{2}\right)^2$

$\therefore x - \frac{5}{2} = \pm\frac{1}{2}$

Taking (+)ve sign,

$x - \frac{5}{2} = \frac{1}{2}$ or, $x = \frac{1}{2} + \frac{5}{2} = \frac{6}{2} = 3$

Taking (-)ve sign,

$x - \frac{5}{2} = -\frac{1}{2}$

or, $x = \frac{5}{2} - \frac{1}{2} = \frac{4}{2} = 2$

Therefore, the roots of x is 2 and 3.

Example 2

Solve by completing the square

(a) $x^2 - 10x + 16 = 0$ (b) $x^2 - 7x + 12 = 0$ (c) $2x^2 - 7x + 6 = 0$

Solution

(a) $x^2 - 10x + 16 = 0$

or, $x^2 - 2 \times x \times 5 + (5)^2 - (5)^2 + 16 = 0$ [$\because (a-b)^2 = a^2 - 2ab + b^2$]

or, $x^2 - 2 \times x \times 5 + (5)^2 - 25 + 16 = 0$

or, $(x-5)^2 - 9 = 0$

or, $(x-5)^2 = 9$

or, $(x-5)^2 = 3^2$

or, $x - 5 = \pm 3$

Taking (+)ve sign,

$x - 5 = 3$ or, $x = 3 + 5 = 8$

Taking (-)ve sign,

$x - 5 = -3$ or, $x = 5 - 3 = 2$

$\therefore x = 8, 2$

(b) $x^2 - 7x + 12 = 0$

or, $x^2 - 2.\frac{7}{2}x + \left(\frac{7}{2}\right)^2 + 12 - \left(\frac{7}{2}\right)^2 = 0$

or, $\left(x - \frac{7}{2}\right)^2 + 12 - \frac{49}{4} = 0$

or, $\left(x - \frac{7}{2}\right)^2 + \frac{48 - 49}{4} = 0$

or, $\left(x - \frac{7}{2}\right)^2 + \left(\frac{-1}{4}\right) = 0$

or, $\left(x - \frac{7}{2}\right)^2 = \left(\frac{1}{2}\right)^2$

or, $\left(x - \frac{7}{2}\right) = \pm\frac{1}{2}$

Taking (+)ve sign,

$x - \frac{7}{2} = \frac{1}{2}$ or, $x = \frac{7}{2} + \frac{1}{2} = \frac{8}{2} = 4$

Taking (-)ve sign,

$x - \frac{7}{2} = -\frac{1}{2}$ or, $x = \frac{7}{2} - \frac{1}{2} = \frac{6}{2} = 3$

$\therefore$ x = 4, 3

(c) $2x^2 - 7x + 6 = 0$

or, $2x^2 - 7x + 6 = 0$

or, $2(x^2 - \frac{7}{2}x + 3) = 0$

or, $x^2 - \frac{7}{2}x + 3 = 0$

or, $x^2 - 2 \times x \times \frac{7}{4} + \left(\frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2 + 3 = 0$

or, $x^2 - 2 \times x \times \frac{7}{4} + \left(\frac{7}{4}\right)^2 + 3 - \left(\frac{49}{16}\right) = 0$

or, $\left(x - \frac{7}{4}\right)^2 - \frac{1}{16} = 0$

or, $\left(x - \frac{7}{4}\right)^2 = \frac{1}{16}$

or, $\left(x - \frac{7}{4}\right)^2 = \left(\pm\frac{1}{4}\right)^2$

or, $\left(x - \frac{7}{4}\right) = \pm\frac{1}{4}$

Taking (+)ve sign,

$x - \frac{7}{4} = \frac{1}{4}$ or, $x = \frac{7}{4} + \frac{1}{4} = \frac{8}{4} = 2$

Taking (-)ve sign, $x - \frac{7}{4} = -\frac{1}{4}$

or, $x = \frac{7}{4} - \frac{1}{4} = \frac{6}{4} = \frac{3}{2}$

$\therefore x = 2, \frac{3}{2}$

(C) Solving quadratic equation by using formula

Activity 3

How shall we find the value of quadratic equation $ax^2 + bx + c = 0$?

Here, $ax^2 + bx + c = 0$

or, $ax^2 + bx = -c$

or, $\frac{ax^2 + bx}{a} = -\frac{c}{a}$ [$\therefore$ dividing both sides by a]

or, $x^2 + \frac{bx}{a} = -\frac{c}{a}$

or, $x^2 + 2 \times x\frac{b}{2a} + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 = -\frac{c}{a}$ [$\therefore$ completing the square]

or, $\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$

or, $\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4xa^2}$

or, $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$

or, $\left(x + \frac{b}{2a}\right)^2 = \left(\frac{b^2 - 4ac}{4a^2}\right)$

or, $x + \frac{b}{2a} = \pm\sqrt{\left(\frac{b^2 - 4ac}{4a^2}\right)}$

or, $x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$

or, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Therefore, the roots of x are $\frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$

Example 3

Solve the given quadratic equations by using formula.

(a) $x^2 - 5x + 6 = 0$ (b) $x\left(x - \frac{2}{7}\right) = \frac{3}{49}$

Solution

(a) Here, comparing $x^2 - 5x + 6 = 0$ to $ax^2 + bx + c = 0$, we get

$a = 1,\ b = -5,\ c = 6$

We know that,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 1 \times 6}}{2 \times 1}$$

$$= \frac{5 \pm \sqrt{25 - 24}}{2}$$

$$= \frac{5 \pm 1}{2}$$

Taking (+)ve sign, $x = \frac{5+1}{2} = \frac{6}{2} = 3$

Taking (-)ve sign, $x = \frac{5-1}{2} = \frac{4}{2} = 2$

Therefore, roots of x are 3 and 2.

(b) $x\left(x - \frac{2}{7}\right) = \frac{3}{49}$

Here, $x\left(x - \frac{2}{7}\right) = \frac{3}{49}$

or, $x^2 - \frac{2}{7}x - \frac{3}{49} = 0$

or, $49x^2 - 14x - 3 = 0$ Comparing $49x^2$ - 14x - 3 = 0 with

$ax^2 + bx + c = 0$ we get

$a = 49,\ b = -14,\ c = -3$

We know that,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-14) \pm \sqrt{(-14)^2 - 4 \times 49 \times (-3)}}{2 \times 49}$$

$$= \frac{14 \pm \sqrt{196 + 588}}{98}$$

Taking (+)ve sign, $x = \frac{14+28}{98} = \frac{42}{98} = \frac{3}{7}$

Taking (-)ve sign, $x = \frac{14-28}{98} = \frac{-14}{98} = -\frac{1}{7}$

Therefore, roots of x are: $\frac{3}{7}$ and $-\frac{1}{7}$

Exercise 7.1

1. **Which of the following are the quadratic equation? Write with reason.**

   (a) $(x-2)^2+1=2x-3$ (b) $x(x+1)+8=(x+2)(x-2)$

   (c) $x(2x+3)=x^2+1$ (d) $(x+2)^3=x^3-4$

   (e) $x^2+3x+1=(x-2)^2$ (f) $(x+2)^3=2x(x^2-1)$

2. **Solve by factorization method.**

   (a) $x^2-3x-10=0$ (b) $2x^2+x-6=0$ (c) $2x^2-x+\frac{1}{8}=0$

   (d) $100x^2-20x+1=0$ (e) $x^2-45x+324=0$ (f) $x^2-27x-182=0$

3. **Solve by completing square.**

   (a) $x^2-6x+9=0$ (b) $9x^2-15x+6=0$ (c) $2x^2-5x+3=0$

   (d) $5x^2-6x-2=0$ (e) $x^2+\frac{15}{16}=2x$ (f) $x^2+\frac{2}{3}x=\frac{35}{9}$

4. **Solve by using formula.**

   (a) $x^2-9x+20=0$ (b) $x^2+2x-143=0$ (c) $3x^2-5x+2=0$

   (d) $2x^2-2\sqrt{2}\,x+1=0$ (e) $x+\frac{1}{x}=3$ (f) $\frac{1}{x}+\frac{1}{(x-2)}=3$,

   (g) $\frac{1}{x+4}-\frac{1}{x-7}=\frac{11}{30}$

5. Ramnaresh Mahato scored a total of 30 marks in two subjects VR. English and Mathematics in the first terminal examination of grade 10. If he scored 2 more marks in Mathematics and 3 fewer marks in English, then the product of his marks would be 210. Find his scores on both subjects.

6. **A rectangular figure of a playground is given here. The length of the longer side of the playground is 30 m more than it's shorter side but its diagonal is 60 m more than its shorter side.**

   a) Find the length and breadth of the playground.

   b) If 12 m $\times$ 3 m size artificial grass turfs have to be placed on the ground, how many turfs are needed?

   c) If the ground has to be fenced 4 times with a barbed wire costing Rs. 15 per meter, how much will be the cost?

Answers

1. (a) Yes (b) no (c) yes (d) yes (e) no (f) no

2. (a) 5, – 2 (b) $-2, \frac{3}{2}$ (c) $\frac{1}{4}, \frac{1}{4}$ (d) $\frac{1}{10}, \frac{1}{10}$ (e) 9, 36 (f) 13, 14

3. (a) 3, 3 (b) $1, \frac{2}{3}$ (c) $1, \frac{3}{2}$ (d) $\frac{3+\sqrt{19}}{5}, \frac{3-\sqrt{19}}{5}$
   (e) $\frac{3}{4}, \frac{5}{4}$ (f) $\frac{5}{3}, -\frac{7}{3}$

4. (a) 4, 5 (b) 11, –13 (c) $1, \frac{2}{3}$ (d) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$
   (e) $\frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}$ (f) $\frac{4+\sqrt{10}}{3}, \frac{4-\sqrt{10}}{3}$ (g) 1, 2

5. 12, 18 or 13, 17

6. (a) 120 m, 90 m or 13, 17 (b) 300 (c) Rs. 25,200

7.2 Word problems related to quadratic equation

Activity 4

Age of Sumitra is 12 years and her sister is 18 years now. How many years after will the product of their age be 280? How can we find it?

	Sumitra's age	Sumitra's sister's age	Product of their ages
Now	12	18	216
1 year later	13	19	247
2 years later	14	20	280

Here,

Now, Sumitra's age = 12 years

Sumitra's Sister's age = 18 years

x years later,

Sumitra's age = $12 + x$

Sumitra's sister's age = $18 + x$

According to the condition given,

$(12 + x)(18 + x) = 280$

or, $216 + 18x + 12x + x^2 = 280$

or, $x^2 + 30x + 216 - 280 = 0$

or, $x^2 + 30x - 64 = 0$

or, $x^2 + 32x - 2x - 64 = 0$

or $x(x + 32) - 2(x + 32) = 0$

or, $(x + 32)(x - 2) = 0$

Either $x + 32 = 0$ $\quad \therefore x = -32$

Or, $x - 2 = 0$ $\quad \therefore x = 2$

Here, $x = -32$ is not a possible solution because age cannot be negative.

Therefore, $x = 2$

2 years later, the product of their age will be 280.

Example 4

If the sum of two positive numbers is 18 and their product is 77, then find these numbers.

Solution

Let these two numbers be x and y.

According to the question,

$x + y = 18$............. (i)

$x \times y = 77$............. (ii)

From equation (i) $y = 18 - x$........(iii)

Placing the value of y in equation (ii)

$x(18 - x) = 77$

or, $18x - x^2 = 77$

or, $18x - x^2 - 77 = 0$

or, $x^2 - 18x + 77 = 0$

or, $x^2 - 11x - 7x + 77 = 0$

or, $x(x - 11) - 7(x - 11) = 0$

or, $(x - 11)(x - 7) = 0$

Either, $(x - 11) = 0 \quad \therefore x = 11$

Or, $(x - 7) = 0 \quad \therefore x = 7$

Placing the value of x in equation (iii),

If $x = 11$ then $y = 18 - x = 18 - 11 = 7$

If $x = 7$ then $y = 18 - x = 18 - 7 = 11$

Therefore, two positive numbers are 7 and 11 or 11 and 7.

Example 5

If 11 is sbutracted from the square of a positive integer, then the result is 38. Find the number.

Solution

Let the positive integer be x, then its square is x^2.

According to the question, $x^2 - 11 = 38$

or, $x^2 - 11 = 38$

or, $x^2 = 38 + 11$

or, $x^2 = 49$

or, $x^2 = (\pm 7)^2$

$\therefore x = \pm 7$

But we need a positive integer, so x = 7 only.

Therefore, 7 is the required positive integer.

Example 6

If the product of two consecutive positive even numbers is 24, then find these numbers.

Solution

Let, the two consecutive even numbers be x and $x + 2$.

According to the question,

$x \times (x + 2) = 24$

or, $x^2 + 2x - 24 = 0$

or, $x^2 + 6x - 4x - 24 = 0$

or, $x(x + 6) - 4(x + 6) = 0$

or, $(x + 6)(x - 4) = 0$

Either, $(x + 6) = 0 \quad \therefore x = -6 \quad$ [$\because$ This is a negative number]

Or, $x - 4 = 0 \quad \therefore x = 4$

Therefore, the required two positive numbers are 4 and 4 + 2 = 6.

Example 7

If the sum of a number and its reciprocal is $\frac{26}{5}$ then find the number.

Solution

Let, the number be x and the reciprocal of that number be $\frac{1}{x}$

According to the question,

$x + \frac{1}{x} = \frac{26}{5}$

or, $\frac{x^2 + 1}{x} = \frac{26}{5}$

or, $5x^2 + 5 = 26x$

or, $5x^2 - 26x + 5 = 0$

or, $5x^2 - 25x - x + 5 = 0$

or, $5x(x-5) - 1(x-5) = 0$

or, $(5x-1)(x-5) = 0$

Either, $(5x-1) = 0$ $\therefore x = \frac{1}{5}$

Or, $x - 5 = 0$ $\therefore x = 5$

Therefore, the required numbers are 5 and $\frac{1}{5}$

Example 8

The sum of the two brothers' age is 34 and the product of their ages is 288, then find their present age.

Solution

Let the age of the elder brother and younger brother be x and y respectively.

According to to question,

$x + y = 34$............ (i)

$x \times y = 288$............ (ii)

From equation (i) $y = 34 - x$........(iii)

Placing the value of y in equation (ii)

or, $x(34 - x) = 288$

or, $34x - x^2 = 288$

or, $x^2 - 34x + 288 = 0$

or, $x^2 - 16x - 18x + 288 = 0$

or, $x(x-16) - 18(x-16) = 0$

or, $(x-16)(x-18) = 0$

Either, $x - 16 = 0$ $\therefore x = 16$

Or, $x - 18 = 0$ $\therefore x = 18$

Placing the value of x in equation (iii)

If $x = 16$ then $y = 34 - x = 34 - 16 = 18$ [$\because$ This is not possible.]

If $x = 18$ then $y = 34 - x = 34 - 18 = 16$

Therefore, the age of elder brother is 18 and the age of younger brother is 16.

Example 9

The product of the digits of a two digit number is 18. If 27 is added to the number, the places of digits are reversed. What is the number? Find it.

Solution

Let, the two digit number $= 10x + y$ [$\because$ where x is ten place and y is once place digit.]

According to the question,

$x + y = 18$

or, $x = \frac{18}{y}$ .... (i)

Again the second condition, $(10x + y) + 27 = 10y + x$

$10x + y + 27 - 10y - x = 0$

or, $9x - 9y + 27 = 0$

or, $9(x - y + 3) = 0$

or, $x - y + 3 = 0$ .... (ii)

Placing the value of x from equation (i) to equation (ii)

or, $\frac{18}{y} - y + 3 = 0$

or, $\frac{18 - y^2 + 3y}{y} = 0$

or, $y^2 - 3y - 18 = 0$

or, $y^2 - 6y + 3y - 18 = 0$

or, $y(y - 6) + 3(y - 6) = 0$

or, $(y - 6)(y + 3) = 0$

Either $y - 6 = 0$ $\therefore y = 6$

Or, $y + 3 = 0$ $\therefore y = -3$

Placing the value of y in equation (ii)

If $y = 6$, then $x = \frac{18}{6} = 3$.

If $y = -3$, then $x = \frac{18}{-3} = -6$.

If $y = 6$ and $x = 3$ then the number is $10x + y = 10x3 + 6 = 36$.

If $y = -3$ and $x = -6$ then the number is $10x + y = 10x(-6) - 3 = -63$.

Example 10

The present age of the father and his son is 42 years and 16 years respectively. Find how many years ago the product of their age was 272.

Solution

Let, x years ago, the age of the father and his son was $42 - x$ and $16 - x$ respectively.

According to the question,

x years ago, the product of their age = 272.

or, $(42 - x)\ (16 - x) = 272$

or, $672 - 42x - 16x + x^2 = 272$

or, $x^2 - 58x + 400 = 0$

or, $x^2 - 50x - 8x + 400 = 0$

or, $x(x - 50) - 8(x - 50) = 0$

or, $(x - 8)\ (x - 50) = 0$

Either, $x - 8 = 0$ $\quad\therefore x = 8$

O, $x - 50 = 0$ $\quad\therefore x = 50$

Here, $x = 50$ years, which is impossible. Therefore, $x = 8$.

Hence, 8 years ago the product of the father and his son's age was 272.

Example 11

The length of the hypotenuse of a right angled triangle is 13m. If the difference of its other two sides is 7m, find the length of the remaining sides.

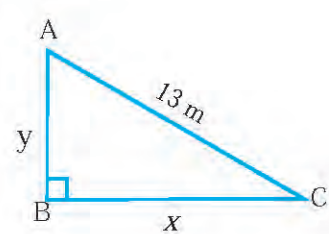
Solution

Here, ΔABC is a right angled triangle where $\angle B = 90^0$
and hypotenuse (h) = AC = 13m

Let base (b) = BC = x and perpendicular (p) = AB = y.

According to the question,

$x - y = 7$ $\quad$ or, $y = x + 7$..........(i)



Now, in the right angled triangle ABC $h^2 = p^2 + b^2$.

Therefore, $AC^2 = AB^2 + BC^2$

or, $13^2 = (x + 7)^2 + x^2$

or, $169 = x^2 + 14x + 49 + x^2$

or, $2x^2 + 14x - 120 = 0$

or, $x^2 + 7x - 60 = 0$

or, $x^2 + 12x - 5x - 60 = 0$

or, $x(x + 12) - 5(x+12) = 0$

or, $(x - 5)(x + 12) = 0$

Either, $(x - 5) = 0 \quad \therefore x = 5$

Or, $x + 12 = 0 \quad \therefore x = 0$

Here, $x = -12$ is impossible because length of a base cannot be negative, therefore $x = 5$.

Hence, base (b) = BC = x = 5m

and perpendicular (p) = AB = y = 5 + 7 = 12m.

Hence, the remaining sides are 5m and 12m.

Example 12

The area of a rectangular land is $50m^2$ and its perimeter is 90m. If the land is to be made square, by what percentage should it be reduced in length?

Solution

Let, the length and breadth of the rectangular land are x and y respectively.

According to question,

Area of a the rectangular land = $50m^2$

or, $xy = 500$..................(i)

Perimeter of the rectangular land = 90m

or, $2(x + y) = 90$

or, $x + y = 45$

or, $y = 45 - x$ ....(ii)

Now, place $y = 45 - x$ from equation (ii) in equation (i).

$xy = 500$

or, $x(45 - x) = 500$

or, $45x - x^2 = 500$

or, $x^2 - 45x + 500 = 0$

or, $x^2 - 25x - 20x + 500 = 0$

or, $x(x - 25) - 20(x - 25) = 0$

or, $(x - 25)(x - 20) = 0$

either, $(x - 25) = 0$. $\therefore x = 25$

or, $x - 20 = 0$ $\therefore x = 20$

If $x = 25$, $y = 45 - x = 45 - 25 = 20$.

If $x = 20$, $y = 45 - x = 45 - 20 = 25$.

Therefore, the length and breadth of the rectangular land are 25m and 20m respectively.

If the land is to be made square, then the length and breadth of the land should be equal.

So, the length should be reduced by 5m.

Therefore, percentage to be reduced in length = $\frac{5}{25} \times 100\% = 20\%$

Example 13

Some students of grade 10 organized a picnic of Rs. 42,000 budget. They decided to collect equal money for picnic. But, 5 students were absent in the picnic day, so each student should collect Rs. 700 more. Based on this context, solve the problem given below.

a) How many students attended the picnic?

b) How much did each participant have to pay? Find it.

Solution

Let, the number of students = x and the amount to be paid by each participant = Rs. $\frac{42000}{x}$

Here, 5 students were absent.

Therefore, the number of students participated = $x - 5$

According to the question,

$$\frac{42000}{x-5} = \frac{42000}{x} + 700$$

or, $\frac{42000}{x-5} - \frac{42000}{x} = 700$

or, $\frac{60}{x-5} - \frac{60}{x} = 1$

or, $60x - 60x + 300 = x(x-5)$

or, $x^2 - 5x - 300 = 0$

or, $x^2 - 20x + 15x - 300 = 0$

or, $x(x-20) + 15(x-20) = 0$

or, $(x-20)(x+15) = 0$

Either, $(x-20) = 0 \quad \therefore x = 20$

Or, $(x+15) = 0 \quad \therefore x = -15$

Here, x denotes the number of students, so $x = -15$ is impossible. Therefore, $x = 20$.

Hence,

a) The total number of students participated in picnic = $20 - 5 = 15$

b) The total amount of money per person = $\frac{42000}{x-5} = \frac{42000}{15}$ = Rs. 2800

Exercise 7.2

1. If 11 is added to the square of a natural number, the sum is 36. Find the number.
2. If 11 is subtracted from the square of a number, the remainder number is 25. Find the number.
3. If 7 is subtracted from the double of a square of a positive number, the remainder is 91. Find the number.
4. If 2 is subtracted from the square of a natural number, the remainder is 7. Find the number.
5. If 11 is subtracted from the square of a number and the remainder is 89, find that number.
6. If 17 is subtracted from the square of a number, the remainder is 55. Find the number.
7. If 3 is subtracted from the double ofthe square of a positive number, the remainder is 285. Find the number.

8. If the sum of a number and its square is 72, find the number.
9. If the product of the two consecutives even numbers is 80, find the numbers.
10. If the product of the two consecutive odd numbers is 225, find the numbers.
11. If the sum of a number and their reciprocal is $\frac{10}{3}$, find the number.
12. If the sum of two natural numbers is 21 and sum of their square is 261, find the numbers.
13. If the age difference between two brothers is 4 years and the product of their ages is 221. Find their age.
14. The sum of the present age of two brothers is 22 and the product of their ages is 120. Find their present age.
15. The age difference between two sisters is 3 years and the product of their age is 180. Find their present age.
16. (a) The present age of a father and his son is 40 years and 13 years respectively. Find how many years ago the product of their age was 198.
    (b) The present age of a mother and her daughter is 34 years and 4 years respectively. Find how many years later the product of their age will be 400.
    (c) The present age of a father and his son is 35 years and 1 year respectively. Find how many years later the product of their age will be 240.
    (d) The present age of a husband and wife is 35 years and 27 years respectively. Find how many years ago the product of their age was 425.
17. (a) The hypotenuse of a right angled triangle is 25m. If the difference between its other two sides is 17m, find the length of the remaining sides.
    (b) The length of hypotenuse of a right angled triangle is double and 6m more than its shortest side. If the length of the remaining side is 2m less than hypotenuse, find the length of all sides.
    (c) Calculate the length and breadth of a rectangular land whose area is $150m^2$ and perimeter is 50m.
    (d) Calculate the length and breadth of a rectangular land whose area is $54m^2$ and perimeter is 30m.
    (e) Calculate the area of a rectangle whose length is 24m and diagonal is 16m more than its breadth.

(f) The area and perimeter of a rectangular land is 2000m^2 and 180m respectively. If the land is to be made square, by what percentage should it be reduced in length or breadth? Calculate.

18. The two digit number is equal to the four times the sum of their digits and three times the product of their digits. Find the number.

19. An institute made a plan to distribute 180 pencils equally to the students enrolled in grade one. On the pencil distribution day, 5 students were absent so that each students got 3 more pencils.

    (a) How many students enrolled in grade one?

    (b) How many pencil did each student receive in total?

Project work

Form three groups of students for preparing a volleyball court of your school. The first group made a volleyball court whose area is 128m^2 and perimeter is 48m. The second group made a volleyball court whose area is 162m^2 and perimeter is 54m. The third group made a volleyball court whose area is 200m^2 and perimeter is 60m. Discuss the dimension of volleyball court in group and conclude which court is suitable for playing volleyball. Present your conclusion in your class.

Answers

1. 5 2. ±6 3. 7 4. 3 5. ±10 6. ±6
7. 12 8. 8 9. 8 and 10 or –10 and –8
10. 3 and 5 or –5 and –3 11. 3 and 1/3 12. 6 and 15
13. 17 years and 13 years 14. 12 years and 10 years 15. 15 years and 12 years
16. (a) 7 years (b) 6 years (c) 5 years (d) 10 years
17. (a) 24 m and 7m (b) 10m, 24m, 26m (c) 15 m and 10 m
    (d) 9 m and 6 m (e) 240 m^2 (f) less then 20%
18. 24 19. (a) 20 person (b) 12 pencil

Lesson 8

Algebraic Fraction

8.0 Review

1) Change the given algebraic fractions into the lowest terms. Check with your friends whether they are corret or not.

(a) $\frac{xy}{x^2y}$ (b) $\frac{x-y}{x^2-y^2}$ (c) $\frac{a+3}{a^2+5a+6}$

(d) $\frac{a-2}{a^2-6a+8}$ (e) $\frac{a-6}{a^2-8a+12}$ (f) $\frac{a+2}{a^2-4a+12}$

2) Simplify the given fraction. Ask your friend to check your answer or not.

(a) $\frac{3}{5}+\frac{1}{5}$ (b) $\frac{2}{3}+\frac{1}{5}$ (c) $\frac{1}{4}+\frac{1}{6}$

(d) $\frac{a}{b}+\frac{2a}{b}$ (e) $\frac{3a}{b}-\frac{ab}{a}$ (f) $\frac{3}{xy}+\frac{2a}{xy^2}$

8.1 Simplification of Algebraic Fractions

Activity 1

Simplify the given algebraic fraction. Discuss with friends about the simplification process.

(a) $\frac{x}{x-y}-\frac{y}{x-y}$ (b) $\frac{x}{x-y}-\frac{y}{x+y}$ (c) $\frac{1}{a-b}-\frac{b}{a^2+b^2}$

When simplifying the above fraction, first confirm that denominators of the given fractions are same or different. If that denominators are same, only operation between numerator are done and a single denominator is placed. only and write denominator in once. If denominators are different, we have to find the LCM of the denominators.

For example:

(a) $\frac{x}{x-y}+\frac{y}{x-y}$ are like fraction.

$\frac{x}{x-y}+\frac{y}{x-y} = \frac{x+y}{x-y}$ [adding the numerators and placing common denominator.]

(b) $\frac{x}{x-y} + \frac{y}{x+y}$ are unlike fraction. Now to make denominator equal,

$$= \frac{x(x+y)}{(x-y)(x+y)} - \frac{y(x-y)}{(x+y)(x-y)}$$

[∵ Multiplied denominator of one fraction to numerator and denominator of another fraction]

$$= \frac{x(x+y) - y(x-y)}{(x+y)(x-y)} = \frac{x^2 + xy - xy + y^2}{(x+y)(x-y)} = \frac{x^2 + y^2}{x^2 - y^2}$$

(c) $\frac{1}{a-b} - \frac{b}{a^2 - b^2}$

$= \frac{1}{a-b} - \frac{b}{(a-b)(a+b)}$ are unlike fraction.

Now, to make denominator same,

$$= \frac{1(a+b)}{(a-b)(a+b)} - \frac{b}{(a-b)(a+b)}$$

to make denominator equal

Denominator of the first fraction $= (a-b) \times (a+b)$

Denominator of the second fraction $= (a-b)(a+b) \times 1$

$$= \frac{a+b-b}{(a-b)(a+b)}$$

$$= \frac{a}{(a-b)(a+b)}$$

$$= \frac{a}{a^2 - b^2}$$

Alternatively,

Denominator of the first fraction = (a – b)

Denominator of the second fraction = (a – b) (a + b)

LCM = (a – b) (a + b)

Now to simplify,

$$= \frac{1}{(a-b)} - \frac{b}{(a-b)(a+b)}$$

$$= \frac{(a+b) - b}{(a-b)(a+b)}$$

[Placing the LCM of the denominator in denominator. LCM is divided by each termdenominator and multiply to numerator of the same fraction.]

$$= \frac{a}{(a-b)(a+b)}$$

$$= \frac{a}{(a^2 - b^2)}$$

Example 1

Simplify: $\frac{x^2}{x+y} - \frac{y^2}{x+y}$

Solution

$= \frac{x^2-y^2}{x+y}$

$= \frac{(x-y)(x+y)}{x+y}$

$= x - y$

Example 2

Simplify: $\frac{1}{x-y} - \frac{1}{x+y}$

Solution

$= \frac{1}{x-y} - \frac{1}{x+y}$

$= \frac{(x+y)-(x-y)}{x^2-y^2}$

$= \frac{x+y-x+y}{x^2-y^2}$

$= \frac{2y}{x^2-y^2}$

Example 3

Simplify: $\frac{x+y}{x-y} + \frac{x-y}{x+y}$

Solution

$= \frac{x+y}{x-y} + \frac{x-y}{x+y}$

$= \frac{(x+y)^2+(x-y)^2}{(x-y)(x+y)}$

$= \frac{x^2+2xy+y^2+x^2-2xy+y^2}{x^2-y^2}$

$= \frac{2(x^2+y^2)}{x^2-y^2}$

Example 4

Simplify: $\frac{a^3+1}{a^2-a+1} + \frac{a^3-1}{a^2+a+1}$

Solution

$= \frac{a^3+1}{a^2-a+1} + \frac{a^3-1}{a^2+a+1}$

$= \frac{(a+1)(a^2-a+1)}{a^2-a+1} + \frac{(a-1)(a^2+a+1)}{a^2+a+1}$

$= (a+1) + (a-1)$

$= 2a$

Example 5

Simplify: $\frac{1}{2a-3b} - \frac{a+b}{4a^2-9b^2}$

Solution

$= \frac{1}{2a-3b} - \frac{a+b}{4a^2-9b^2}$

$= \frac{1}{2a-3b} - \frac{a+b}{(2a-3b)(2a+3b)}$

$= \frac{(2a+3b)-(a+b)}{(2a-3b)(2a+3b)}$

$= \frac{(a+2b)}{4a^2-9b^2}$

Calculating the multiplication of $4a^2 - 9b^2$

$= (2a)^2 - (3b)^2$

$= (2a+3b)\ (2a-3b)$

Example 6

Simplify: $\frac{4x^2+y^2}{4x^2-y^2}-\frac{2x-y}{2x+y}$

Solution

$$=\frac{4x^2+y^2}{4x^2-y^2}-\frac{2x-y}{2x+y}$$

$$=\frac{4x^2+y^2}{(2x-y)(2x+y)}-\frac{2x-y}{2x+y}$$

$$=\frac{4x^2+y^2-(2x-y)^2}{(2x-y)(2x+y)}$$

$$=\frac{4x^2+y^2-4x^2+4xy-y^2}{4x^2-y^2}$$

$$=\frac{4xy}{4x^2-y^2}$$

Calculating the multiplication of $4x^2-y^2$

$=(2x)^2-(y)^2$

$=(2x+y)\ (2x-y)$

Example 7

Simplify: $\frac{x}{x-y}+\frac{x}{x+y}+\frac{2xy}{x^2+y^2}$

Solution

$$=\frac{x}{x-y}+\frac{x}{x+y}+\frac{2xy}{x^2+y^2}$$

$$=\frac{x(x+y)+x(x-y)}{x^2-y^2}+\frac{2xy}{x^2+y^2}$$

$$=\frac{x^2+xy+x^2-xy}{x^2-y^2}+\frac{2xy}{x^2+y^2}$$

$$=\frac{2x^2}{x^2-y^2}+\frac{2xy}{x^2+y^2}$$

$$=\frac{2x^2(x^2+y^2)+2xy\,(x^2-y^2)}{(x^2-y^2)(x^2+y^2)}$$

$$=\frac{2x^4+2x^2y^2+2x^3y-2xy^3}{(x^4-y^4)}$$

Example 8

Simplify: $\frac{1}{2(x-y)} - \frac{1}{2(x+y)} - \frac{y}{x^2-y^2}$

Solution

$$\frac{1}{2(x-y)} - \frac{1}{2(x+y)} - \frac{y}{x^2-y^2}$$

$$= \frac{1}{2(x-y)} - \frac{1}{2(x+y)} - \frac{y}{(x-y)(x+y)}$$

$$= \frac{(x+y)-(x-y)-2y}{2(x-y)(x+y)}$$

$$= \frac{x+y-x+y-2y}{2(x-y)(x+y)}$$

$$= \frac{0}{2(x-y)(x+y)}$$

$$= 0$$

Example 9

Simplify: $\frac{a-1}{a^2-4a+3} + \frac{a-2}{a^2-8a+12} + \frac{a-5}{a^2-8a+15}$

Solution

$$= \frac{a-1}{a^2-4a+3} + \frac{a-2}{a^2-8a+12} + \frac{a-5}{a^2-8a+15}$$

$$= \frac{a-1}{(a-1)(a-3)} + \frac{a-2}{(a-6)(a-2)} + \frac{a-5}{(a-5)(a-3)}$$

$$= \frac{1}{(a-3)} + \frac{1}{(a-6)} + \frac{1}{(a-3)}$$

$$= \frac{a-6+a-3+a-6}{(a-6)(a-3)}$$

$$= \frac{3a-15}{(a-2)(a-3)}$$

$$= \frac{3(a-5)}{(a-2)\ (a-3)}$$

$a^2 - 4a + 3$

$= a^2 - 3a - 1a + 3$

$= a(a-3) - 1(a-3)$

$= (a-3)\ (a-1)$

$a^2 - 8a + 15$

$= a^2 - 5a - 3a + 15$

$= a(a-5) - 3(a-5)$

$= (a-5)\ (a-3)$

$a^2 - 8a + 12$

$= a^2 - 6a - 2a + 12$

$= a(a-6) - 2(a-6)$

$= (a-6)\ (a-2)$

Example 10

Simplify: $\frac{pr^2+q}{2r-1}+\frac{pr^2-q}{2r+1}+\frac{4pr^3}{1-4r^2}$

Solution

$=\frac{pr^2+q}{2r-1}+\frac{pr^2-q}{2r+1}+\frac{4pr^3}{1-4r^2}$

$=\frac{pr^2+q}{2r-1}+\frac{pr^2-q}{2r+1}-\frac{4pr^3}{4r^2-1}$

$=\frac{pr^2+q}{2r-1}+\frac{pr^2-q}{2r+1}-\frac{4pr^3}{(2r-1)(2r+1)}$

$=\frac{(pr^2+q)(2r+1)+(pr^2-q)(2r-1)-4pr^3}{(2r-1)(2r+1)}$

$=\frac{(2pr^3+pr^2+2rq+q)+2pr^3-pr^2-2rq+q-4pr^3}{4r^2-1}$

$=\frac{2q}{4r^2-1}$

Calculating the multiplication of

$4r^2-1$

$=4r^2-1$

$=(2r)^2-(1)^2$

$=(2r-1)\ (2r+1)$

Example 11

Simplify: $\frac{a-b}{a^2-ab+b^2}+\frac{a+b}{a^2+ab+b^2}-\frac{2a^3}{a^4-a^2b^2+b^4}$

Solution

$=\frac{a-b}{a^2-ab+b^2}+\frac{a+b}{a^2+ab+b^2}-\frac{2a^3}{a^4-a^2b^2+b^4}$

$=\frac{(a-b)(a^2+ab+b^2)+(a+b)(a^2-ab+b^2)}{(a^2-ab+b^2)(a^2+ab+b^2)}-\frac{2a^3}{a^4-a^2b^2+b^4}$

$=\frac{a^3-b^3+a^3+b^3}{(a^4+a^2b^2+b^4)}-\frac{2a^3}{a^4-a^2b^2+b^4}$

$=\frac{2a^3}{(a^4+a^2b^2+b^4)}-\frac{2a^3}{a^4-a^2b^2+b^4}$

$=\frac{2a^3\left(a^4-a^2b^2+b^4\right)-2a^3\left(a^4+a^2b^2+b^4\right)}{(a^4+a^2b^2+b^4)\,(a^4-a^2b^2+b^4)}$

$=\frac{2a^7-2a^5b^2+2a^3b^4-2a^7-2a^5b^2-2a^3b^4}{(a^4+a^2b^2+b^4)\,(a^4-a^2b^2+b^4)}$

$=\frac{-4a^5b^2}{(a^8+a^4b^4+b^8)}$

Exercise 8.1

1. Change into the simplest form.

(a) $\dfrac{x^2-5x}{x^2-25}$ (b) $\dfrac{x^2-b^2}{(a+b)^2}$ (c) $\dfrac{x^2-5x+6}{x^2-7x+12}$

2. Simplify:

(a) $\dfrac{a}{a-b}+\dfrac{b}{b-a}$ (b) $\dfrac{1}{b-c}-\dfrac{b+c}{b^2-c^2}$ (c) $\dfrac{1}{m-n}+\dfrac{1}{m+n}$

(d) $\dfrac{m+n}{m-n}+\dfrac{m-n}{m+n}$ (e) $\dfrac{1}{m+n}+\dfrac{n}{m^2-n^2}$ (f) $\dfrac{3}{x^2-4}+\dfrac{1}{(x-2)^2}$

(g) $\dfrac{a^3+b^3}{a^2-ab+b^2}+\dfrac{a^3-b^3}{a^2+ab+b^2}$ (h) $\dfrac{4x^2+25y^2}{4x^2-25y^2}-\dfrac{2x-5y}{2x+5y}$ (i) $\dfrac{4x^3}{x^4+a^4}-\dfrac{8x^7}{x^8-a^8}$

(j) $\dfrac{x}{x-y}-\dfrac{x}{x+y}+\dfrac{2xy}{x^2+y^2}$ (k) $\dfrac{3}{a+3}+\dfrac{4}{a-3}+\dfrac{9a}{2(9-a^2)}$

(l) $\dfrac{1}{x+2y}-\dfrac{1}{x-2y}+\dfrac{2x}{4y^2-x^2}$ (m) $\dfrac{a}{(a-b)(a-c)}+\dfrac{b}{(b-a)(b-c)}+\dfrac{c}{(c-b)(c-a)}$

(n) $\dfrac{y-z}{x^2-(y-z)^2}+\dfrac{z-x}{y^2-(z-x)^2}+\dfrac{x-y}{z^2-(x-y)^2}$

(o) $\dfrac{x^2-(a-b)^2}{(x+b)^2-a^2}+\dfrac{a^2-(x-b)^2}{(x+a)^2-b^2}+\dfrac{b^2-(x-a)^2}{(a+b)^2-x^2}$

(p) $\dfrac{1}{p^2+7p+12}+\dfrac{2}{p^2+5p+6}-\dfrac{3}{p^2+6p+8}$

(q) $\dfrac{x+3}{x^2+3x+9}+\dfrac{x-3}{x^2-3x+9}-\dfrac{54}{x^4+9x^2+81}$ (r) $\dfrac{1}{x^2-5x+6}+\dfrac{2}{4x-x^2-3}-\dfrac{3}{x^2-3x+2}$

(s) $\dfrac{b+2}{1+b+b^2}-\dfrac{b-2}{1-b+b^2}-\dfrac{2b^2}{1+b^2+b^4}$ (t) $\dfrac{1}{1-b+b^2}-\dfrac{1}{1+b+b^2}-\dfrac{2b}{1-b^2+b^4}$

(u) $\dfrac{a+c}{a^2+ac+c^2}+\dfrac{a-c}{a^2-ac+c^2}+\dfrac{2c^3}{a^4+a^2c^2+c^4}$

3. Simplify:

(a) $\dfrac{1}{4(1-\sqrt{x})}-\dfrac{1}{4(1+\sqrt{x})}+\dfrac{2\sqrt{x}}{4(1-x)}$ (b) $\dfrac{1}{8(1-\sqrt{x})}-\dfrac{1}{8(1+\sqrt{x})}+\dfrac{2\sqrt{x}}{8(1-x)}$

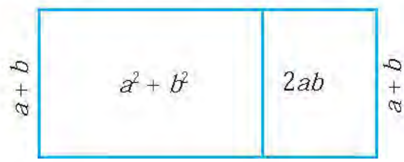
(c) $\dfrac{1}{(a+1)^2}+\dfrac{1}{(a-1)^2}-\dfrac{2}{a^2-1}$

4. If $\dfrac{a}{2x+1}+\dfrac{1}{x+2}=\dfrac{4x+5}{2x^2+5x+2}$ what is the value of a? Find it.

5. If $\dfrac{a}{2x-3}+\dfrac{b}{3x+4}=\dfrac{x+7}{6x^2-x-12}$ what are the values of a and b? Find it

Project work

Divide a rectangular piece of paper into two parts from its length side making its breadth equal. Consider the area and breadth of first piece is $a^2 + b^2$ and $a + b$ respectively. Similarly, the area and breadth of second piece of paper is 2ab and $a + b$ respectively. Now, consider $a > b$,



a) Find the total length of both pieces of paper. Express in terms of a and b.

b) If a = 5m and b = 3m, then find the relation of area, length and breadth of the rectangular piece of paper and present in your classroom.

Answers

1. (a) $\frac{x}{a+5}$ (b) $\frac{a+b}{a-b}$ (c) $\frac{x-2}{x-4}$

2. (a) 1 (b) 0 (c) $\frac{2m}{m^2-n^2}$

(d) $\frac{2(m^2+n^2)}{m^2-n^2}$ (e) $\frac{m}{m^2-n^2}$ (f) $\frac{4(x-1)}{(x+2)(x-2)^2}$

(g) $2a$ (h) $\frac{20xy}{4x^2-20y^2}$ (i) $\frac{4x^3}{a^4-x^4}$ (j) $\frac{4x^3y}{x^4-y^4}$

(k) $\frac{5a+6}{2(a^2-9)}$ (l) $\frac{2}{2y-x}$ (m) 0 (n) 0 (o) 1

(p) $\frac{1}{(p+2)(p+3)(p+4)}$ (q) $\frac{2(x-3)}{x^2-3x+9}$ (r) $\frac{4}{3x-x^2-2}$

(s) $\frac{4}{1+b^2+b^4}$ (t) $\frac{-4b^3}{1+b^4+b^8}$ (u) $\frac{2(a+c)}{a^2+ac+c^2}$

3. (a) $\frac{\sqrt{x}}{1-x}$ (b) $\frac{\sqrt{x}}{2(1-x)}$ (c) $\frac{4}{(a^2-1)^2}$

4. $a = 2$

5. $a = 1, b = -1$

Lesson 9

Indices

9.0 Review

We studied about simplification of indices in the previous grades. Here, we discuss about exponential equation.

Fill in the blank spaces in the given table. Which value of x satisfies the given condition?

(a) $2^x = 2$

x	-3	-2	-1	0	1	2	3
2^x	$2^{-3} = \frac{1}{8}$	......	......	......	......	......	......

(b) $5^{x+1} = 125$

x	-3	-2	-1	0	1	2	3
5^{x+1}	$5^{-3+1} = \frac{1}{25}$	......	......	......	......	......	......

(c) $3^x = \frac{1}{9}$

x	-3	-2	-1	0	1	2	3
3^x	$3^{-3} = \frac{1}{27}$	......	......	......	......	......	......

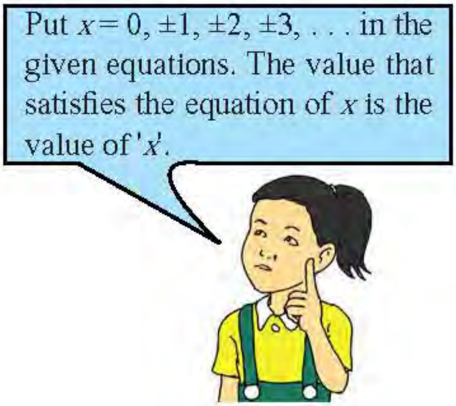
9.1 Exponential Equations

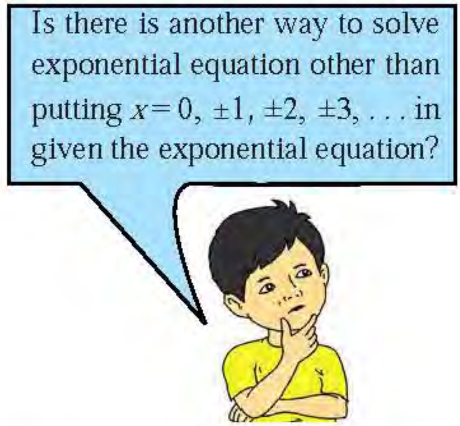
Activity 1

How can we solve the given exponential equations? Discuss in groups.

(a) $2^x = 4$ (b) $3^{x-1} = 81$

(c) $3^{x+1} + 3^x = \frac{4}{27}$ (d) $3^x + \frac{1}{3^x} = 3\frac{1}{3}$





(a) In $2^x = 4$ put $x = 0, \pm 1, \pm 2, \pm 3, \ldots$ respectively. x =2 satisfies the equation. Therefore, $x = 2$.

Alternatively,

Here, $2^x = 4$

or, $2^x = 2^2$

$\Rightarrow x = 2$

(b) $3^{x-1} = 81$

or, $3^{x-1} = 3^4$

$\Rightarrow x - 1 = 4$

$\therefore$ x = 5

Verification, $\boldsymbol{x} = 5$

$3^{x-1} = 81$

LHS 3^{5-1}

$= 3^4 = 81$

$\therefore$ LHS = RHS

So, $x = 5$ is correct.

(c) $3^{x+1} + 3^x = \frac{4}{81}$

or, $3^x \times 3^1 + 3^x = \frac{4}{81}$

or, $3^x(3 + 1) = \frac{4}{81}$

or, $3^x (4) = \frac{4}{81}$

or, $3^x = \frac{1}{81}$

or, $3^x = 3^{-4}$

$\Rightarrow x = -4$

Verification, $\boldsymbol{x} = -4$

$3^{x+1} + 3^x = \frac{4}{81}$

LHS $3^{-4+1} + 3^{-4}$

$= 3^{-3} + 3^{-4}$

$= \frac{1}{3^3} + \frac{1}{3^4}$

$= \frac{1}{27} + \frac{1}{81}$

$= \frac{4}{81}$ = RHS

The value of $x = -4$ is correct.

(d) $3^x + \frac{1}{3^x} = 3\frac{1}{3}$ What is the difference between this and previous exponential equation?

or, $3^x + \frac{1}{3^x} = 3\frac{1}{3}$

or, $\frac{(3^x)^2 + 1}{3^x} = \frac{10}{3}$

or, $3 \times (3^x)^2 + 3 = 10 \times 3^x$

or, $3 \times (3^x)^2 - 10 \times 3^x + 3 = 0$

This is the quadratic equation of 3^x.

Now, putting the value of a in equation (i)

$3^x = a$ .... (i)

Now $3a^2 - 10a + 3 = 0$

or, $3a^2 - 9a - a + 3 = 0$

or, $3a(a - 3) - 1(a - 3) = 0$

or, $(a - 3)(3a - 1) = 0$

either, $(a - 3) = 0$ $\therefore a = 3$

or, $(3a - 1) = 0$ $\therefore a = \frac{1}{3}$

Therefore, the value of x are 1 and -1.

If $a = 3$, $3^x = 3^1$ $\Rightarrow x = 1$

If $a = \frac{1}{3}$, $3^x = \frac{1}{3} = 3^{-1}$ $\Rightarrow x = -1$

Example 1

Solve: $7^x = 49$

Solution

Here, $7^x = 49$

or, $7^x = 7^2$

$\Rightarrow x = 2$

Example 2

Solve: $4^{x-2} = 0.25$

Solution

Here, $4^{x-2} = 0.25$

or, $(2)^{2(x-2)} = \frac{1}{4}$

or, $(2)^{2(x-2)} = \left(\frac{1}{2}\right)^2$

or, $(2)^{2(x-2)} = 2^{-2}$

or, $(2)^{2(x-2)} = 2^{-2}$

$\Rightarrow 2(x-2) = -2$

or, $x - 2 = -1$

$\therefore x = 1$

Alternative Method

Here, $4^{x-2} = 0.25$

or, $4^{x-2} = \frac{1}{4}$

or, $4^{x-2} = (4)^{-1}$

or, $(4)^{(x-2)} = (4)^{-1}$

$\Rightarrow (x-2) = -1$

or, $x = -1 + 2$

$\therefore x = 1$

Example 3

Solve: $3^{5x-4} + 3^{5x} = 82$

Solution

Here, $3^{5x-4} + 3^{5x} = 82$

or, $3^{5x} \times 3^{-4} + 3^{5x} = 82$

or, $3^{5x}\left(\frac{1}{81} + 1\right) = 82$

or, $3^{5x}\left(\frac{82}{81}\right) = 82$

or, $3^{5x} = 81$

or, $3^{5x} = 3^4$

$\Rightarrow 5x = 4$

$\therefore x = \frac{4}{5}$

Example 4

Solve : $3^{x-1} + 3^{x-2} + 3^{x-3} = 13$

Solution

Here, $3^{x-1} + 3^{x-2} + 3^{x-3} = 13$

or, $3^x \times 3^{-1} + 3^x \times 3^{-2} + 3^x \times 3^{-3} = 13$

or, $\frac{1}{3} \times 3^x + \frac{1}{9} \times 3^x + \frac{1}{27} \times 3^x = 13$

or, $3^x\left(\frac{1}{3} + \frac{1}{9} + \frac{1}{27}\right) = 13$

or, $3^x\left(\frac{9+3+1}{27}\right) = 13$

or, $3^x\left(\frac{13}{27}\right) = 13$

or, $3^x = 27$

or, $3^x = 3^3$

$\Rightarrow x = 3$

Example 5

Solve: $2^x + \frac{1}{2^x} = 2\frac{1}{2}$

Solution

Here, $2^x + \frac{1}{2^x} = 2\frac{1}{2}$

or, $2^x + \frac{1}{2^x} = \frac{5}{2}$

Let $2^x = a$..................(i)

so, $a + \frac{1}{a} = \frac{5}{2}$

or, $\frac{a^2+1}{a} = \frac{5}{2}$

or, $2(a^2 + 1) = 5a$

or, $2a^2 - 5a + 2 = 0$

or, $2a^2 - 4a - a + 2 = 0$

or, $2a(a - 2) - 1(a - 2) = 0$

or, $(a - 2)(2a - 1) = 0$

Either, $(a - 2) = 0$ $\quad \therefore a = 2$

Or, $(2a - 1) = 0$ $\quad \therefore a = \frac{1}{2}$

Substituting the value of a in eq^n(i), we get

$a = 2$ then $2^x = 2^1$ $\quad \Rightarrow x = 1$

$a = \frac{1}{2}$ then $2^x = \frac{1}{2} = 2^{-1}$ $\Rightarrow x = -1$

Hence, the values of x are 1 and -1.

Example 6

Solve: $5 \times 4^{x+1} - 16^x = 64$

Solution

Here, $5 \times 4^{x+1} - 16^x = 64$

or, $5 \times (4^x \times 4) - 4^{2x} = 64$

or, $20 \times 4^x - (4^x)^2 = 64$

Let $4^x = a$..................(i)

Hence, $20a - a^2 = 64$

or, $a^2 - 20a + 64 = 0$

or, $a^2 - 16a - 4a + 64 = 0$

or, $a(a-16) - 4(a-16) = 0$

or, $(a-4)(a-16) = 0$

Either, $(a-4) = 0$ $\therefore a = 4$

or, $(a-16) = 0$ $\therefore a = 16$

Now, putting the values of a in eq^n (i), we get

If $a = 4$ then $4^x = 4^1$ $\Rightarrow x = 1$

If $a = 16$ then $4^x = 16 = 4^2$ $\Rightarrow x = 2$

Hence, the values of x are 1 and 2.

Example 7

If $x^2 + 2 = 3^{\frac{2}{3}} + 3^{\frac{-2}{3}}$, then prove that $3x(x^2 + 3) = 8$

Solution

Here, $x^2 + 2 = 3^{\frac{2}{3}} + 3^{\frac{-2}{3}}$

or, $x^2 = 3^{\frac{2}{3}} + 3^{\frac{-2}{3}} - 2$

or, $x^2 = \left(3^{\frac{1}{3}}\right)^2 - \left(3^{\frac{-1}{3}}\right)^2 - 2 \times 3^{\frac{1}{3}} \times 3^{\frac{-1}{3}}$ $[\because 3^{\frac{1}{3}} \times 3^{\frac{-1}{3}} = 1]$

or, $x^2 = \left(3^{\frac{1}{3}} - 3^{\frac{-1}{3}}\right)^2$

$\Rightarrow x = 3^{\frac{1}{3}} - 3^{\frac{-1}{3}}$ ....(i)

Cubing on both sides of eq^n (i)

or, $x^3 = (3^{\frac{1}{3}} - 3^{\frac{-1}{3}})^3$

or, $x^3 = (3^{\frac{1}{3}})^3 + (3^{\frac{-1}{3}})^3 - 3 \times 3^{\left(\frac{1}{3}\right)} \times 3^{-\left(\frac{1}{3}\right)}\left(3^{\frac{1}{3}} - 3^{-\frac{1}{3}}\right)$

or, $x^3 = 3 - 3^{-1} - 3 \times 1 \times x$

or, $x^3 = 3 - \frac{1}{3} - 3x$

or, $x^3 = \frac{9 - 1 - 9x}{3}$

or, $3x^3 = 8 - 9x$

or, $3x^3 + 9x = 8$ *Proved.*

Exercise 9.1

1. Fill in the blank spaces in the following table and show it to your teacher.

(a)

x	-3	-2	-1	0	1	2	3
7^x	……	……	……	……	……	……	……

(b)

x	-3	-2	-1	0	1	2	3
5^{-x}	……	……	……	……	……	……	……

2. Solve and examine

(a) $3^x = 9$ (b) $5^{x-1} = 25$ (c) $\frac{1}{5^{2x-4}} = 125$

(d) $4^{x-2} = 0.125$ (e) $\left(\frac{3}{5}\right)^x = \left(1\frac{2}{3}\right)^3$ (f) $2^x \times 3^{x+1} = 18$

3. Solve:

(a) $4^{\frac{1-x}{1+x}} = 4^{\frac{1}{3}}$ (b) $\sqrt[2x+4]{4^{x+8}} = \sqrt[6]{128}$

(c) $2^{x+1} + 2^{x+2} + 2^x + 3 = 448$ (d) $3^{x+1} - 3^x = 162$

(e) $4^{x+1} - 8 \times 4^{x-1} = 32$ (f) $4 \times 3^{x+1} - 3^{x+2} - 3^{x-1} = 72$

(g) $3^{x+2} + 3^{x+1} + 2 \times 3^x = 126$ (h) $2^x + 3^{x-2} = 3^x - 2^{x+1}$

(i) $8^{x-1} - 23^{x-2} + 8 = 0$ (j) $\left(\frac{1}{4}\right)^{2-\sqrt{5x+1}} = 4 \times 2^{\sqrt{5x+1}}$

4. Solve:

(a) $5^x + \frac{1}{5^x} = 5\frac{1}{5}$

(b) $7^x + \frac{1}{7^x} = 7\frac{1}{7}$

(c) $9^x + \frac{1}{9^x} = 9\frac{1}{9}$

(d) $4^x + \frac{1}{4^x} = 16\frac{1}{16}$

(e) $5^x + 5^{-x} = 25\frac{1}{25}$

(f) $81 \times 3^x + 3^{-x} = 30$

5. **Solve:**

(a) $4 \times 3^{x+1} - 9^x = 27$

(b) $3 \times 2^{p+1} - 4^p = 8$

(c) $5^{2x} - 6 \times 5^{x+1} + 125 = 0$

(d) $2^{x-2} + 2^{3-x} = 3$

(e) $5^{x+1} + 5^{2-x} = 126$

(f) $3^{2y} - 4 \times 3^y + 3 = 0$

6. Solve $16^x - 5 \times 4^{x+1} + 64 = 0$. Prove that the value of x satisfies $5^x + \frac{125}{5^x} = 30$

7. a) If $x = 3^{\frac{1}{3}} + 3^{\frac{-1}{3}}$, then prove that $3x(x^2 - 3) = 10$

b) If $x = 2^{\frac{1}{3}} - 2^{\frac{-1}{3}}$, then prove that $2x^3 + 6x - 3 = 0$

Answers

2. (a) 2 (b) 3 (c) $\frac{1}{2}$ (d) $\frac{1}{2}$ (e) -3 (f) 1

3. (a) $\frac{1}{2}$ (b) 34 (c) 5 (d) 4 (e) 2 (f) 3 (g) 2 (h) 3
(i) 2 (j) 7

4. (a) ± 1 (b) ± 1 (c) ± 1 (d) ± 2 (e) ± 2 (f) 1, 3

5. (a) 1, 2 (b) 1, 2 (c) 1, 2 (d) 2, 3 (e) -1, 2 (f) 0, 1

7. 1, 2

Mixed Exercise

1. The commission of two employees of Nepal Pustak Pasal in five months is given below.

Months					
Name	**Baishakh**	**Jestha**	**Ashadh**	**Shrawan**	**Bhadra**
Employee A	Rs. 5000	Rs. 6000	Rs. 7000	Rs. 8000	Rs. 9000
Employee B	Rs. 2000	Rs. 3000	Rs. 4500	Rs. 6750	Rs. 10125

Answer the following questions based on the above table.

a) Which employee received commission in arithmetic sequence? Write with reason.

b) What is the total amount received by employee A at the end of five months? Calculate using formula.

2. Bishal borrowed Rs. 45,000 from his friend, Sunil on the condition that he would pay the sum in 6 instalments. He continued to pay Rs. 1000 more in each instalment than the previous one. Similarly, Sita also borrowed Rs. 63,000 from her friend Om kumari on the condition that she would also pay in 6 instalments. But, she continued to pay double in each instalment than the previous instalment.

a. Find, how much money do Bishal and Sita pay in the first installment.

b. Find the difference between the first and the last installments paid by Bishal and Sita.

c. In which installment do Bishal and Sita pay same amount of money? Calculate.

3. Two birds are migrated to a community forest on the first day. The birds that came the previous day had migrated with the double number of their friends the next day. If the birds continue to migrate at this rate, then

a. How many birds will migrate on the tenth day? Find it.

b. How many birds in total will migrate in the ten days?

4. In Nabraj's piggy bank, his father deposited money from the 1st baisakh to the 7th baisakh. Each day, we deposited double the amount he had deposited the previous day. On the seventh day, there was a total of Rs. 635 in his piggy bank. Than,

a. How much money did Nabraj's father deposit in Nabraj's piggy bank on the first day?

b. How many rupees were deposited on the seventh day?

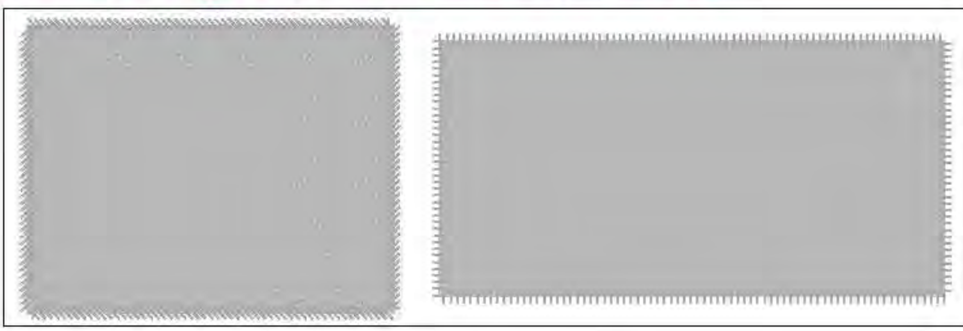
5. Sunil's father decided to deposit some amount of money on the occasion of Sunil's every birthday. Accordingly, Rs. 500 was deposited on the occasion of the first birthday, Rs. 1000 on the second birthday and Rs. 1500 on the third birthday. In this way, the deposit amount increased by Rs. 500 on every birthday.

a. How much amount would be deposited on the occasion of Sunil's 16th birthday? Find it.

b. What is the total amount accumulated upto Sunil's 16th birthday?

c. How many birthdays would Sunil wait to collect Rs. 1 Lakh? Write with a reason .

6. Harisaran could not estimate the length of the barbed wire to fence his two rectangular field. The area of his both land is 360 m^2. The difference between the length and breadth of the first land is 22m and the difference between the length and breadth of the second land is 9m.



Based on this, answer the following questions.

a. Find the length and breadth of both fields.

b. Does if require equal length of wire to fence both of the plots?

c. Which plot of land costs more by how much to fence around with the wire that costs Rs. 10 per meter? Calculate.

7. Sita and Ram are wife and husband. The present age of Ram is 30 years and Sita is 25 years.

a. What were Sita and Ram's ages x years ago?

b. If the product of their age was 500 x year ago, then find the value of x.

c. How many years later, the sum of their age will be 99? Calculate.

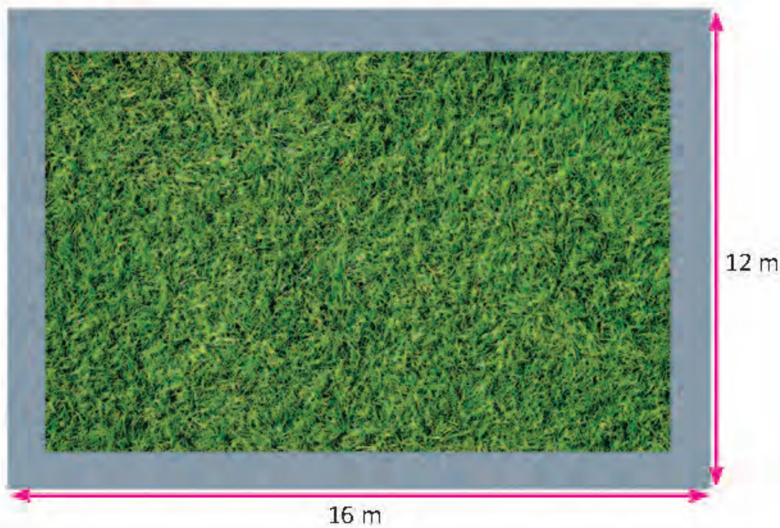
8. **The first term of an arithmetic sequence is 2. If the sum of the next five terms is equal to four times the sum of the first five terms. Then,**

   a. Find the common difference.

   b. Prove that $t_{20} = -112$.

   c. Find the sum of the first five terms.

9. **A footpath of equal width has been constructed around 16m x 12m grassland as shown in the figure. The total area of the grassland and footpath is 320m².**



   a. Let the width of the footpath be x. Write the equation based on the above context.

   b. Find the width of the footpath.

10. **In a rectangular field, the longer side is 40m more than the shorter side but the diagonal of the field is 40m more than its longer side.**

   a. Let the length of the shorter side be x and write the equation based on the above context.

   b. Find the length of the shorter side, longer side and diagonal of the field.

   c. Find how much it will cost to surround the land four times with a barbed fence at the rate of Rs. 10 per meter.

   d. How many pieces of land of 20m x 15m can be prepared on that rectangular field? Calculate.

11. Ramesh and Sita are brother and sister. Ramesh's present age is 30 years and Sita's is 25 years.

a. What were ramesh and sita's ages x years ago?

b. If the product of their age was 644 x years ago, then find the value of x.

c. How many years later will the product of their age be 864? Calculate.

12. Two cars leave a cross road at the same time. One is travelling towards the north and the other is travelling towards the west. When the car travelling towards the north has travelled a distance of 24 miles, the distance between the two cars was four miles more than three times the distance of the car travelling to the west.

a. Write the equation based on above context.

b. Find how far the car travelling to the west has travelled.

c. Find the actual distance between the two cars.

13. A bus travelled a distance of 90km at the same speed. If the speed of the bus was 15km/hr more than the previous speed, the total travelling time would have been reduced by 30 minutes.

a. Let the speed of the bus be x and write the equation based on above context.

b. What was the initial speed of the bus? Find it.

14. Simplify

(a) $\frac{1}{a-b}-\frac{2b}{a^2-b^2}$

(b) $\frac{a-4}{a^2-4a+16}+\frac{a+4}{a^2+4a+16}+\frac{128}{a^4+16a^2+256}$

(c) $\frac{2a-6}{a^2-9a+20}-\frac{a-1}{a^2-7a+12}-\frac{a-2}{a^2-8a+15}$

(d) $\frac{a+b}{2ab}(a+b-c)+\frac{b+c}{2bc}(b+c-a)+\frac{c+a}{2ac}(c+a-b)$

15. Solve:

(a) $3^{x+2}+3^{2-x}=82$

(b) $\frac{3^{2x+1}}{3^x}=\frac{82}{9}$

16. **Prove the following relations.**

(a) If $x = 1 + 2^{\frac{1}{3}} + 2^{\frac{2}{3}}$, then prove that $x(x^2 - 3x - 3) = 1$

(b) If $x = 3 + 3^{\frac{1}{3}} + 3^{\frac{2}{3}}$, then prove that $x(x^2 - 9x + 8) = 12$

(c) If $x = 2 - 2^{\frac{1}{3}} + 2^{\frac{2}{3}}$, then prove that $x(x^2 - 6x + 18) = 22$

Answers

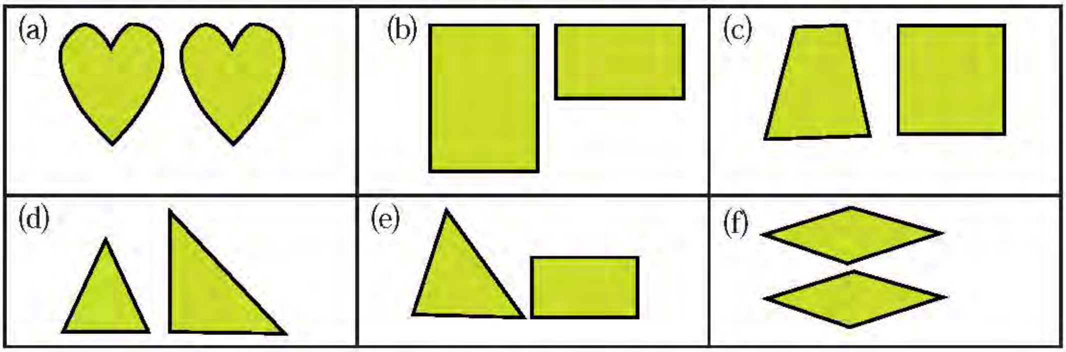
1. (a) staff A (b) Rs. 6000, Rs. 3000 (c) Rs. 8625
2. (a) Rs. 5000, Rs. 1000 (b) Rs.. 4000, Rs. 22000 (c) Fourth
3. (a) Rs. 024 (b) Rs. 2046
4. (a) Rs. 5 (b) Rs. 320
5. (a) Rs. 8,000 (b) Rs. 68,000 (c) 20^{th} years
6. (a) 20 m, 18 m and 24 m, 15 m (b) No, 76 m and 78 m
   (c) second Rs. 20
7. (a) $(30 - x)$ years and $(25 - x)$ years (b) 5 years (c) 22 years
8. (a) -6 (c) -50
9. (a) $(16 + 2x)(12 + 2x) = 320$ (b) 2 m
10. (b) 120 m, 160 m and 200 m (c) Rs. 33600 (d) 64
11. (a) $30 - x$ and $25 - x$ (b) 2 years (c) 2 years
12. (b) 7 km (c) 25 km
13. (a) $\frac{90}{x} - \frac{90}{x+15} = \frac{1}{2}$ (b) 45 km/h
14. (a) $\frac{1}{a+b}$ (b) $\frac{2(a-4)}{a^2 - 4a + 16}$ (c) $\frac{5}{(a-3)(a-4)(a-5)}$ (d) 3
15. (a) ± 2 (b) ± 2 (c) $\frac{2(a-4)}{a^2 - 4a + 16}$

Lesson 10

Triangle and Quadrilaterals

10.0 Review

Observe the given figures and discuss the questions given.



a. Do the pair of pictures match when they are overlapped?
b. Is the area of the pair of figures equal?
c. Which pair of figures are congruent and which are not?
d. Are all the figures having equal area congruent?

Discuss the above questions in groups and present the conclusion in your classroom.

10.1 Area of triangle and quadrilaterals

Activity 1

Observe the given figures and answer the questions.

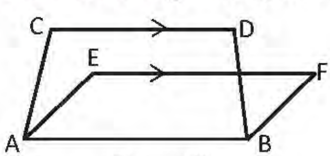


Figure (i)

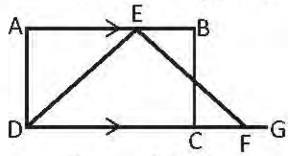


Figure (ii)

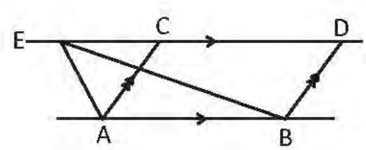


Figure (iii)

based on above figures,

a. In which of the given figures are the quadrilaterals formed with the same base and different parallel lines? Name the quadrilaterals.

b. In which of the given figures are the triangles and quadrilaterals formed with the different bases and the same parallel lines? Name the triangles and quadrilaterals.

c. In figure (c), identify the triangles and quadrilateral standing on the same base and lying between parallel lines.

A. The relation of parallelogram standing on the same base and lying between the same parallel lines

Activity 2

In the given figure, what will be the area of parallelograms ABCD and ABEF? What is the relation between them?

Here, the base and the height of both parallelograms are 7cm and 3cm respectively.

Hence, the area of parallelogram ABCD = base × height

$= AB \times DG$

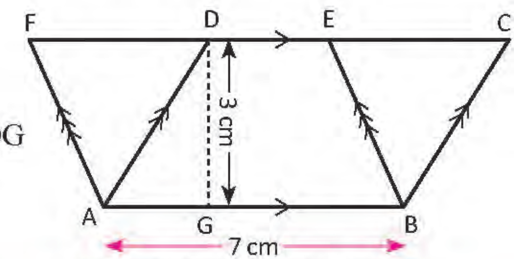
$= 7 \times 3 \ cm^2$

$= 21 \ cm^2$

Again, the area of parallelogram = AB × DG

$= 7 \times 3 \ cm^2$

$= 21 \ cm^2$



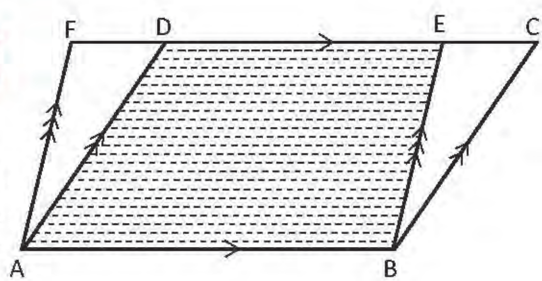
Hence, the area of parallelogram ABCD = area of parallelogram ABEF.

The area of parallelograms standing on the same base and lying between the same parallel lines are equal. Area of parallelogram ABCD = Area of parallelogram ABEF.

Theoretical Proof

Theorem 1

Parallelograms standing on the same base and lying between the same parallel lines are equal in area.



Given: parallelograms ABCD and ABEF are standing on the same base AB and lying between the same parallel lines AB and CF.s

To be proved: Area of parallelogram ABCD = area of parallelogram ABEF.

Proof

	Statements	Reasons
1.	In ΔADF and ΔBCE i) $\angle ADF = \angle BCE$ (A) ii) $\angle AFD = \angle BEC$ (A) iii) $AD = BC$ (S))	1. (i) Being opposite sides of parallelogram ABCD. (ii) Corresponding angle being AB//BC. (iii) Corresponding AF//BE
2.	$\Delta ADF \cong \Delta BCE$	AAS axiom.
3.	Area of ΔADF = Area of ΔBCE	Congruent triangles are equal in area.
4.	Area of ΔADF + area of trapezium ABED = area of ΔBCE + area of trapezium ABED	Adding the same trapezium on both sides.
5.	Area of parallelogram ABCD = Area of parallelogram ABEF.	From statement (4) (Whole part axiom)
Parallelograms standing on the same base and lying between the same parallel lines are equal.		

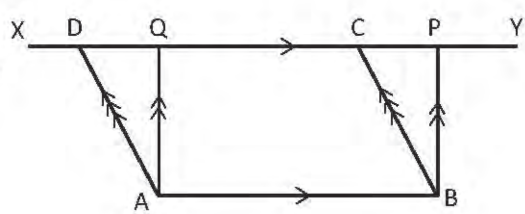
Proved.

Example 1

In the given figure, AB//XY. ABPQ is a rectangle and ABCD is a parallelogram. Prove that the area of parallelogram ABCD = area of rectangle ABPQ.

Solution

Parallelogram ABCD and rectangle ABPQ are standing on the same base AB and lying between the same parallel lines XY//AB.



To be proved: area of parallelogram ABCD = area of rectangle ABPQ.

Proof

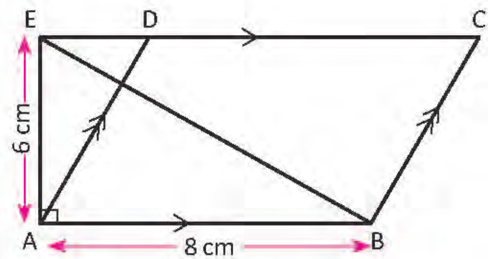
	Statements		Reasons
1.	Area of parallelogram ABCD = AB × AQ	1.	Area of parallelogram = base × height.
2.	Area of rectangle ABPQ = AB × AQ	2.	Area of rectangle = length × breadth
3.	Area of parallelogram ABCD = area of rectangle ABPQ.	3.	From statements (1) and (2)

Proved.

B. The relation of triangle and parallelogram standing on the same base and lying between the same parallel lines

Activity 3

Find the area of ΔABE and parallelogram ABCD from the given figure. Discuss with your friends and find the relation between them.



Here,

ΔABE is a right angled triangle with the base AB = 8 cm and the height AE = 6 cm.

Hence, the area of ΔABE = $\frac{1}{2}$ × base × height = $\frac{1}{2}$ × 8 cm × 6 cm = 24 cm^2.

Again, the base of parallelogram ABCD, AB = 8 cm and the height = (AE) = $6m^2$

Hence, the area of parallelogram ABCD = base × height = 8 cm × 6 cm = 48 cm^2

Therefore, the area of ΔABE = $\frac{1}{2}$ area of parallelogram ABCD.

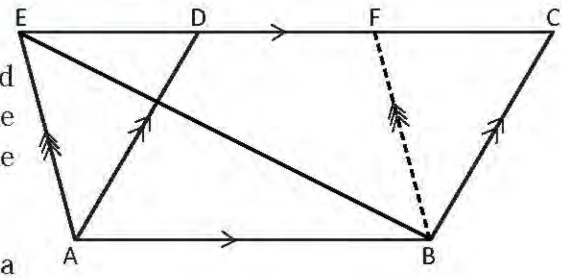
or, Area of parallelogram ABCD = 2 × area of ΔABC

The area of a triangle is half of the area of a parallelogram standing on the same base and lying between the same parallels. Area of ΔABE = $\frac{1}{2}$ area of parallelogram ABCD.

Theoretical Proof

Theorem 2

The area of a triangle is half of the area of a parallelogram standing on the same base and lying between the same parallels.



Given: Parallelogram ABCD and triangle ABE are standing on the same base AB and lying between the same parallels AB and EC.

To be proved: area of $\Delta ABC = \frac{1}{2}$ area of parallelogram ABCD

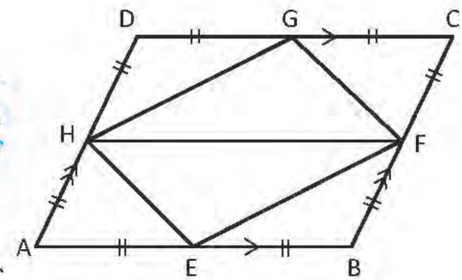
Construction: Draw AE//BF. Now, ABFE is a parallelogram.

Proof:

	Statements		Reasons
1.	Area of parallelogram ABFE = area of parallelogram ABCD	1.	Both being on the same base AB and between the same parallels AB and CE.
2.	Area of $\Delta ABE = \frac{1}{2}$ area of parallelogram ABFE	2.	Diagonal EB bisects the parallelogram ABFE.
3.	Area of $\Delta ABE = \frac{1}{2}$ area of parallelogram ABCD	3.	From statements (1) and (2).
Hence, the area of a triangle is half the area of a parallelogram standing on the same base and lying between the same parallels.			

Proved.

Example 2



If E, F, G and H are the mid points of sides of the parallelogram ABCD. Then prove that the area of parallelogram EFGH = $\frac{1}{2}$ area of parallelogram ABCD.

Solution

Given: E, F, G and H are the mid points of sides AB, BC, CD and DA respectively of parallelogram ABCD given along sides.

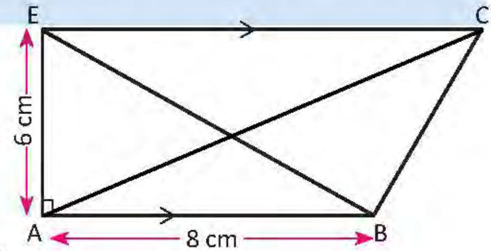
To be proved: area of parallelogram EFGH = $\frac{1}{2}$ area of parallelogram ABCD.

Proof

	Statements	Reasons
1.	AH = $\frac{1}{2}$ AD	H is the midpoint of AD
2.	BF = $\frac{1}{2}$ BC	F is the midpoint of BC
3.	AD = BC	Opposite sides of parallelogram ABCD
4.	Hence, AH = BF and AH // BF	From statements (1), (2) and (3), AD//BC
5.	Hence, ABFH being a parallelogram	By AH//BF and AH = BF
6.	Similarly, CDHF being a parallelogram	By CF//DH and CF = DH
7.	Now, the area of ΔHEF = $\frac{1}{2}$ area of parallelogram ABFH	Both being on the same base HF and between the same parallel lines HF and AB.
8.	Again, the area of ΔHGF = $\frac{1}{2}$ area of parallelogram CDHF	Both being on the same base HF and between the same parallel lines HF and CD.
9.	Area of ΔHEF + area of ΔHGF = $\frac{1}{2}$ area of parallelogram ABFH + $\frac{1}{2}$ area of parallelogram CDHF or, Area of parallelogram HEFG = $\frac{1}{2}$ (area of parallelogram ABFH + area of parallelogram CDFH) $\therefore$ or, Area of parallelogram HEFG = $\frac{1}{2}$ area of parallelogram ABCD	From statements (7) and (8) using addition law.
	Proved.	

C. The relation of triangles standing on the same base and lying between same parallel

Activity 3



Find the area of ΔABE and ΔABC based on the given figure. Compare the area of triangles and find the relation.

Here, ABE is a right angled triangle with the base AB = 8 cm and height AE = 6cm (by AB//EC)

Area of ABE = $\frac{1}{2}$ × perpendicular × base = $\frac{1}{2}$ × 6cm × 4 cm = 24 cm^2

Therefore, area of ΔABC = $\frac{1}{2}$ base × height = $\frac{1}{2}$ × 8 cm × 6 cm = 24 cm^2

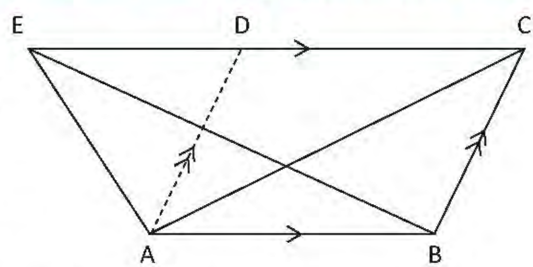
Hence, the area of ΔABE = the area of ΔABC

The area of triangles standing on the same base and lying between the same parallel lines are equal.

Theoretical Proof

Theorem 3

The area of triangles standing on the same base and lying between the same parallel are equal.



Given: Triangles ABE and ABC are standing on base AB and lying between parallel lines EC and AB.

To be prove: The area of ΔABE = the area of ΔABC

Construction: Draw AD//BC. So that, ABCD is a parallelogram.

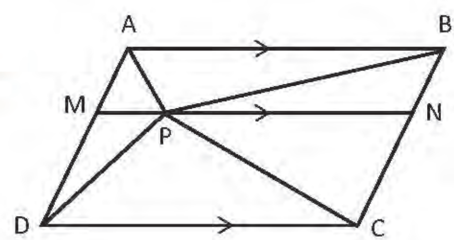
Proof

	Statements	Reasons
1.	Area of $\Delta ABC == \frac{1}{2}$ area of parallelogram ABCD	Diagonal AC bisects parallelogram ABCD.
2.	Area of $\Delta ABE = \frac{1}{2}$ area of parallelogram ABCD	ΔABE and parallelogram ABCD are standing on the same base and lying between the same parallels.
3.	Area of ΔABC = area of ΔABE	From statements (1) and (2) above
The area of triangles standing on same base and lying on the same parallel are equal.		

Proved.

Example 3

In the given figure, ABCD is a parallelogram. Prove that area of ΔAPB + area of $\Delta PCD = \frac{1}{2}$ area of parallelogram ABCD.



Solution

Given: ACBD is a parallelogram.

To be Prove: Area of ΔAPB + area of $\Delta PCD = \frac{1}{2}$ area of parallelogram ABCD.

Construction: Draw MN//CD, where MN passes through the point P.

Here, ABNM and MNCD are parallelograms.

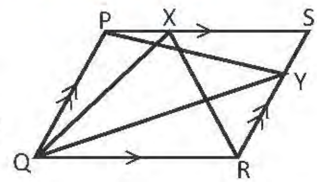
Proof

	Statements	Reasons
1.	Area of $\Delta APB = \frac{1}{2}$ area of parallelogram ABNM	Triangle and parallelogram standing on the same base AB and lying between AB//MN
2.	Area of $\Delta PCD = \frac{1}{2}$ area of parallelogram MNCD	Triangle and parallelogram standing on the same base AB and lying between CD//MN
3.	Area of ΔAPB + area of $\Delta PCD = \frac{1}{2}$ area of parallelogram ABNM + $\frac{1}{2}$ area of parallelogram MNCD	Adding statements (1) and (2)
4.	Area of ΔAPB + area of $\Delta PCD = \frac{1}{2}$ area of parallelogram ABCD	From statement (3) and by whole part axiom.

Proved.

Example 4

In the figure, PQRS is a parallelogram. In which X and Y are any two points on PS and SR respectively. Prove that area of ΔPQY = area of ΔQRX.



Solution

Given: PQRS is a parallelogramthere. X and Y are any two points on PS and SR respectively. PQY and QXR are two triangle.

To be Prove: area of ΔPQY = area of ΔQRX.

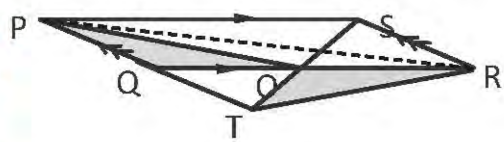
Proof

	Statements	**Reasons**
1.	Area of $\Delta PQY = \frac{1}{2}$ area of parallelogram PQRS	The triangle PQY and the parallelogram PQRS standing on the same base PQ and between the same parallels PQ and SR.
2.	Area of $\Delta QRX = \frac{1}{2}$ area of parallelogram PQRS	ΔQRX and parallelogram PQRSS are standing on the same base QR and between the same parallels QR and PS.
3.	Area of ΔPQY = area of ΔQRX	From statements (1) and (2) above.

Proved.

Example 5

In the given figure, PQRS is a parallelogram in which S is joined with any point O of side QR. SO and PQ are produced upto the point 'T'. Then, prove that the area of the ΔPQO = the area of ΔRTO.



Solution

Given: The point O is on the side QR in the parallelogram PQRS. PQ and SO are produced up to the point T.

To be Prove: area of ΔPQO = area of ΔRTO

Construction: Draw a diagonal PR in parallelogram PQRS.

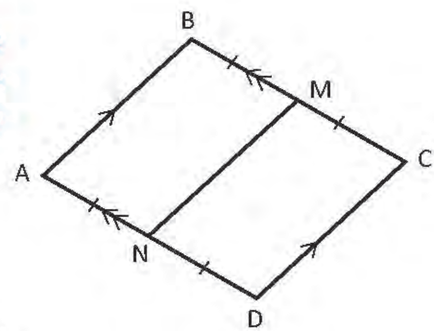
Proof

	Statements	Reasons
1.	Area of ΔPRS = area of ΔPQR	Diagonal PR bisects parallelogram PQRS.
2.	Area of ΔPRS = area of ΔTRS	Standing on the same base SR and lying between SR//QR.
3.	Area of ΔPQR = area of ΔTRS	From statements (1) and (2) above.
4.	Area of ΔPOR = area of ΔSOR	Triangles standing on the same base OR and lying between PS//QR.
5.	$\Delta PQR - \Delta POR = \Delta TRS - \Delta SOR$	(Equal axiom.) subtracting statement (4) from statement (3)
6.	Area of ΔPQO = area of ΔRTO	Remainder axiom, from statement (5)

Proved.

Example 6

ABCD is a parallelogram. M and N are the mid points of BC and AD respectively. Prove that MN divides parallelogram ABCD into two equal parallelograms.



Solution

Given: ABCD is a parallelogram. M and N are the midpoints of BC and AD respectively.

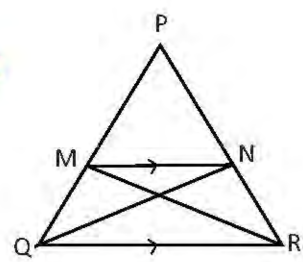
To be Prove: area of parallelogram ABMN = area of parallelogram CDNM.

Proof

	Statements	Reasons
1.	AD = BC and AD//BC	Being opposite sides of a parallelogram.
2.	Again, BM = MC and AN = ND	By M and N are midpoints of BC and AD respectively.
3.	AN = BM and AN//BM	From Statements (1) and (2).
4.	ABMN is a parallelogram	By AN = BM and AN//BM
5.	Again, DN = CM and DN//CM	Same as above
6.	CDNM is a parallelogram	By DN = CM and DN//CM
7.	area of parallelogram ABMN = area of parallelogram CDNM	Parallelogram standing on equal bases AN = ND and lying between BC//AD.

Proved.

Example 7



In the given figure, MN//QR, prove that area of ΔPQN = area of ΔPRM.

Solution

Given: In triangle PQR, MN//QR.

To be prove: area of ΔPQN = area of ΔPRM.

Proof

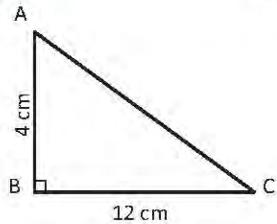
	Statements	Reasons
1.	Area of ΔMNQ = area of ΔMNR	Triangles standing on the same base MN and lying between QR// MN
2.	$\Delta PMN + \Delta MNQ = \Delta PMN + \Delta MNR$	Adding ΔPMN on both sides of statement (1)
3.	Area of ΔPNQ = area of ΔPRM	From statement (2) [by whole part axiom]

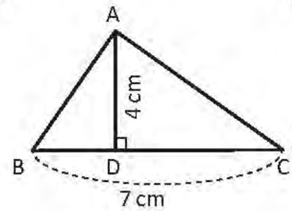
Proved.

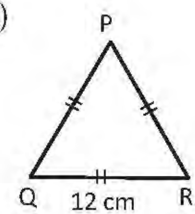
Exercise 10.1

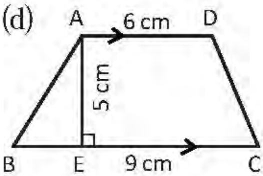
A) Solve:

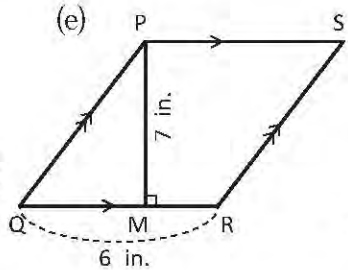
1. Find the area of given figures.

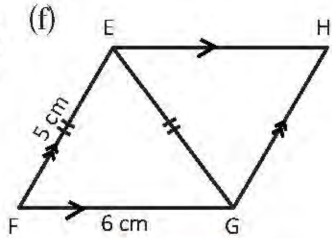
(a)


(b)


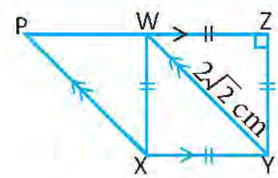
(c)


(d)


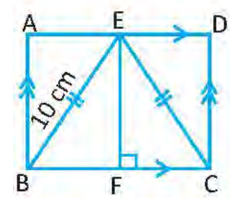
(e)


(f)


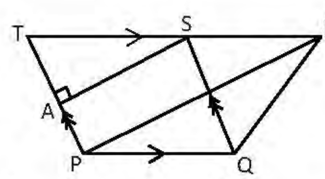
2. (a) In the given figure, WXYZ is a square. If WY $= 2\sqrt{2}$cm then find the area of parallelogram PXYW.



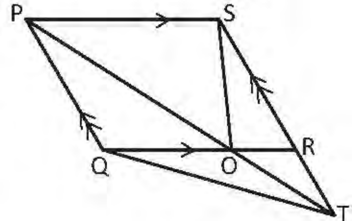
(b) The given figure, if BE = EC, EF⊥BC, BE = 10 cm and AD = 16 cm, find the area of parallelogram ABCD.



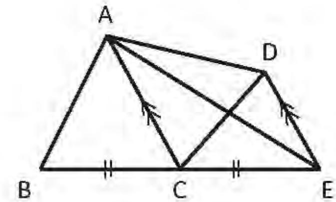
(c) The given figure, PQST is a parallelogram. If SA⊥TP, SA = 8cm and area of ΔPQR = 64cm^2 then find the length of TP.



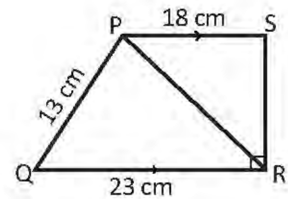
(d) In the given figure, PQRS is a parallelogram. Taking any point O on QR, join PO produced PO and SR to the point T. Q and T are joined. Proved that the area of ΔQOT = area of ΔROS.



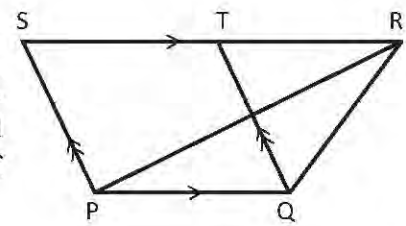
(e) In the given figure, AC//DE and BC = EC. If, area of ΔACE is $24cm^2$, then find the area of quadrilateral ABCD.



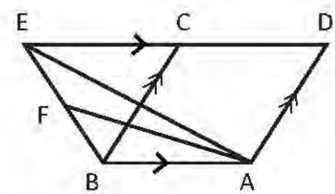
3. (a) In trapezium PQRS, PS//QR, PQ = 13cm, PS = 18cm, QR = 23cm and $SR \perp QR$, then find the area of ΔPSR.



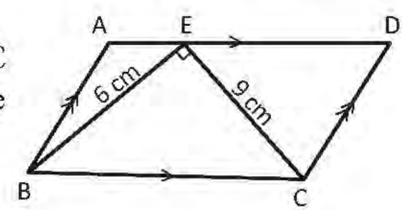
(b) In the adjoining figure, PQ//SR, PS//QT, area of the trapezium PQRS is $95cm^2$ and area of ΔQRT is $15cm^2$. Find the area of ΔRPQ.



(c) In the adjoining figure, DE//AB, AD//BC and F is the midpoint of BE. If the area of ΔAFE is $12cm^2$ then find the area of parallelogram ABCD.

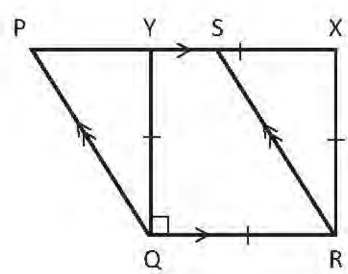


(d) In the given parallelogram, ABCD, $\angle BEC = 90°$, BE = 6cm, CE = 9cm, then find the area of parallelogram ABCD.

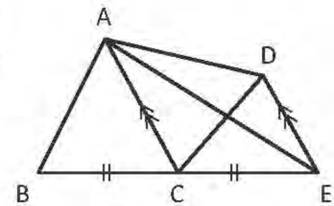


4. Prove that:

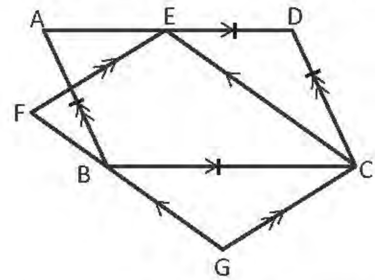
(a) In the given figure, PQRS is a parallelo gram and QRXY is a square. Prove that the area of the parallelogram PQRS and the area of square QRXY are equal.



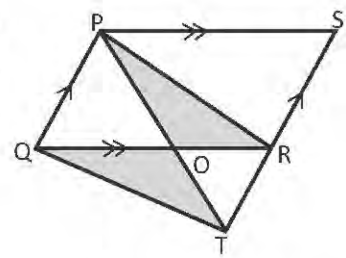
(b) In the given figure, ABCD is a quadrilateral. Drawn PE so that AC//DE. Prove that the area of quadrilateral ABCD = area of ΔABE.



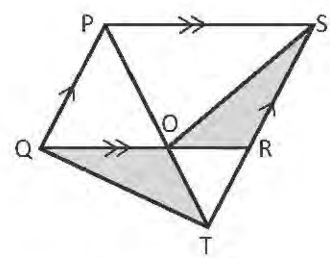
(c) In the given figure, ABCD and EFGC are parallelograms. Prove that the area of parallelogram ABCD = area of parallelogram EFGC.



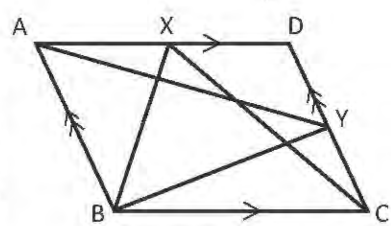
(d) In the given figure, PQRS is a parallelogram and SR is extended upto T. Join P and T where PT intersects QR at O. Prove that area of ΔPOR = area of ΔQOT.



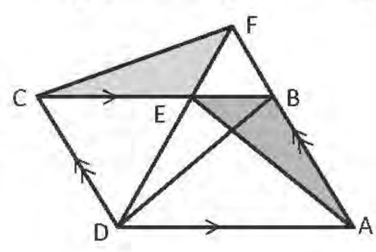
(e) In the given figure, PQRS is a parallelogram in which SR is extended upto T and PT and ST are joined where PT meets QR at O. Prove that the area of ΔQOT = area of ΔROS.



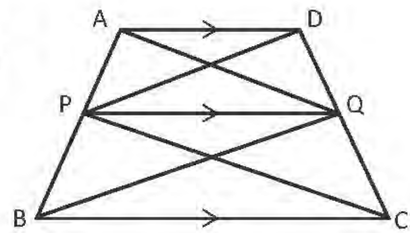
(f) In the adjoining figure, ABCD is a parallelogram in which X and Y are any two points in which triangles XBC and YAB are made. Prove that area of ΔABY = area of ΔABX + area of ΔCDX.



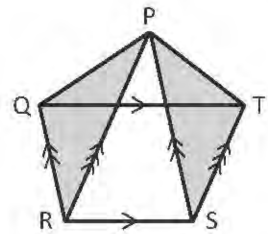
(g) In the adjoining figure, ABCD is a parallelogram where E is any point on BC, DE and AB are produced upto F. Join C and F. Prove that area of ΔCEF = area of ΔABE.



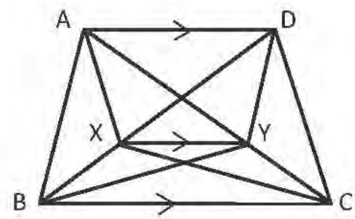
(h) In the given figure, ABCD is a trapezium in which AD//PQ//BC. Prove that the area of ΔAQB and the area of ΔDPC are equal.



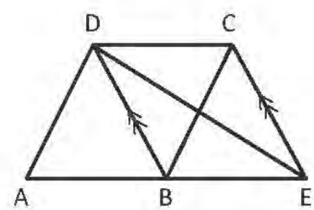
(i) In the given figure, QT//RS, PR//TS and PS//QR. Prove that the area of ΔPQR = the area of ΔPTS.



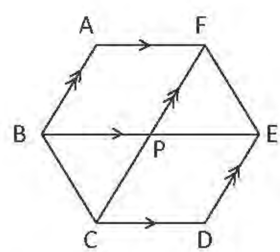
(j) In the given figure, ABCD is a trapezium in which Y and X are any points on AC and BD respectively where AD//XY//BC. Prove that the area of ΔAXC and the area of ΔBYD are equal.



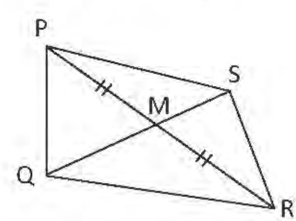
(k) In the adjoining figure, ABCD is a quadrilateral in which DB//CE. Prove that the area of ΔADE is equal to the area of quadrilateral ABCD.



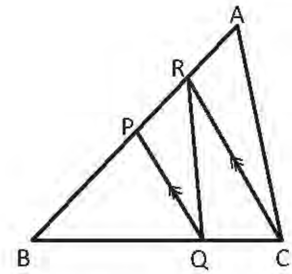
(l) In the given figure, ABCDEF is a hexagon in which AF//BE//CD and AB//CF//DE. If the area of parallelograms ABPF and CDEP are equal, then prove that EF//BC.



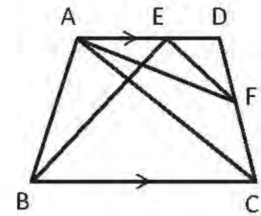
(m) In the given figure, the diagonal QS of the quadrilateral PQRS bisects another diagonal PR. Prove that area of ΔPQS is equal to half the area of the quadrilateral PQRS.



(n) In the given figure, P is midpoint of AB and Q is any point in BC. If BC//PQ then prove that area of $\Delta BQR = \frac{1}{2}\Delta ABC$ area of ΔABC.



(o) In the given figure, AD//BC. If area of ΔABE and ΔACF are equal then prove that EF//AC.



Project work

Prepare the chart using different colors on graph paper or a square grid, Show the following relationships, which are standing on the same base and lying between the same parallel lines

(a) Relation of parallelograms.

(b) Relation of parallelogram and triangle, and

(c) Relation of triangles.

Answers

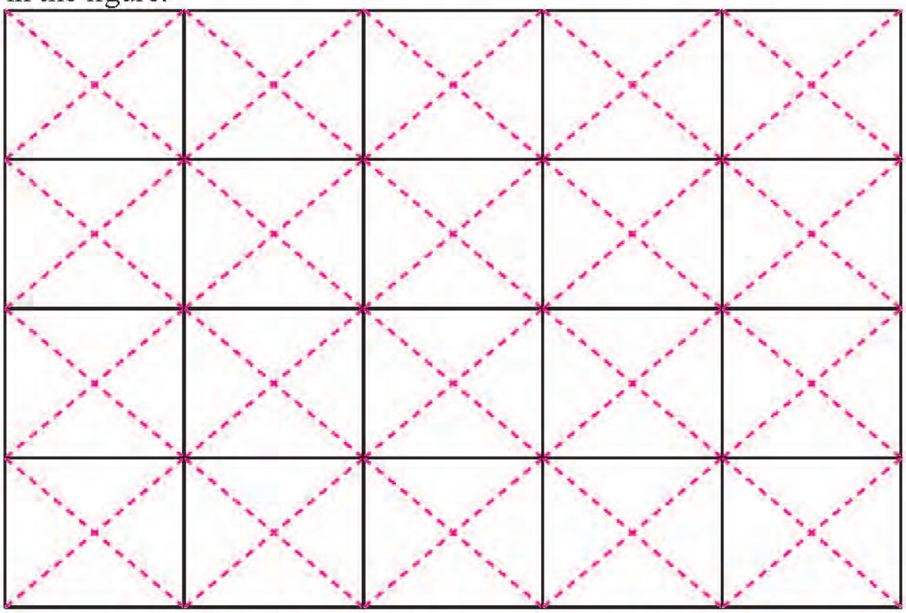
1. (a) 24 cm^2 (b) 21 cm^2 (c) $36\sqrt{3}$ cm^2
   (d) 37.5 cm^2 (e) 42 sq.inch (f) 24 cm^2
2. (a) 4 cm^2 (b) 96 cm^2 (c) 16 cm^2
   (d) Show it to the teacher. (e) 48 cm^2
3. (a) 138 cm^2 (b) 40 cm^2 (c) 48 cm^2
   (d) 54 cm^2 4. Show it to the teacher.

Lesson 11

Construction

11.0 Review

Draw a figure as given below in a square grid. Find and shade the following condition in the figure.



(a) The parallelogram standing on the same base and lying between the same parallels are equal in area.

(b) A triangle which is equal to one half of the area of a parallelogram standing on the same base and lying between the same parallels.

(c) A pair of triangles standing on the same base and lying between same parallels are equal in area.

11.1. Construction of triangle and quadrilaterals with equal areas

Steps:

(a) Draw a rough sketch using a ruler and a pencil according to the given condition.

(b) Place the values on the rough sketch.

(c) Using a compass and a ruler, construct and name correct figures according to the given condition and data following the given steps.

(A) Construction of Parallelograms equal in area

Activity 1

Construct a parallelogram ABCD having AB = 4cm, BC = 5.5cm and ∠ABC = 60°. How can we construct a parallelogram whose area is equal to the area of parallelogram ABCD?

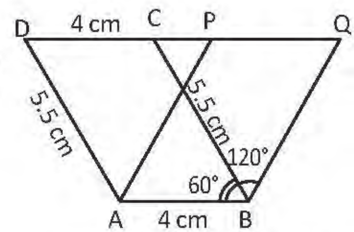
(a) Parallelogram ABQP having an angle 120^0

(b) Parallelogram ABQP having one side 6 cm

(a) Parallelogram ABQP having an angle 120°

i. Draw a rough sketch of parallelogram ABCD according to the given data.

ii. Draw AB = 4cm



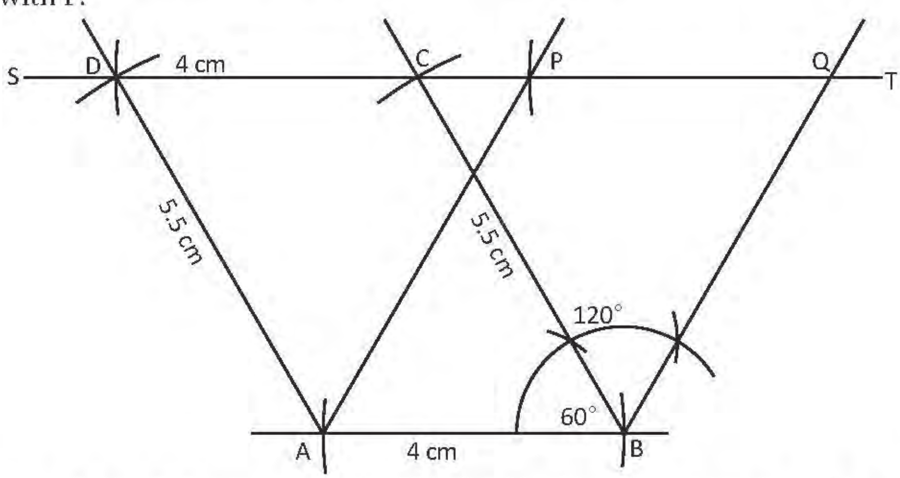
iii. Draw an angle 60° at the point B. Again, take an arc of radius 5.5cm from the point B cutting the line from 60° at B and name the point C.

iv. Now, let us give name the point D where 5.5cm arc from A and 4cm from C are intersected.

v. Now, parallelogram ABCD is prepared after joining D with C and A with D.

vi. Extend DC up to T. Construct an angle 120° at point B and name the point Q where the line ST meets.

vii. Take an arc equal to AB and cut QS from Q and name the point P. Join A with P.

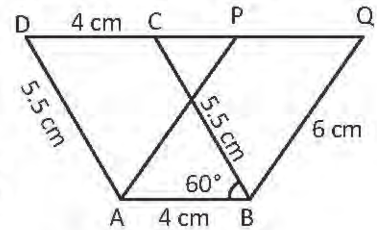


The area of parallelogram ABQP is equal to the parallelogram ABCD. How? Discuss with your friend.

Parallelograms ABCD and ABQP are standing on the same base and lying between the same parallels AB//ST are equal in area.

(b) Parallelogram ABQP having one side 6 cm

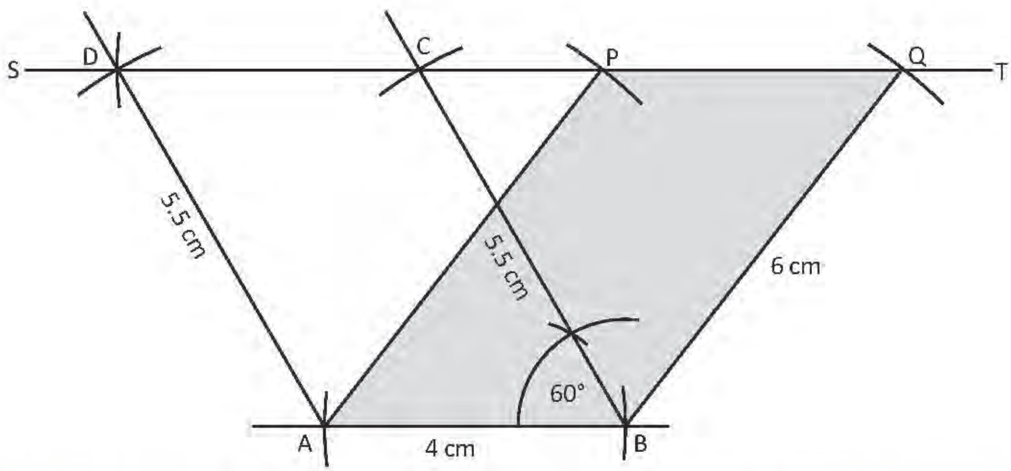
i. Let's draw a rough sketch of a parallelogram ABCD and place the value.



ii. Let's construct a parallelogram ABCD in which AB = 4cm, BC = 5.5cm and $\Delta ABC = 60°$. Extend CD upto ST.

iii. Take an arc of radius 6cm from point A on the line ST cutting it at a point and name it P. Again, take an arc of radius AP, from the point B on the line ST cutting it at a point and name it Q.

iv. Join A and P, B and Q by using a ruler and a pencil.



Why are the area of parallelograms ABQP and ABCD equal? Discuss with friends.

Parallelograms ABCD and ABQP standing on the same base AB and lying between the same parallels AB//ST are equal in area.

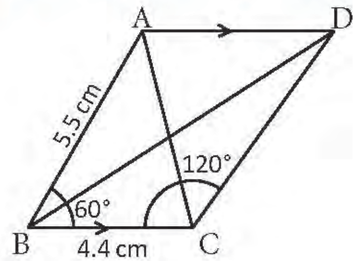
(B) Construction of triangles equal in area

Activity 2

How can we construct a triangle which is equal in area with the triangle ABC having ∠ABC = 60°, BC = 4.4 cm and AB = 5 cm?

a) ΔABC having an angle 120°.

b) ΔDBC having a side 6.2cm.



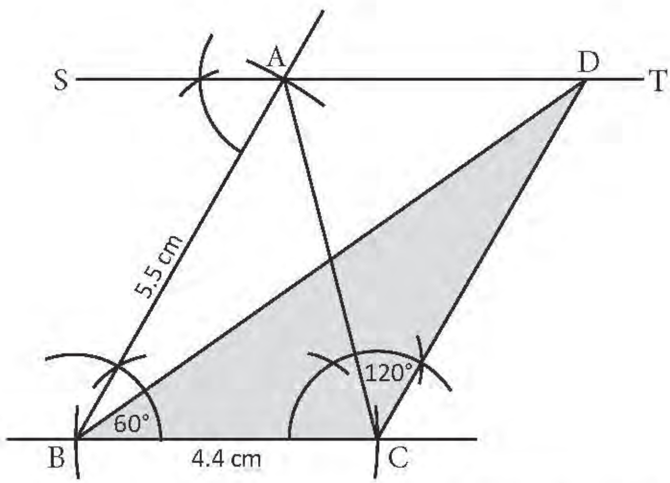
a) ΔABC having an angle 120°.

i. Let's draw a rough sketch of a triangle and place the values..

ii. Construct a ΔABC having BC = 4.4cm, ∠ABC = 60° and AB = 5.5cm.

iii. Draw BC//ST from A making ∠ABC = ∠BAS.

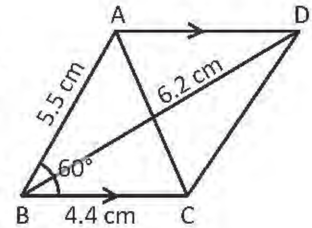
iv. Join D and C.



The area of the ΔDBC is equal to the area of the ΔABC. How? Discuss with your friends.

The triangles standing on the same base and lying between the same parallels BC//ST are equal in area.

b) A triangle DBC having one side 6.2cm



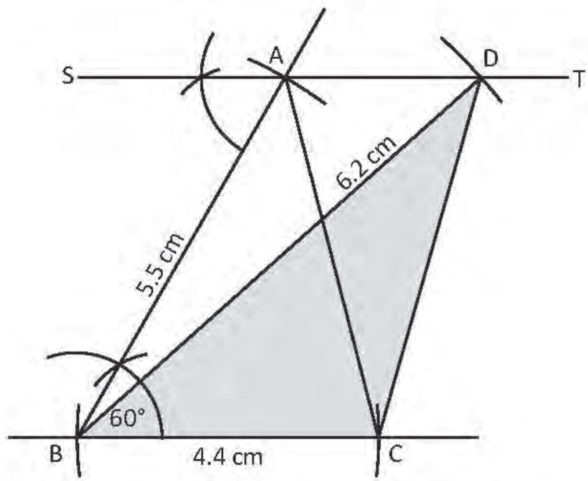
i. Let's draw a rough sketch of ΔABC as and place the values.

ii. Construct a triangle ABC in which BC = 4.4cm, $\angle ABC = 60°$ and AB = 5.5cm.

iii. Draw BC//ST from A making $\angle ABC = \angle BAS$.

iv. Now, take an arc of radius 6.2cm and center B cutting ST at a print and name it D.

v. Join D and C.



The area of ΔBCD is equal to the area of ΔABC. How? Discuss with your friends.

The triangle standing on the same base and lying between the same parallels BC//ST are equal in area.

(C) Construction of triangles and parallelograms equal in area

Activity 3

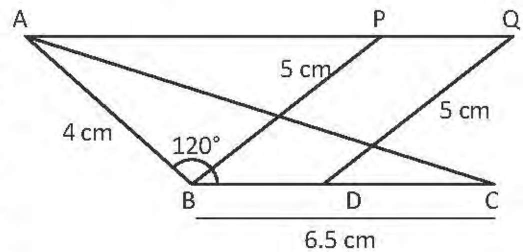
Construct a triangle ABC in which AB = 4cm, BC = 6.5cm and ∠ABC = 120°. How can we construct a parallelogram having a side PB = 5cm and equal to the area of ∆ABC?

i) Let's draw a rough sketch of a triangle ABC and place the values.

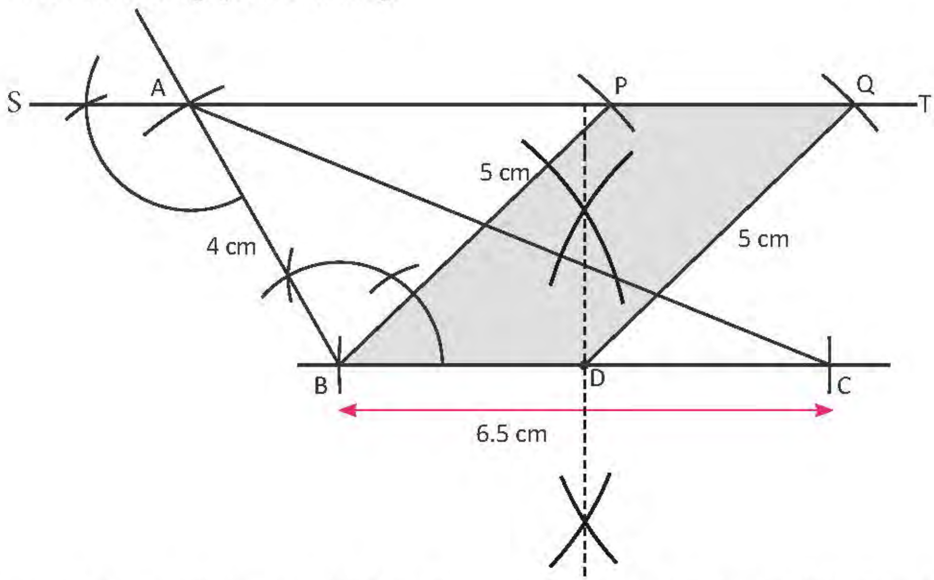
ii) Construct a ∆ABC in which AB = 4cm, BC = 6.5cm and ∠ABC = 120°.

iii) Draw ST//BC from 'A' making ∠ABC = ∠BAS.

iv) Bisect BC and mark the midpoint D. Now, take an arc of radius 5cm from the point B on the line ST cutting it at a point and name it P.



v) Take an arc of radius BP, from the point D on the line PT cutting it at a point and name it. Q. Join D and Q.



The area of the parallelogram PBD is Q equal to the area of the ∆ABC. How? Discuss with your friend.

The area of a triangle is half of the area of the parallelogram standing on the same base and lying between the same parallels. If the parallelogram is constructed on the half of the base of triangle, then the area of the triangle is equal to the area of parallelogram.

Activity 4

Construct a parallelogram ABCD in which AB = 5cm, BC = 4cm and ∠ABC = 60^0. How can construct a triangle PBE with a side PB = 6cm and equal area to the parallelogram ABCD?

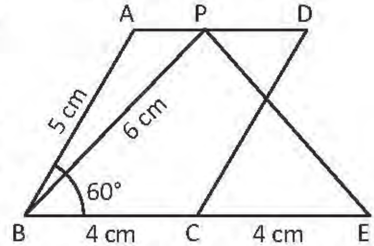
i) Draw a rough sketch of parallelogram and place the values.

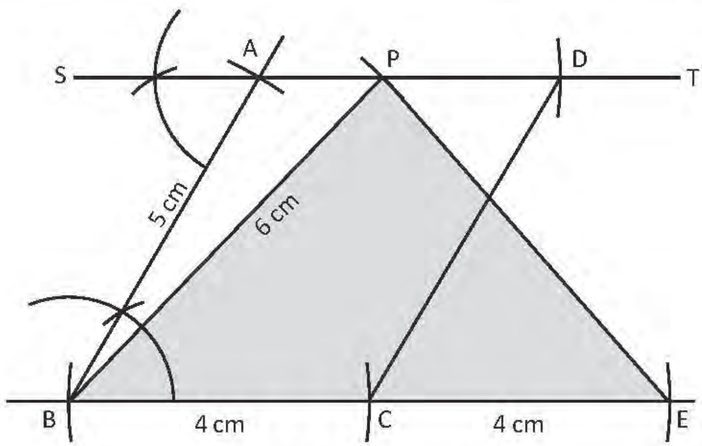
ii) Construct a parallelogram in which AB = 5cm, BC = 4cm and ∠ABC = 60°.

iii) Produce BC to E making BC = CE. Produce AD up to ST.

iv) Take an arc of radius 6.2cm from the point B on the line ST cutting it at a point and name it P.

v) Join B and P, also P and E.





The area of the triangle BPE is equal to the area of the parallelogram ABCD. How? Discuss with your friends.

The area of a parallelogram is double of the triangle standing on the same base and lying between the same parallels. If the triangle base is double of the base of the parallelogram, then their area are equal.

(D) Construction of a triangle and a quadrilateral equal in area

Activity 5

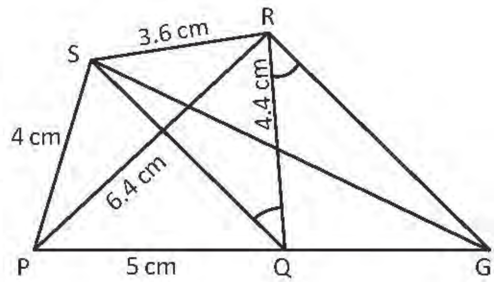
Construct a quadrilateral PQRS in which PQ = 5cm, PS = 4cm, QR = 4.4cm, RS = 3.6cm and diagonal PR = 6.4cm. How to construct a triangle PSG whose area is equal to the quadrilateral PQRS?

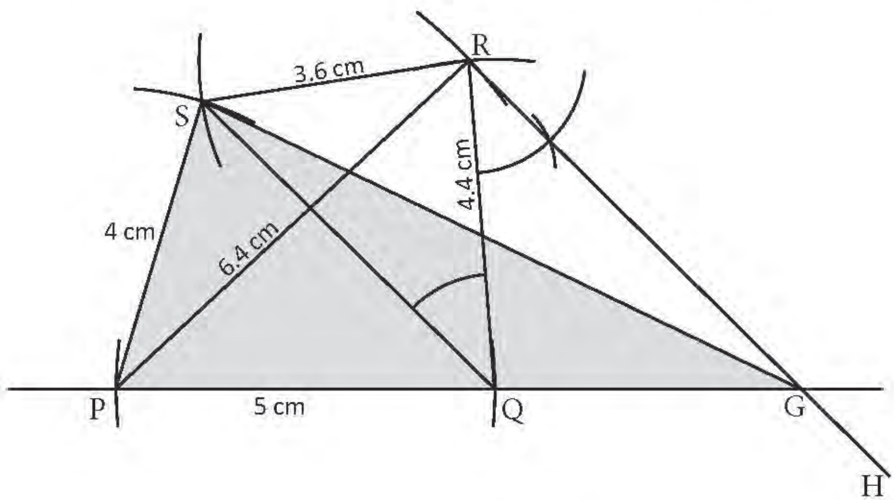
i) Draw a rough sketch of quadrilateral PQRS according to the given data.

ii) Construct a quadrilateral PQRS in which PQ = 5cm, PS = 4cm, QR = 4.4cm, RS = 3.6cm and diagonal PR = 6.4 cm.

iii) Draw a diagonal SQ.

iv) Draw SQ//RH from R making ∠SQR = ∠QRH. Now, extend PQ and name the point H where the line meets RH. Join S and G.





The area of a triangle PSG is equal to the area of the quadrilateral PQRS. How? Discuss with your friends.

Area of ΔSQR = area of ΔSQG,
Adding ΔPSQ on both sides [∴The triangles on the same base and same parallels RH//SQ.]
Area of (ΔSQR + ΔPSQ) = area of (ΔSQG + ΔPSQ)
∴ area of quadrilateral PQRS = area of ΔPSG.

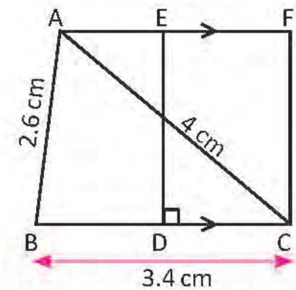
Example 1

Construct a triangle ABC in which AB = 2.6cm, BC = 3.4cm and CA = 4cm. Again construct a rectangle whose area is equal to ΔABC.

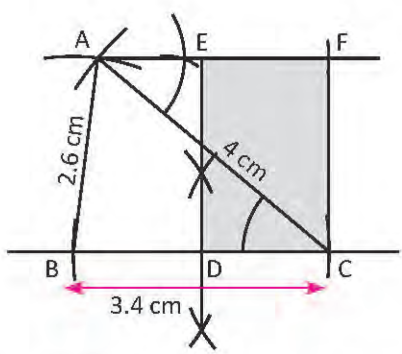
Solution

Here, ΔABC having AB = 2.6cm, BC = 3.4cm and CA = 4cm.

Rough sketch



The construction of required CDEF whose area is equal to the area of ΔABC is below.



Hence, CDEF is the required rectangle whose area is equal to the area of ΔABC.

Exercise 11

1. (a) Construct a parallelogram whose one angle is $45°$ and the area is equal to the parallelogram with AB = 4 cm, AD = 6 cm and $\angle$BAD= $60°$.

   (b) Construct a parallelogram whose one angle is $75°$ and the area is equal to the parallelogram with AB = 5 cm, AD = 6 cm and diagonal BD = 6 cm.

   (c) Construct a parallelogram whose diagonal is 7.2 cm and area is equal to the parallelogram whose diagonals are 6cm and 4.8 cm and the angle between its diagonals is $30°$.

   (d) Construct the parallelogram ABXY equal in area to the parallelogram ABCD with AB = 5 cm, AD = 6cm and diagonal BD = 8 cm.

   (e) Construct a parallelogram whose one angle is $30°$ which equals area of a parallelogram PQRS having PQ = 4.2 cm, QR = 6 cm and $\angle$PQR = $60°$.

2. (a) Construct a triangle ABC in which BC = 6.4cm, AB = 5.6cm and AC = 6cm. Then, construct another triangle having a side 7cm and equal in area to the triangle ABC.

   (b) Construct a ΔLMN in which LM = 4.3 cm, $\angle$NLM = $30°$ and $\angle$LMN = $45°$, then construct another triangle OLM having OM = 7.5 cm and which is equal in area to the ΔLMN.

   (c) Construct a triangle PQR in which PQ = 4.5 cm, QR = 7c m and PR = 6cm. Then, construct another triangle having a side 8cm and which is equal in area to the triangle PQR.

   (d) Construct triangle equal in area to the triangle ABC with AB = 4.2 cm, BC = 5.2 cm and CA = 3.5 cm.

   (e) Construct a triangle MBC with one side 8 cm and whose area is equal to the area of a triangle ABC having AB = 7.2 cm, BC = 5.9 cm and CA = 6.1 cm.

3. (a) Construct a triangle PQR in which PQ = 6.5 cm, QR = 6 cm and 5.5 cm then construct a parallelogram RSTI having $\angle$TSR = $75°$ and is equal in area to the triangle PQR.

   (b) Construct a ΔABC in which AC = 5 cm, BC = 4.8 cm and $\angle$ABC = $45°$. Then, construct a parallelogram CDEF having CD = 7.5cm and is equal in area to the ΔABC.

   (c) Construct a ΔABC in which AB = 4 cm, BC = 3.2 cm and AC = 3.5 cm. Then, construct a parallelogram BXYE having BE = 5 cm and is equal in area to the ΔABC.

(d) Construct a rectangle equal in area to the triangle XYZ having XY = 4 cm, YZ = 6.8 cm and ZX = 6.5 cm.

(e) Construct a triangle PQR in which PQ = 7.1 cm, $\angle RPQ = 60^\circ$ and PR = 5.7 cm. Then, construct a parallelogram having one side 7.5 cm and is equal in area to the triangle PQR.

4. (a) Construct a parallelogram in which AB = 6 cm, BC = 4.5 cm and $\angle DAB = 60^\circ$, then construct the ΔAEF having FE = 7.5cm and is equal in area to the parallelogram.

(b) Construct a parallelogram ABCD in which AB = 6cm, BC = 4cm and $\angle BAD = 45^\circ$. Then, construct a triangle APQ having $\angle APQ = 60^\circ$ and is equal in area to the parallelogram ABCD.

(c) Construct a triangle equal in area to the parallelogram PQRS in which PQ = 5 cm, diagonal PR = 6 cm and diagonal QS = 8 cm.

(d) Construct a triangle equal in area to the parallelogram having EF = 5 cm, GF = 4 cm and $\angle EFG = 120^\circ$.

(e) Construct a triangle equal in area to the parallelogram IJKL in which IJ = 5 cm, IK = 6 cm and JL = 8 cm.

5. (a) Construct a quadrilateral PQRS in which PQ = QR = 5.5 cm, RS = SP = 4.5cm and $\angle SPQ = 75^\circ$. Then, construct a triangle PST equal in area to the quadrilateral PQRS.

(b) Construct a quadrilateral ABCD in which AB = 4.5 cm, BC = 5.5 cm, CD = 5.7 cm and DA = 4.9 cm and diagonal BD = 5.9 cm. Then, construct a triangle DAE equal in area to the quadrilateral ABCD.

(c) Construct a ΔQRT equal in area to the quadrilateral PQRS in which PQ = 5 cm, QR = 7 cm, RS = 4.5 cm and SP = 5.4 cm, QS = 6.5 cm.

(d) Construct a rhombus PQRS in which the diagonals PR = 6c m and QS = 8 cm. Then, construct a ΔPSA which is equal in area to the rhombus PQRS.

(e) Construct a triangle whose one angle is 60° and area is equal to the rectangle in which length is 6cm and breadth is 4.5 cm.

Project work

Formulate groups with five students in each group. Using bamboo sticks , wheat straw or juice pipe, all the groups should make a sample of each of the above construction and paste it on the cardboard, and present it in the classroom.

Lesson 12

Circle

12.0 Review

Experimentally verify the given statements.

(a) The perpendicular drawn from the centre of a circle bisects the chord.

(b) The line joining the centre of a circle to the midpoint of the chord is perpendicular to the chord.

(c) Chords which are equidistant from the centre of a circle are equal.

12.1 Central Angle and Inscribed Angle

Activity 1

Observe the given circles. O is the centre of the given circles. Based on this, discuss the questions below.

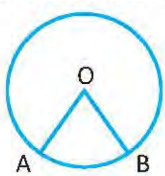


Figure (a)

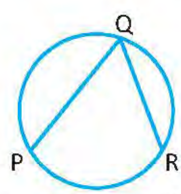


Figure (b)

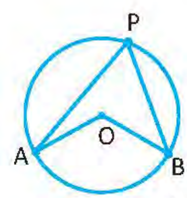


Figure (c)

(a) Where is the $\angle AOB$ formed? What is it called?

(b) Where is the $\angle PQR$ formed? What is it called?

(c) What is the difference between $\angle AOB$ and $\angle PQR$? Compare it.

(d) In figure (c), what are arc APB and AB called?

1. The angle formed by two radii at the centre is called the central angle. In the given figure, $\angle AOB$ is called the central angle.
2. The angle formed by joining two chords of a circle at the circumference is called the circumference (inscribed) angle. In the figure, $\angle PQR$ is called the circumference angle.
3. If an arc is smaller than a semicircle, it is called a minor arc and if it is larger than a semicircle, it is called a major arc. Here, $\overset{\frown}{APB}$ is a major arc and $\overset{\frown}{AB}$ is a minor arc.

12.2 Relation between central angle and its corresponding arc

Activity 2

Draw circles with the centre O using a compass. Discuss the relationship between the central angles and its opposite arc.

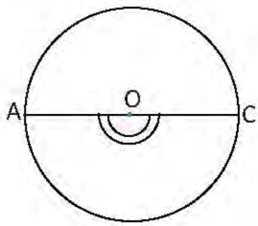


Figure 1

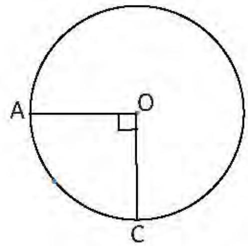


Figure 2

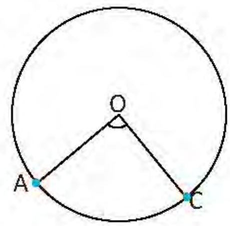


Figure 3

(a) When the central angle is 180°, discuss what part of the circumference is the arc opposite to it.

(b) When the central angle is one fourth of the circle, discuss what part of the circumference is the arc opposite to it.

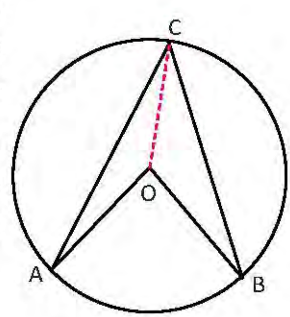
(c) When the central angle is one sixth of the circle, discuss what part of the circumference is the arc opposite to it.

(d) Is there a direct relationship between the central angle and its opposite arc?

$\angle AOC \hat{=} \widehat{AOC}$ read as arc AOC equals in degree measure. There is direct a relation between the central angle and its opposite arc. The symbol $\hat{=}$ is also read as equal influence.

12.3 Relation between inscribed angle and its corresponding

Activity 3



Draw a circle with centre O by using a compass and pencil where $\angle AOB$ is a central angle and $\angle ACB$ is a circumference angle. O and C are joined. Are the radii of the same circle OA, OB and OC equal?

Now, what types of triangles are triangle OAC and triangle OBC? Discuss which sides and angles of these triangles are equal.

Now, in an isosceles triangle OAC,

$\angle OAC + \angle OCA + \angle AOC = 180°$

or, $2\angle OCA = 180° - \angle AOC$ .... (i) $\quad [\angle OAC = \angle OCA]$

Again, in an isosceles triangle OBC,

$\angle OCB + \angle OBC + \angle BOC = 180°$

or, $2\angle OCB = 180° - \angle BOC$ .... (ii) $\quad [\angle OCB = \angle OBC]$

Adding equation (i) and (ii), we get

$2(\angle OCA + \angle OCB) = 360° - (\angle AOC + \angle BOC)$

or, $2\angle ACB = 360° - \text{Reflex } \angle AOB$

or, $2\angle ACB = \angle AOB$

or, $2\angle ACB \triangleq \overset{\frown}{AB}$

The relationship between the double the angle at the circumference and its opposite arc has the equal influence. It is denoted by $2\angle ACB \triangleq \overset{\frown}{AB}$

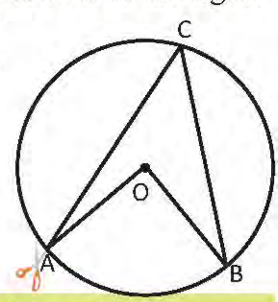
12.4 Relation between the central angle and inscribed angle

(a) Relation between the central angle and inscribed angle based on the same arc

Activity 4

(i) Using paper

Draw a central angle and inscribed angle in a chart paper as shown in the figure. Take out the central angle by cutting with a scissor. Now, fold the central angle making two equal parts and measure the inscribed angle and find the conclusion.



The central angle is double of the inscribed angle based on the same arc.
$\angle AOB = 2\angle ACB$

(ii) Experimental verification

Draw the circles having different radii as shown in the figure.

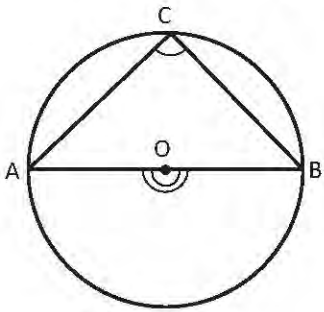


Figure 1

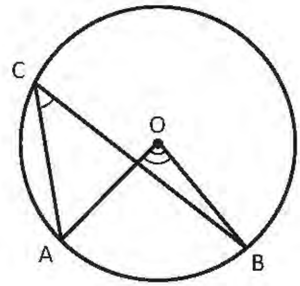


Figure 2

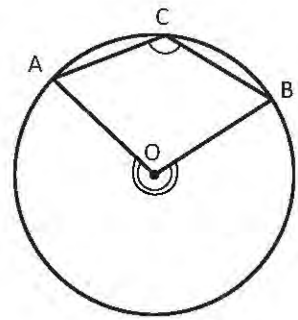


Figure 3

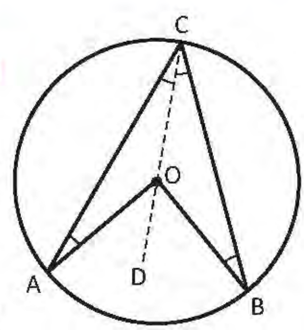
Measure the central angle AOB and the inscribed angle ACB based on the same arc AB of each circle and fill in the table below.

Figure No.	$\angle AOB$	$\angle ACB$	Result
1.			
2.			
3.			
Conclusion:			

(iii) Theoretical proof

Theorem 1

The central angle is double of the inscribed angle based on the same arc.



Given: O is the centre of the circle. The central angle $\angle AOB$ and inscribed angle $\angle ACB$ are based on the same arc AB.

To be prove: $\angle AOB = 2\angle ACB$

Construction: Points C and O are joined and produced CO to the point D.

Proof

S.N.	Statement	Reason
1.	In ΔAOC, (i) $\angle OAC = \angle OCA$ (ii) $\angle AOD = \angle OAC + \angle OCA$ (iii) $\angle AOD = \angle OCA + \angle OCA$ $= 2\angle OCA$	(i) OA = OC (radii of the same circle), So base angles of isosleces Δ. (ii) Exterior and opposite interior angle of ΔAOC (iii) from statement (i) and (ii)
2.	In ΔBOC, $\angle BOD = 2\angle OCB$	Same as above facts and reasons.
3.	$\angle AOD + \angle BOD = 2\angle OCA + 2\angle OCB$	From statements 1 (iii) and (2) above.
4.	$\therefore \angle AOB = 2\angle ACB$	From statement (3), by whole partaxiom.
Conclusion: The central angle is double of the inscribed angle based on the same arc.		

Proved.

The inscribed angle is half of the centre angle based on the same arc. That is, the central angle is twice the angle on the circumference. As in the above figure, $\angle AOB = 2\angle ACB$.

Activity 5

How to show the angle in the circumference of the semicircle is a right angle by experimental verification.

(ii) Experimental verification

Draw the circles having different radii as shown in the figures. The angle $\angle ACB$ is on the circumference based on the diameter. Write the conclusion by taking the value of $\angle ACB$ in the table below.

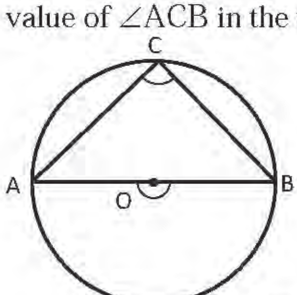


Figure 1

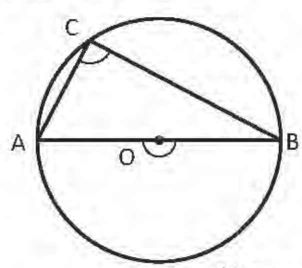


Figure 2

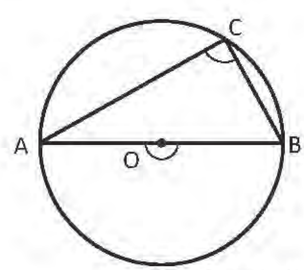
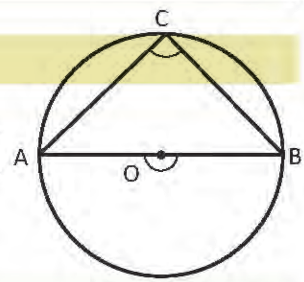


Figure 3

Measure the angle ∠ACB of the semicircle of each circle and fill in the table below.

Figure No.	∠ACB	Result
1.		
2.		
3.		
Conclusion:		

(ii) Theoretical proof



Given: O is the centre of a circle in which AOB is the diameter of the circle. ∠ACB is the angle of the circumference based on the diameter.

To Prove: ∠ACB = 90°

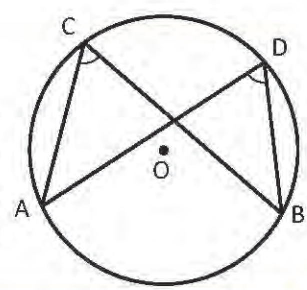
	Statement	Reason
1.	$\angle ACB = \frac{1}{2}\angle AOB$	Inscribed angle is half of the central angle standing on the same arc.
2.	∠AOB =180°	Being ∠AOB is a straight angle
3.	$\angle ACB = \frac{1}{2} \times 180^\circ = 90^\circ$	From statement (1) and (2) above
Conclusion: An angle in a semicircle is a right angle.		

Proved.

(b) Relation between the angles in the circumference based on the same arc

Activity 6

(i) Using paper



On a chart paper, draw two angles in the circumference of the circle as shown in the figure. With the help of scissors, cut one angle from the circumference and fold it to the other angle. Write a conclusion based on this.

Inscribed angles are equal when they base on the same arc. ∠ACB = ∠ADB

(ii) Experimental verification

Draw the circles having different radii as shown in the figure. The angle ∠ACB and ∠ADB are the angles on the circumference based on the same arc AB.

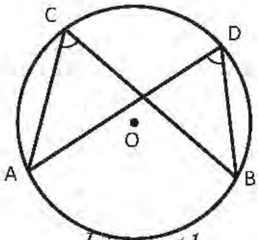


Figure t1

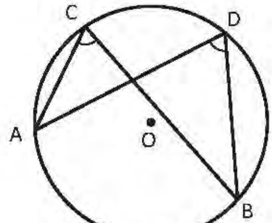


Figure 2

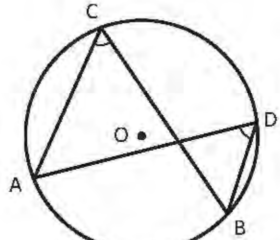


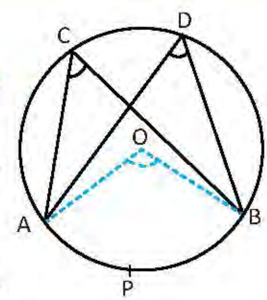
Figure 3

Measure the circumference angles ∠ACB and ∠ADB based on the same arc AB of each circle. Fill in the table below and write the conclusion.

Figure No.	∠ACB	∠ADB	Result
1.			
2.			
3.			
Conclusion:			

(iii) Theoretical proof

Theorem 2



The angles on the circumference of a circle, based on the same arc are equal.

Given: O is the centre of a circle in which inscribed angles ∠ACB and ∠ADB are based on the same arc AB.

To prove: ∠ACB = ∠ADB

Construction: Join centre O of the circle with the points A and B successively.

Proof

	Statements	Reasons
1.	∠AOB = 2∠ACB	The circumference angle and the central angle are based on the same arc APB.
2.	∠AOB = 2∠ADB	The circumference angle and the central angle are based on the same arc APB.
3.	2∠ACB = 2∠ADB or, ∠ACB = ∠ADB	From statement (1) and (2)
Conclusion: The angles on the circumference of a circle, based on the same arc are equal.		

Proved.

12.5 The Relation between opposite angles of cyclic quadrilateral

(i) Experimental verification

Draw circles having different radii as shown in the figure. Draw the cyclic quadrilateral ABCD in each circle.

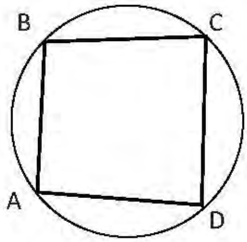


Figure 1

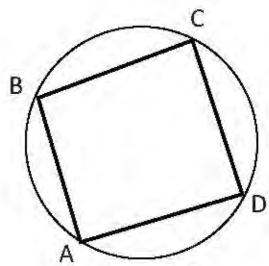


Figure 2

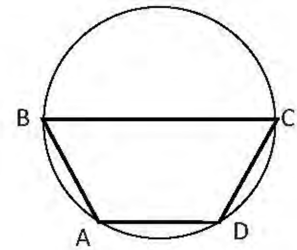


Figure 3

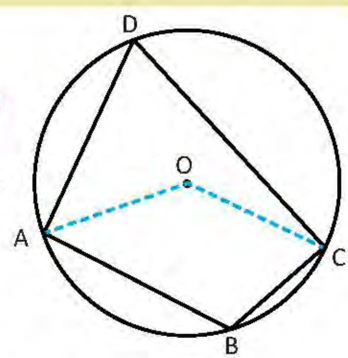
Measure the circumference angles and fill in the table below and write the conclsion.

Figure No.	∠DAB	∠ABC	∠BCD	∠ADC	∠DAB + ∠BCD	∠ABC + ∠ADC	Result
1.							
2.							
3.							
Conclusion:							

(ii) Theoretical proof

Theorem 3

The opposite angles of a cyclic quadrilateral are supplementary.



Given: O is the centre of a circle. ABCD is a cyclic quadrilateral.

To prove: $\angle ABC + \angle ADC = 180^\circ$

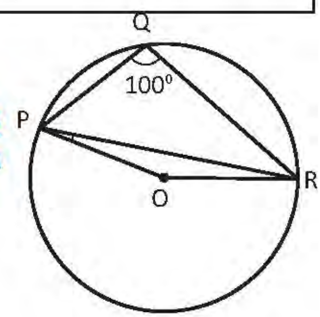
$\angle BCD + \angle BAD = 180^\circ$

Construction: Join centre O of the circle with the points A and C successively.

Proof

	Statements	Reasons
1.	Obtuse $\angle AOC = 2\angle ADC$	The central angle and the circumference angle based on the same arc APB.
2.	Reflex $\angle AOC = 2\angle ABC$	The central angle and the circumference angle based on the same arc ADC.
3.	$2\angle ADC + 2\angle ABC$ = Obtuse $\angle AOC$ + Reflex $\angle AOC$ or, $2(\angle ADC + \angle ABC) = 360°$ or, $\angle ADC + \angle ABC = \frac{360°}{2} = 180°$ $\therefore \angle ADC + \angle ABC = 180°$	From statement (1) and (2), the sum of the angles around the point O is 360°
4.	Similarly, $\angle DAB + \angle DCB = 180°$	Similar as above.

Example 1



In the given figure, $\angle PQR = 100°$ and the points P, Q and R are the circumference points of the circle with the centre O. What is the value of $\angle OPR$< Find.

Solution

According to the figure,

(i) Reflex angle POR $= 2 \times \angle PQR = 2 \times 100° = 200°$

[$\because$ The central angle and the inscribed angle based on the same arc PR]

(ii) Reflex angle POR + obtuse angle POR = 360° [$\because$ Sum of the angles around the point O]

$200°$ + obtuse $\angle POR = 360°$

$\angle POR = 360° - 200° = 160°$

(iii) Again, ΔPOR is an isosceles triangle. So that, $\angle OPR = \angle ORP$

$\angle OPR + \angle ORP + \angle POR = 180°$ [$\because$ The sum of the angles of a triangle]

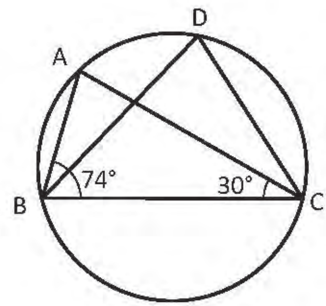
or, $\angle OPR + \angle OPR + 160° = 180°$ [$\because$ $\angle OPR = \angle OPR$]

or, $2\angle OPR = 180° - 160° = 20°$

or, $\angle OPR = \frac{20°}{2} = 10°$

Example 2

In the adjoining figure, $\angle ABC = 74°$ and $\angle ACB = 30°$, then find the value of angle $\angle BDC$.



Solutions

Here,

(i) In the triangle ABC, $\angle ABC + \angle ACB + \angle BAC = 180°$

[$\because$ Sum of interior angles of triangle.]

or, $74° + 30° + \angle BAC = 180°$

or, $104° + \angle BAC = 180°$

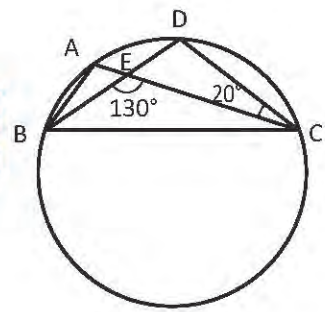
or, $\angle BAC = 180° - 104° = 76°$

$\therefore \angle BDC = 76°$

(ii) BDC = BAC [$\because$ Inscribed angles standing on the same arc]

Example 3

In the adjoining figure, A, B, C and D are four points on the circumference of the circle. The chords AC and BD are intersecting at a point E. If $\angle BEC = 130°$ and $\angle ECD = 20°$, find the value of $\angle BAC$.



Solution

(i) $\angle BEC = 130°$ and $\angle ECD = 20°$

$\angle BEC + \angle CED = 180°$ [$\because$ The sum of the angles in a straight line]

or, $\angle CED = 180° - \angle BEC = 180° - 130° = 50°$

(ii) Again, $\angle EDC + \angle CED + \angle ECD = 180°$ [$\because$ The sum of the angles of a triangle]

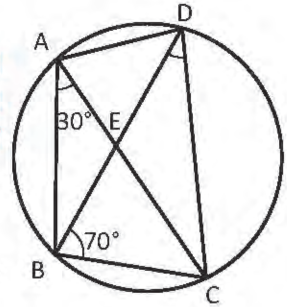
or, $\angle EDC = 180° - 50° - 20° = 110°$

(iii) or, $\angle EDC = \angle BAC$ [$\because$ Inscribed angles standing on the same arc BC]

or, $\angle BAC = 110°$

Example 4

In a cyclic quadrilateral ABCD, the diagonals AC and BD are intersecting at a point E. If $\angle DBC = 70°$ and $\angle BAC = 30°$, then find the value of $\angle BCD$. Also, if AB = BC, what is the value of $\angle ECD$?



Solution

In a cyclic quadrilateral ABCD, the diagonals AC and BD are intersecting at a point E.

(i) $\angle DAC = \angle DBC = 70°$ [Inscribed angles standing on the same arc CD]

(ii) $\angle DAB = \angle DAC + \angle BAC = 70° + 30° = 100°$ [By whole part axiom]

(iii) Again $\angle BCD + \angle DAB = 180°$ [The sum of opposite angles of a cycle quadrilateral]

or, $\angle BCD + 100° = 180°$

or, $\angle BCD + 100° = 180° – 100° = 80°$

(iv) Again $\angle BAC = \angle ACB = 30°$ [Being AB = BC]

$\angle BCD = \angle BCA + \angle ACD = 80°$

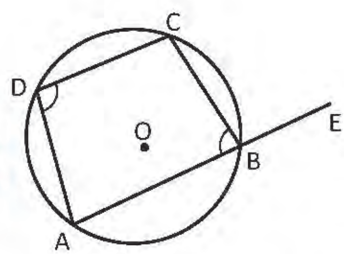
or, $30° + \angle ACD = 80°$

or, $\angle ACD = 80° – 30° = 50°$

$\therefore \angle AX\Delta = \angle ECD = 50°$

Example 5

In the given figure alongside, ABCD is cyclic quadrilateral. If the side AB is produced to the point E then prove that $\angle ADC = \angle CBE$.



Solution

Given: ABCD is a cyclic quadrilateral of a circle with centre O. The side of cyclic quadrilateral AB is produced to the point E

To Prove: $\angle ADC = \angle CBE$

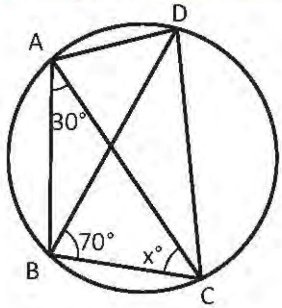
Proof:

	Statements	Reasons
1.	$\angle ADC + \angle ABC = 180°$	The sum of opposite angles of a cyclic quadrilateral
2.	$\angle ABC + \angle CBE = 180°$	Being straight angle
3.	$\angle ADC + \angle ABC = \angle ABC + \angle CBE$ or, $\angle ADC = \angle CBE$	From statement (1) and (2) above.

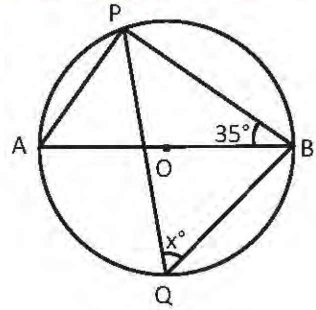
Proved.

Exercise 12

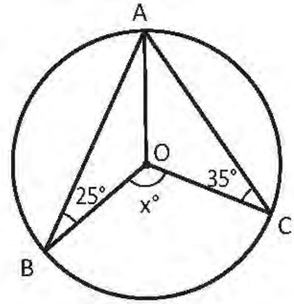
1. If O is the centre of the following circles, find the value of *x*.



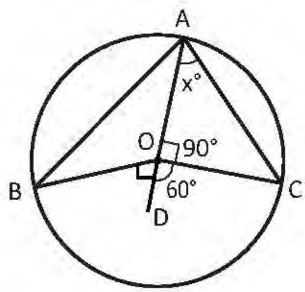
(a)



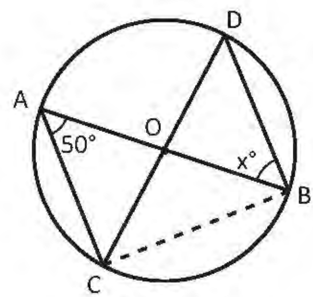
(b)



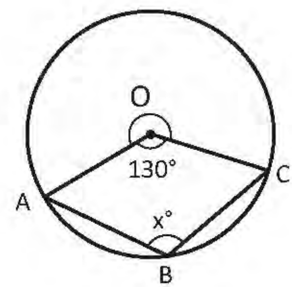
(c)



(d)

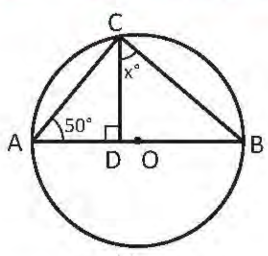


(e)

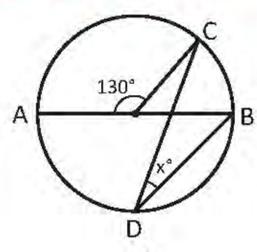


(f)

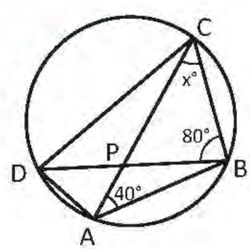
2. In the following figure, find the value of *x*.



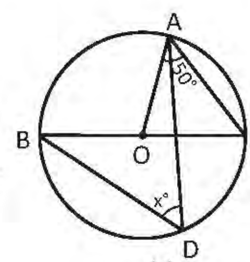
(a)



(b)

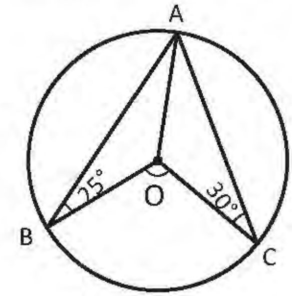


(c)

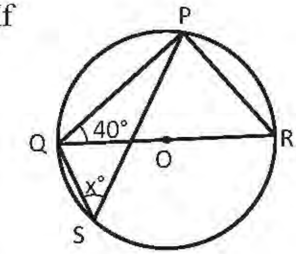


(d)

3. (a) In the adjoining figure, O is the centre of circle, $\angle OBA = 25°$ and $\angle OCA = 30°$ find the value of obture $\angle BOC$.



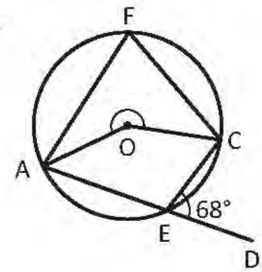
(b) In the adjoining figure, O is the centre of the circle. If $\angle PQR = 40^\circ$ and $\angle PSQ = x^\circ$, find the value of x.

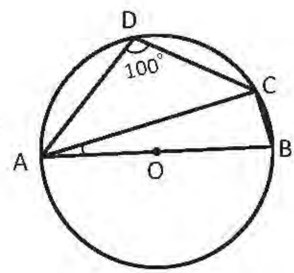


(c) In the given figure, O is the centre of the circle. FAEC is a cyclic quadrilateral. If $\angle CED = 68^\circ$, then

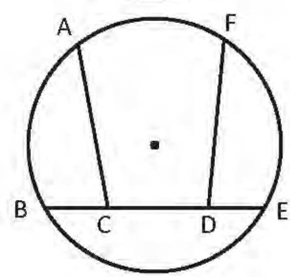
(i) Find the value of $\angle AFC$.

(ii) Find the reflex $\angle AOC$.



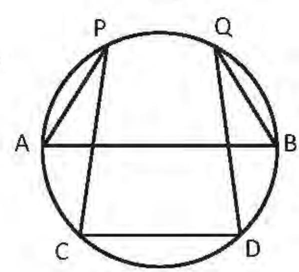


(d) In the given figure, AOB is a diameter of the circle. If $\angle ADC = 100^\circ$, then find the value of $\angle BAC$.

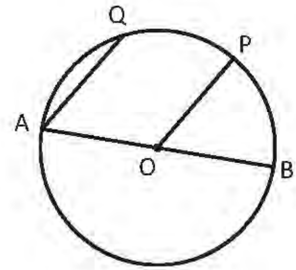


4. (a) In the adjoining figure, BC = DE and $\overset{\frown}{AB} = \overset{\frown}{FE}$ Prove that $\angle ACB = \angle FDE$.

(b) In the given figure, if $\angle APC = \angle BQD$, then prove that AB//CD.



(c) In the adjoining figure, O is the centre of the circle. If arc PQ = arc PB, then prove that AQ// OP.

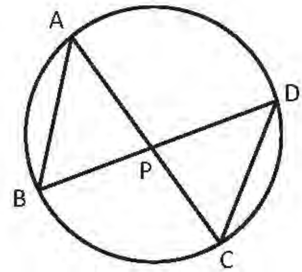


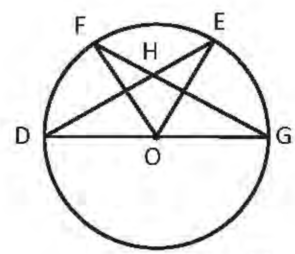
(d) In the given figure, chords AC and BD are intersected at a point P. If PB = PC then prove that:

(i) Chord AB = chord DC.

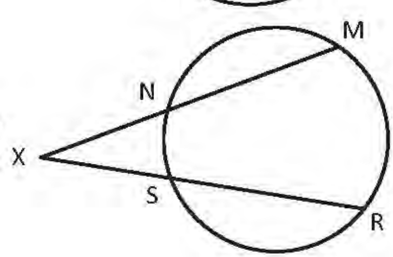
(ii) Chord AC = chord BD.

(ii) Arc ABC = arc BCD.

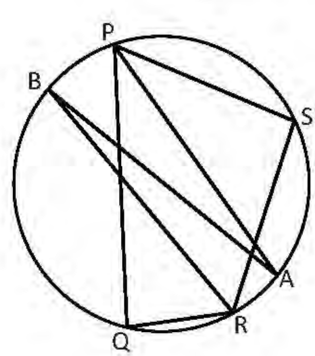




5. In the figure, O is the centre of the circle. If the chords DE and FG are intersected at a point H, prove that: $\angle DOF + \angle EOG = 2\angle EHG$.

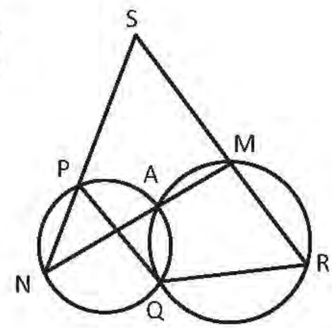


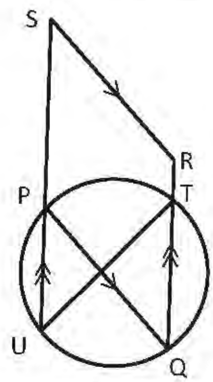
6. In the figure, chords MN and RS of the circle intersect externally at the point X. Prove that: $\angle MXR \overset{\circ}{=} \frac{1}{2}(\overset{\frown}{MR} - \overset{\frown}{NS})$.



7. PQRS is a cyclic quadrilateral. If the bisectors of the $\angle QPS$ and $\angle QRS$ meet the circle at A and B respectively, prove that AB is a diameter of the circle.

8. In the given figure, NPS, MAN and RMS are straight lines. Prove that PQRS is a cyclic quadrilateral.





9. In the given figure, PQRS is a parallelogram. Prove that UTRS is a cyclic quadrilateral.

Practical work and project

1. Make models of paper to show the relationship between the central angle and the inscribed angle, and the arcs and chords related to them. Present them in the classroom.
2. Draw three pairs of equal circles ABP and CDQ having centres X and Y respectively. Join chords AB and CD making equal arcs AB and CD. Measure AB and CD and enter the result in a table.
   - Does chord AB = chord CD?
   - Are the angles subtended by chords AB and CD at the centre equal?
   - Is the angle subtended by the chord AB at the circumference of a circle half of the central angle? Fine thread or wire and tracing paper can be used for this work.

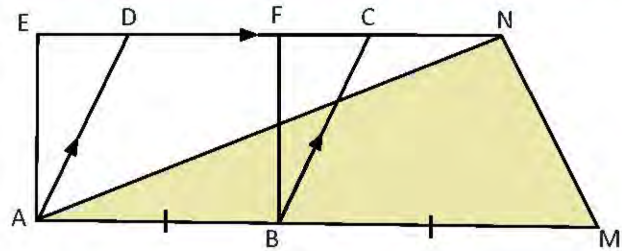
Answers

1. (a) $80°$ (b) $55°$ (c) $120°$ (d) $75°$ (e) $50°$ (f) $115°$
2. (a) $50°$ (b) $25°$ (c) $60°$ (d) $50°$
3. (a) $110°$ (b) $50°$ (c) (i) $68°$ (ii) $136°$ (d) $10°$

Show the answers from 4 to 9 to the teachers.

Mixed Exercise

1. **In the given figure, ABCD is a parallelogram and ABFE is a rectangle.**

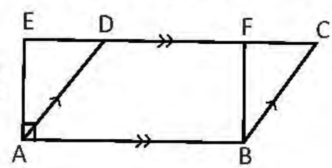


   (a) What is the relationship between the area of the parallelogram and rectangle? Write.

   (b) In the figure, if AB = BM, then write the relationship between the parallelogram ABCD and triangle AMN.

2. **A parallelogram ABCD and a rectangle ABFE are on the same base AB and between the same parallel lines AB and EC.**

   (a) What is the relationship between the area of the parallelogram ABCD and rectangle ABFE? Write it.



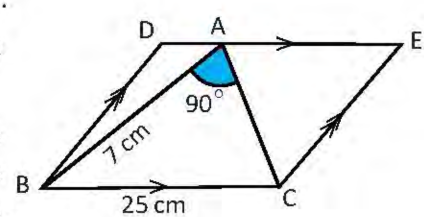
   (b) If the area of the rectangle ABFE is 35 cm^2, what is the area of the parallelogram ABCD? Find.

   (c) Construct a parallelogram ABCD having the side AB = 7 cm, BC = 5 cm and $\angle ABC = 120°$. Construct a rectangle ABFE whose area is equal to the area of that parallelogram.

   (d) Are the triangle AED and triangle BFC congruent? Write with reason.

3. **A parallelogram BCED and a triangle ABC are on the same base BC and between the same parallel lines BC and DE, where $\angle BAC = 90°$, AB = 7 cm and BC = 25 cm.**

   (a) What is the measurement of AC? Find.



   (b) What is the area of parallelogram BCED? Find.

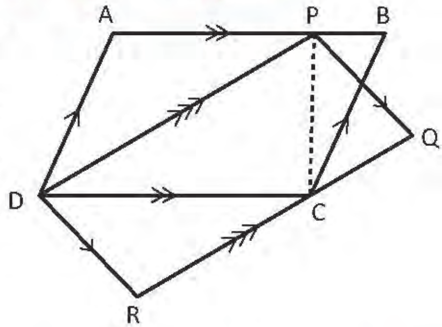
   (c) Theoretically prove that the relationship between the area of the parallelogram ABCD and triangle ABC.

   (d) Construct a triangle ABC, where AC = 5 cm, AB = 4 cm and $\angle BAC = 45°$. Also construct a parallelogram ADMN whose area is equal to the area of the triangle.

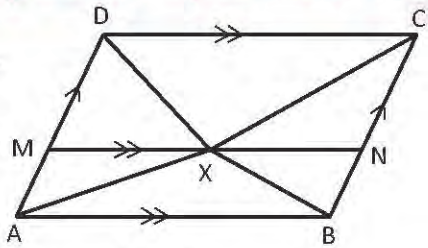
4. In the given figure, ABCD and PQRD are two parallelograms.

(a) Find the relation between the parallelograms ABCD and PQRD.

(b) If the base and height of the parallelogram ABCD are 8 cm and 7 cm respectively, find the area of the parallelogram PQRD.



5. In the given figure, ABCD is a parallelogram. X is a point inside it. If MN//AB then,



(a) Prove that the sum of the area of triangles XCD and XAB is equal to the half of the area of the parallelogram ABCD.

(b) What is the relationship among the triangle AXD, triangle BCX and parallelogram ABCD?

6. (a) Construct a parallelogram ABCD having AB = 5 cm, BC = 4 cm and $\angle ABC = 60^\circ$. Also construct a triangle PBE equal in area to the parallelogram having a side PB = 5.6 cm.

(b) According to question (a) find the height of the parallelogram ABCD and then find the area of triangle PBE.

7. (a) Construct a parallelogram ABCD having AB = 7 cm, BC = 5 cm and $\angle ABC = 120^\circ$. Also, construct the rectangle ABFE equal in area to the parallelogram.

(b) Find the side BF of the rectangle ABFE formed according to question no. (a) and also find the area of the parallelogram ABCD.

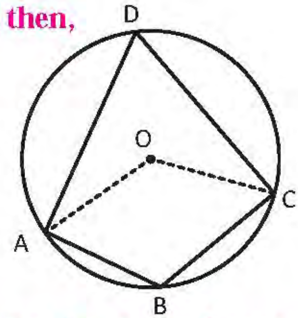
8. If a circle with centre O has a central angle $\angle BOC$ and inscribed angle $\angle BDC$ based on the same arc BC, answer the following questions.

(a) Write the relation between $\angle BOC$ and $\angle BDC$.

(b) Experimentally verify that the relationship between the central angle $\angle BOC$ and inscribed angle $\angle BDC$.

(c) The measurement of the central angle is $(7x^\circ)$ and the inscribed angle is $(3x+5)^\circ$, then find the value of x.

9. In the given figure, ABCD is a cyclic quadrilateral then,

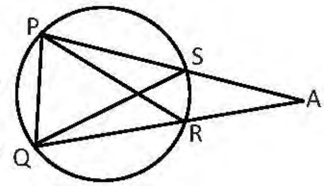


(a) Write the relation of $\angle ABC$ and $\angle ADC$.

(b) Prove that $\angle ADC = \frac{1}{2} \angle AOC$.

(c) If $\angle ABC = 120°$, what is the value of $\angle AOC$?

10. O is the centre of the given circle and P, Q, R and S are the circumference points on it. If AP = AQ then,

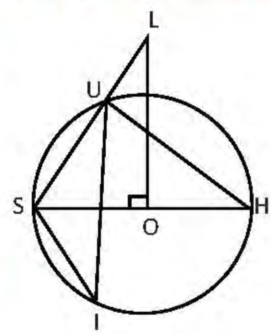


(a) Write the name of the inscribed angles based on the arc PQ.

(b) If $\angle PSQ = 60°$ then what is the measurement of the angle $\angle PRQ$?

(c) Prove that: PR = QS.

11. In the given figure, O is the centre of the circle and SH is a diameter. S, I, H and U are the circumference points and L is an external point HereLO is perpendicular to SH.



(a) What is the measurement of angle $\angle SUH$? Write with reason.

(b) Prove that: $\angle SIU = \angle OLS$.

(c) If $\angle USH = 50°$, then what is the measurement of $\angle SIU$? Find.

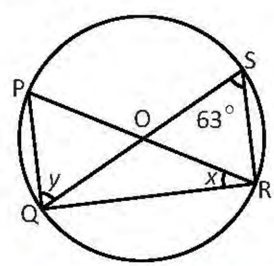
12. In the given figure, O is the centre of circle, $\angle PQS = y$, $\angle QSR = 63°$ and $\angle PRQ = x$.

(a) What is the measurement of angle $\angle PQR$? Write with reason.

(b) What is the value of angle $\angle POS$? Find.

(c) Prove that: $x + y = 90°$.

(d) Prove that: ΔQOR is an isosceles triangle.



Answers

Show to your teacher.

Lesson 13

Statistics

13.0 Review

The marks obtained in Mathematics by 27 students of class 10 in the second terminal examination are given below. On the basis of this, answer the following questions.

25, 15, 30, 22, 27, 12, 25, 30, 22, 24, 15, 23, 19, 27, 28, 17, 19, 22, 25, 15, 14, 13, 28, 26, 18, 20, 22

(a) What is the average marks obtained in Mathematics?

(b) How many students have scored less and more than the average marks?

(c) Find the median, the first quartile and the third quartile on the basis of the data given above.

(d) What is the same mark obtained by maximum number of students? How many have got it? What is it called?

13.1 Mean

Activity 1

In a survey conducted among the students of class 10 of a community school about how much rupees they bring each day for their tiffin. The following results were obtained.

3 bring Rs. 25	6 bring Rs. 30	7 bring Rs. 35
4 bring Rs. 40	4 bring Rs. 45	1 bring Rs. 50

(a) Present the above data in table

(b) Find one day's average expenditure of the grade 10 students.

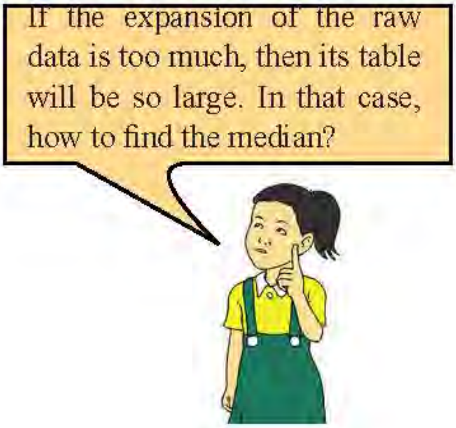
To find the average value (mean) from the data given above, the following formula is applied.

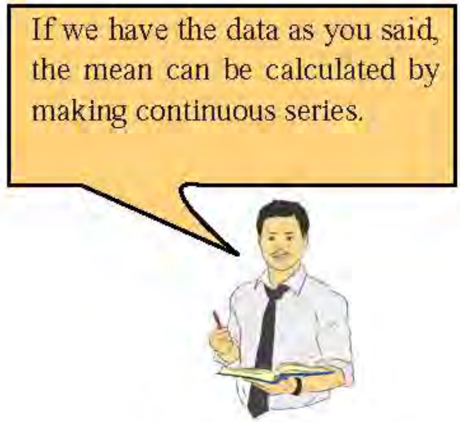
$$\text{Mean } \overline{X} = \frac{(f_1 x_1 + f_2 x_2 + \ldots\ldots + f_n x_n)}{f_1 + f_2 + \ldots\ldots + f_n}$$

$$= \frac{\sum fx}{\sum f}$$

$$= \frac{\sum fx}{n}$$

Activity 2





For example, the obtained marks of 40 students of class 9 of a school in Mathematics are as follows.

25, 10, 31, 22, 37, 42, 45, 37, 32, 34, 45, 40, 29, 27, 28, 17, 19, 22, 25, 33

15, 14, 13, 28, 36, 38, 41, 42, 39, 25, 24, 31, 21, 22, 25, 26, 35, 36, 39, 49

The median can be calculated by placing the above data in a continuous series.

(a) The lowest obtained mark is 10 and the highest obtained mark is 49. We can make a table with the class interval of 10 as given below.

Description of the marks obtained by class 9 students in Mathematics

Marks obtained (X)	Tally bars	No. of students (f)
10 - 20	卌 |	6
20 - 30	卌 卌 ||||	14
30 - 40	卌 卌 |||	13
40 - 50	卌 ||	7
Total number of students		40

To find the mean of the continuous series first of all we have to find the mid value of each class interval.

$$\text{Mid-value (m)} = \frac{\text{Lower limit of a clss interval + Upper limit of a class interval}}{2}$$

After that, as in the discrete series, mean is by placing 'm' in the place of x. calculated.

Description of the marks obtained by class 9 students in mid-value.

Marks obtained (X)	No. of students (f)	Mid value (m)	fm
10-20	6	$\frac{10+20}{2} = 15$	90
20-30	14	$\frac{20+30}{2} = 25$	350
30-40	13	$\frac{30+40}{2} = 35$	455
40-50	7	$\frac{40+50}{2} = 45$	315
	N = 40		$\sum fm = 1210$

$\overline{X} = \frac{\sum fxm}{N} = \frac{1210}{40} = 30.25$

The above table is called frequency table. The number of students of each class interval is called frequency of that class interval. This method to find the mean is called direct method.

Alternative method

Description of the marks obtained by class 9 students in Mathematics

Marks obtained X	No. of students f	Mid-value m	$d = m - A$	fd
10 – 20	6	$\frac{10+20}{2} = 15$	$15 - 25 = -10$	-60
20 – 30	14	$\frac{20+30}{2} = 25$	$25 - 25 = 0$	0
30 – 40	13	$\frac{30+40}{2} = 35$	$35 - 25 = 10$	130
40 – 50	7	$\frac{40+50}{2} = 45$	$45 - 25 = 20$	140
	N = 40			$\sum fd = 210$

Let, assumed mean (A) = 25.

The deviation between mid-value and assumed mean = d

Mean $\overline{X} = A + \frac{\sum fd}{N}$

$= 25 + \frac{210}{40}$

$= 25 + 5.25 = 30.25$

The actual mean of the data can also be calculated by considering the assumed mean. The actual mean is calculated by considering any mid-value and any number as assumed mean. For this, the deviation between mid-value and assumed mean (d) should be found.

Example 1

If $\sum fm = 2700$ and N = 50, then find mean $\overline{X}$.

Solution

Here, $\sum fm = 2700$

$N = 50$

Mean $\overline{X} = ?$

We know that $\overline{X} = \frac{\sum fm}{N}$

$= \frac{2700}{50}$

$= 54$

Hence, mean $(\overline{X}) = 54$

Example 2

The weight of 100 students of Ganesh Secondary School is given in the table below. Find the mean weight of the students from this data.

Weight in (Kg)	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
No. of students	18	27	20	17	6

Solution

Here, the details of the weight of students

Weight in (Kg) (X)	No. of students (f)	Mid value (m)	fm
20 – 30	27	$\frac{20+30}{2}=25$	675
30 – 40	20	$\frac{30+40}{2}=35$	700
40 – 50	17	$\frac{40+50}{2}=45$	765
50 – 60	6	$\frac{50+60}{2}=55$	330
	$\sum f = N = 88$		$\sum fm = 2740$

We know that, mean $\overline{X} = \frac{\sum fm}{N} = \frac{2740}{88} = 31.14$

Hence, the mean weight of students $(\overline{X}) = 31.14$

Alternative method

Here, let the assumed mean (A) = 35

Description of the weight of students

Marks obtained (X)	No. of students (f)	Mid value (m)	d = m-35	fd
10 – 20	18	15	–20	–360
20 – 30	27	25	–10	–270
30 – 40	20	35	0	0
40 – 50	17	45	10	170
50 – 60	6	55	20	120
	N = 88			$\sum fd = -340$

We know that, mean $(\overline{X}) = A + \frac{\sum fd}{N}$

$= 35 + \frac{-340}{88}$

$= 35 - 3.86 = 31.14$

Hence, the mean weight of the students $(\overline{X}) = 31.14$

Example 3

The assumed mean of any data (A) = 40, $\sum fd = 20$ and N = 10, then find the mean $\overline{X}$.

Solution

Here, A = 40,

$\sum fd = 20$ and

N = 10

mean $(\overline{X})$ = ?

We know that, mean $(\overline{X}) = A + \frac{\sum fd}{N}$

$= 40 + \frac{20}{10}$

$= 40 + 2$

$= 42$

Hence, the mean $(\overline{X})$ = 42

Example 4

The data prepared on the basis of the weight of the people of Janajagriti Tole are given in the table. Find the average weight (mean) based on the table.

Weight (Kg)	0-10	10-20	20-30	30-40	40-50	50-60
No. of people	10	18	25	20	12	5

Solution

Here, let assumed mean (A) = 25 and length of class interval (h) = 10

Description of the weight of people of Janajagriti Tole

Weight in Kg. (X)	No. of people (f)	Mid value (m)	$d = m - 25$	fd
0-10	10	5	–20	–200
10-20	18	15	–10	–180
20-30	25	25	0	0
30-40	20	35	10	200
40-50	12	45	20	240
50-60	5	55	30	150
	$\sum f = N = 90$			$\sum fd = 210$

We know that, mean $(\overline{X}) = A + \frac{\sum fd}{N}$

$$= 25 + \frac{210}{90}$$

$$= 25 + 2.33 = 27.33$$

Hence, the mean weight of the people $(\overline{X}) = 27.33$

Example 5

The data prepared based on the height of students of class 11 and 12 of Shanti Secondary School is given in the table. If the average height of the students $\overline{X}$ = 157.75 cm, find the value of p.

Height (cm)	140-145	145-150	150-155	155-160	160-165	165-170	170-175
No. of students	2	5	8	p	7	5	9

Solution

Here, average height of the students $(\overline{X}) = 157.75$

Description of the height of the students of class 11 and 12 of Shanti Secondary School

Height (cm) (X)	No. of students (f)	Mid-value (m)	fm
140-145	2	142.5	285
145-150	5	147.5	737.5
150-155	8	152.5	1220
155-160	p	157.5	157.5p
160-165	7	162.5	1137.5
165-170	5	167.5	837.5
170-175	3	172.5	517.5
	$\sum f = N = 30 + p$		$\sum fm = 4735 + 157.5\,p$

We know that, mean $(\overline{X}) = \frac{\sum fm}{N}$

$$157.75 = \frac{4735 + 157.5p}{30 + p}$$

or, $4732.5 + 157.75p = 4735 + 157.5p$

or, $157.75p - 157.50p = 4735 - 4732.5$

or, $0.25\ p = 2.5$

or, $p = 10$

Hence, the required value of p = 10.

Example 6

The number of people entering into the Balaju Park from 7 am to 8 am according to their age is given below. Construct a frequency table of class interval of 10 from the given data and find the average age of people visiting the park.

7, 22, 32, 47, 59, 16, 36, 17, 23, 39, 49, 31, 21, 24, 41, 12, 49, 21, 9, 8, 51, 36, 35, 18.

Solution

Here, tabulating the given data in the frequency distribution table

Height (cm) (X)	Tally Bars	frequency (f)	Mid-value (m)	*fm*
0-10	|||	3	5	15
10-20	||||	4	15	60
20-30	~~||||~~	5	25	125
30-40	~~||||~~ |	6	35	210
40-50	||||	4	45	180
50-60	||	2	55	110
		$\sum f = N = 24$		$\sum fm = 700$

We know that, mean $(\overline{X}) = \frac{\sum fm}{N}$

$= \frac{700}{24}$

$= 29.17$

$(\overline{X}) = 29.17$

Hence, the average age of people $(\overline{X}) = 29.17$ years.

Exercise 13.1

1. Find the mean in the following condition:

(i) 35, 36, 42, 45, 48, 52, 58, 60

(ii) 13.5, 14.2, 15.8, 15.2, 16.9, 16.5, 17.4, 19.3, 15.3,15.9

(iii)

X	5	8	10	12	14	16
f	4	5	8	10	2	2

(iv) Details of the goals scored by players in the national football league.

Goals	12	13	14	15	16	17
Number of students	2	4	6	12	10	6

2. Find the mean of the following data by direct method and short-cut method.

(a) The details age of passengers travelling in a bus

Age (year)	0-10	10-20	20-30	30-40	40-50
No. of persons	5	9	15	7	4

(b) The details of the marks obtained by class 10 students in science.

Marks obtained	10-20	20-30	30-40	40-50	50-60	60-70
No. of students	1	4	10	8	7	5

(c) Details of daily wages of workers

Wages (Rs)	200-400	400-600	600-800	800-1000	1000-1200
No. of workers (Rs.)	3	7	10	6	4

(d) The details of the marks obtained by class 10 students in Mathematics.

Marks obtained	0-10	10-20	20-30	30-40	40-50	50-60
No. of students	7	5	6	12	8	2

3. Find the unknown value of the following data.

(a) $\overline{X} = 49, \sum fm = 980, N = ?$

(b) $\overline{X} = 102.25, N = 8, \sum fm = ?$

(c) $A = 100, \overline{X} = 90, \sum fd = ?, N = 10$

(d) $\overline{X} = 41.75, \sum fd = 270, N = 40, A = ?$

4. (a) In the given condition, if the mean ($\bar{X}$) is 32.5, find the value of k.

Marks obtained	0-10	10-20	20-30	30-40	40-50	50-60
No. of students	5	10	k	35	15	10

(b) In the given condition, if the mean ($\bar{X}$) is 46.2, find the value of p.

X	0-20	20-40	40-60	60-80	80-100
f	35	400	350	p	65

(c) In the given condition, if the mean ($\bar{X}$) is 36.4, find the value of y.

Marks obtained	16-24	24-32	32-40	40-48	48-56	56-64
No. of students	6	8	y	8	4	2

(d) In the given condition, the mean of daily expenditure ($\bar{X}$) is 264.67, find the value of the unknown frequency.

Daily expenditure	0-100	100-200	200-300	300-400	400-500	500-600
No. of students	20	30	?	20	18	12

5. Prepare a frequency distribution table from the given raw data and find the mean ($\bar{X}$).

(a) 15, 51, 32, 12, 32, 33, 23, 43, 35, 46, 57, 19, 59, 25, 20, 38, 16, 45, 39, 40 (Class interval 10)

(b) 25, 15, 24, 42, 22, 35, 34, 41, 33, 38, 54, 50, 36, 40, 27, 18, 35, 16, 51, 31, 23, 9, 16, 23, 31, 51, 7, 30, 17, 40, 60, 32, 50, 10, 23, 12, 21, 28, 37, 20, 58, 39, 10, 41, 13 (Class interval 5)

6. (a) Find the mean of the following data.

Marks obtained	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	8	10	14	10	8	10

(b)

Daily wages (Rs.)	0-50	50-100	100-150	150-200	200-250	250-300
No. of workers	1	2	3	4	1	2

Project work

Ask the age of 100 people of your community and present it in the frequency distribution table with a suitable class interval. Present that frequency distribution table in histogram. Find the mean by direct method and short-cut method and present in classroom.

Answers

1. (a) 47	(b)16	(c) 10.32	(d) 15.05	
2. (a) 24 year	(b) 43.86	(c) Rs. 706.67	(d) 28.75	
3. (a)) 20	(b) 818	(c) –100	(d) 35	
4. (a) 25	(b) 150	(c) 12	(d) 50	
5. (a) 34.5	(b) 31.29	6. (a) 30	(b) Rs. 155.77	(c) 34.5

13.2 Median

Activity 3

The marks obtained in Mathematics by class 10 students in the first terminal examination are given below.

21, 23, 28, 14, 10, 18, 19, 29, 27, 25, 19, 17, 18, 20, 21, 17, 15, 16,

28, 23, 24, 17, 16, 19, 14, 24, 23, 27, 14, 15, 21, 24, 26, 24, 18

On the basis of the above data, solve the following questions and check your answer with your friends

(a) What is the average mark of the students?

(b) Find the median value of the student's marks by making individual series and discrete series.

(c) Does the median value obtained from different series differ?

The median is the statistical value that divides the given data exactly into two parts.

Example 1

The marks obtained by class 8 students in Mathematics are given below. Find the median of the data.

Marks obtained	17	18	22	2 6	30	32
No. of students	3	4	8	10	7	5

Solution

Here,

Details of the marks obtained by class 8 students

Marks obtained (X)	No. of students (f)	Cumulative frequency (cf)
17	3	3
18	4	7
22	8	15
26	10	25
30	7	32
32	5	37
	$\sum f = N = 37$	

We know that,

The position of median $= \frac{N+1}{2}$ th term

$= \frac{37+1}{2}$ th term

$= 19$ th term

From the above table, the value of the 19th item is 26. So, the median of the marks obtained by class 8 students is 26.

Activity 4

We were able to find the median from the data given in discrete series. Now, if the data is given in a continuous series, how to find the median?

Median from the continuous series of data

Like,

Marks obtained (X)	0-8	8-16	16-24	24-32	32-40	40-48	48-56
No. of students (f)	6	10	16	18	12	10	8

The median of the continuous series of data can be found in the following steps:

(a) Make a less than cumulative frequency table. (Less than the upper value of each class interval)

(b) Identification of the position of the median. The position of median $= \frac{N}{2}$ th item

(c) Finding the class interval of median. The class interval of the position of median is the median class interval.

(d) Finding the median by using the following formula.

$$\text{Median } (M_d) = L + \frac{\frac{N}{2} - cf}{f} \times h$$

Where, L = Lower limit of the median class

N = Total numbers of data

cf = Cumulative frequency of the class preceding the median class

f = Frequency of the median class

h = Length (width) of class interval

Here, constructing the table for the median,

Marks obtained (X)	No. of students (*f*)	Marks	Less than cumulative frequency (cf)
0-8	6	Less than 8	6
8-16	10	Less than 16	6 + 10 = 16
16-24	16	Less than 24	16 + 16 = 32
24-32	18	Less than 32	32 + 18 = 50
32-40	12	Less than 40	50 + 12 = 62
40-48	10	Less than 48	62 + 10 = 72
48-56	8	Less than 56	72 + 8 = 80
	N = 80		

Total number of students (N) = 80

The position of median = $\frac{N}{2}$ th class = $\frac{80}{2}$ th class = 40 th class

40^{th} item lies in the class interval (24 - 32).

Now, the lower limit of the median class (L) = 24

Cumulative frequency of the class preceding the median class (c*f*) = 32

Frequency of the median class (*f*) = 18

Length of class interval (h) = 32 - 24 = 8

We know that, median $(M_d) = L + \dfrac{\frac{N}{2} - cf}{f} \times h$

$$= 24 + \frac{40-32}{18} \times 8$$

$$= 24 + \frac{64}{18}$$

$$= 24 + 3.56 = 27.56$$

Example 2

Find the median from the given data

Weight (Kg)	20-30	30-40	40-50	50-60	60-70	70-80
No. of students (f)	16	12	10	16	18	12

Solution: Here,

Details of the students' weight

Marks obtained (X)	No. of students (f)	Less than cumulative frequency	Cumulative frequency
20 - 30	16	Less than 30 = 16	16
30-40	12	Less than 40 = 16 + 12	28
40-50	10	Less than 50 = 16 + 12 + 10	38
50-60	16	Less than 60 = 16 + 12 + 10 + 16	54
60-70	18	Less than 70 = 16 + 12 + 10 + 16 +18	72
70-80	12	Less than 80 = 16 + 12 + 10 + 16 +18 +12	84
	$\sum f = N = 84$		

Total number of students (N) = 84

The position of median = $\frac{N}{2}$ th item = $\frac{84}{2}$ th item = 42 th item

42^{th} item lies in the class interval (50 - 60).

Now, the lower limit of the median class (L) = 50

Cumulative frequency of the class preceding the median class (cf) = 38

Frequency of the median class (f) = 16

Length of class interval (h) = 60 – 50 = 10

We know that, median (M_d) $= L + \dfrac{\frac{N}{2} - cf}{f} \times h$

$= 50 + \frac{42 - 38}{16} \times 10$

$= 50 + \frac{40}{60}$

$= 50 + 2.5 = 52.5$

Example 3

The table given below is prepared based on the number of people voluntarily contributing to the public works in the village. If the median value of the given data is 93.6, find the value of the missing frequency 'y'.

Days (X)	0-30	30-60	60-90	90-120	120-150	150-180
No. of workers (f)	5	y	22	25	14	4

Solution: Here,

Table to find the cumulative frequency

Days (X)	No. of workers (f)	Less than cumulative frequency (cf)
0 - 30	5	5
30-60	y	5 + y
60-90	22	27 + y
90-120	25	52 + y
120-150	14	66 + y
150-180	4	70 + y
	N = (70 + y)	

Total number of workers N = 70 + y

Median (M_d)= 93.6

Median lies in the class interval (90-120)

Now, the lower limit of the median class (L) = 90

Cumulative frequency of the class preceding the median class (cf) = 27 + y

Frequency of the median class (f) = 25

Length of median class interval (h) = 120 - 90 = 30

We know that, median $(M_d) = L + \frac{\frac{N}{2} - cf}{f} \times h$

or, $93.6 = 90 + \frac{\frac{70+y}{2} - (27+y)}{25} \times 30$

or, $93.6 - 90 = \frac{70 + y - 2(27 + y)}{2 \times 25} \times 30$

or, $3.6 = \frac{70 + y - 54 - 2y}{50} \times 30$

or, $3.6 = \frac{(16 - y) \times 3}{5}$

or, $3.6 \times 5 = 48 - 3y$

or, $3y = 48 - 18$

or, $3y = 30$

or, $y = 10$

Hence, missing frequency $(y) = 10$

Activity 5

Below given is a frequency distribution table based on the height of trees in a garden.

Height (ft)	4-6	7-9	10-12	13-15	16-18	19- 21	22-24
No. of trees	2	3	10	7	4	3	2

(a) How to make the above data into continuous series?

(b) What is the median of the above data? Find.

In the data given here, the class intervals are not continuous. So, to make the class interval continuous, the correction factor should be found as follows.

Correction factor = $\frac{\text{Lower limit of the second class interval-Upper limit of the first class interval}}{}$

$= \frac{7 - 6}{2} = 0.5$

The correction factor is subtracted from the lower value and added to the upper value of each class interval to convert the class interval into continuous series.

Like, in the class interval = 4 - 6

Lower value 4 - 0.5 = 3.5 and upper value 6+ 0.5 = 6.5

Now, we should make class interval 3.5 - 6.5

Table to find the median

Height (cm) X	No. of trees (f)	Less than cumulative frequency (cf)
3.5 - 6.5	2	2
6.5 - 9.5	3	5
9.5 - 12.5	10	15
12.5 - 15.5	7	22
15.5 - 18.5	4	26
18.5 - 21.5	3	29
21.5 - 24.5	2	31
	N = 31	

The position of median = $\frac{N}{2}^{th}$ class

$= \frac{31}{2} = 15.5^{th}$ class

The class interval having 15.5^{th} term = (12.5 - 15.5)

Now, the lower limit of the median class (L) = 12.5

Cumulative frequency of the class preceding the median class (cf) = 15

Frequency of the median class (f) = 7

Length of the median class interval (h) = 15.5 - 12.5 = 3

We know that, median $(M_d) = L + \frac{\frac{N}{2} - cf}{f} \times h$

$= 12.5 + \frac{15.5 - 15}{7} \times 3$

$= 12.5 + \frac{0.5 \times 3}{7}$

$= 12.5 + \frac{1.5}{7}$

$= 12.5 + 0.21 = 12.71$

$\therefore$Hence, the median height of trees = 12.71 ft.

Example 5

Construct a frequency distribution table with a class interval of 10 from the given data and find the median.
21, 9, 34, 42, 17, 54, 13, 38, 23, 39, 49, 29, 38, 44, 21, 42, 19, 7, 29, 8, 55, 36, 39, 13.

Solution

Frequency distribution table

Class interval (X)	Tally Bars	Frequency f	Less than cumulative frequency cf
0-10	|||	3	3
10-20	||||	4	3+4=7
20-30	~~||||~~	5	7+5=12
30-40	~~||||~~ |	6	12+6 =18
40-50	||||	4	18+4 =22
50-60	|||	2	22+2 = 24
		N = 24	

The position of median $= (\frac{N}{2})^{\text{th}}$ class

$= (\frac{24}{2})^{\text{th}} = 12^{\text{th}}$ item

The class interval having 12th term = (20 – 30)

Now, the lower limit of the median class (L) = 20

Cumulative frequency of the class preceding to median class (cf) = 7

Frequency of the median class (f) = 5

Length of median class interval (h) = 30 - 20= 10

We know that, median $(M_d) = L + \frac{\frac{N}{2} - cf}{f} \times h$

$= 20 + \frac{12 - 7}{5} \times 10$

$= 20 + 10$

$= 30$

Exercise 13.2

1. **Find the median from the given data.**

(a) 2.5, 4.5, 3.6, 4.9, 5.4, 2.9, 3.1, 4.2, 4.6, 2.2, 1.5

(b) 100, 105, 104, 197, 97, 108, 120, 148, 144, 190, 148, 22, 169, 171, 92, 100

(c)

Marks obtained	18	25	28	29	34	40	44	46
No. of students	3	6	5	7	8	12	5	4

(d)

Class interval (x)	102	105	125	140	170	190	200
Frequency (f)	10	18	22	25	15	12	8

2. **Find the median from the given data.**

(a)

Weight (Kg)	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No. of students	3	5	7	11	10	3	1

(b)

Height (cm)	140-145	145-150	150-155	155-160	160-165	165-170	170-175
Frequency	2	5	8	10	7	5	3

(c)

Expenditure (per day) Rs	less than100	100-200	200-300	300-400	400-500	more than 500
Frequency	22	34	52	20	19	13

(d)

Marks obtained	less than 20	less than 40	less than 60	less than 80	less than 100
No. of students	21	44	66	79	90

3. **Find the missing frequency from the data given below.**

(a) Median = 35

Marks obtained	20-25	25-30	30-35	35-40	40-45	45-50
No. of students	2	5	8	k	4	5

(b) Median = 132.5

Wages (Rs.)	100-110	110- 120	120-130	130-140	140-150	150-160
No. of workers	5	6	p	4	7	5

(c) Median = 39

Age (yrs)	20-25	25-30	30-35	35-40	40-45	45-50	50-55	55-60
No. of people	50	70	100	300	?	220	70	60

4. Find the median of the following data.

(a)

Marks obtained	50-60	60-70	70-80	80-90	90-100
No. of students	2	3	6	5	4

(b)

Marks obtained	< 20	< 40	< 50	< 80	< 100
No. of students	9	23	43	55	60

(c)

Income (Rs)	< 600	< 700	< 800	< 900	< 1000
No. of workers	30	98	152	177	200

(d)

Temp (°c)	0-9	10-19	20-29	30-39	40-49
Days	8	10	20	15	7

5. (a) The marks obtained by 30 students in a class test are as follows.

22, 56, 62, 37, 48, 30, 58, 42, 29, 39, 37, 50, 38, 41, 32, 20, 28, 16, 43, 18, 40, 52, 44, 27, 35, 45, 36, 49, 55, 40

Construct a frequency distribution table with the class interval of 10 from the above data and find the mean median.

(b) The height of 40 students of class 10 in cm is given below. Construct afrequencydistributiontablewiththeclassintervalof5fromthedataandfind the mean and median.

142, 145, 151, 157, 159, 160, 165, 162, 156, 158, 155, 141, 147, 149, 148, 159, 154, 155, 166, 168, 169, 172, 174, 173, 176, 161, 164, 163, 149, 150, 154, 153, 152, 164, 158, 159, 162, 157, 156, 155

Project work

Ask the ages of 100 people in your community. Find the median age by presenting the obtained data with a class interval of 10.

Answer

1. (a) 3.6	(b) 121	(c) 34	(d) 140
2. (a) 64.5 kg	(b) 157.5cm	(c) 246.15 lbg	(d) 40.9
3. (a) 6	(b) 3	(c) 150	
4. (a) 78.33	(b) 47	(c) 703.70	(d) 25.5
5. (a) 39, 40	(b) 158, 155.42		

13.3 Mode

Activity 6

The temperature of 20 days of a city is as follows find which temperature is repeated maximum.

70, 76, 76, 74, 70, 70, 72, 74, 78, 80, 74, 74, 78, 76, 78, 76, 74, 78, 80, 76

The maximum number of times repeated value of the given data is called mode.

Activity 7

Mode from continuous series

We can find the mode of grouped data in the following steps:

(a) Since, the number of repeated values is the mode, first find the class interval with the highest frequency.

(b) Finding the frequency of the model class is f_1, the frequency of the class preceding the model class is f_0, the frequency of the class succeeding the model class is f_2.

(c) Finding the width of mode l class interval

(d) The following formula is using to find the mode

$$\text{Mode} = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

Where, L = Lower limit of the model class

f_1 = Frequency of the model class

f_0 = Frequency of the class preceding the model class

f_2 = Frequency of the class succeeding the model class

h = Width (size) of model class interval

Example 1

Find the mode from the data given below.

Weight (kg)	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No. of students	3	5	7	11	10	3	1

Solution: Here,

Weight (kg)	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No. of students	3	5	7	11	10	3	1

Here, the highest frequency is 11, so its corresponding class is 60 - 70.

Lower limit of the model class (L) = 60

Frequency of the model class (f_1) = 11

Frequency of the class preceding the model class (f_0) =7

Frequency of the class succeeding the model class (f_2) = 10

Width (size) of model class interval (h) = 70 – 60 = 10

We know that, mode (Mo) = $L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

$= 60 + \frac{11 - 7}{2 \times 11 - 7 - 10} \times 10$

$= 60 + \frac{4}{5} \times 10$

$= 60 + 8 = 68$

Hence, mode (Mo) = 68

Exercise 13.3

1. Find the mode of the following data

(a) 29 cm, 34 cm, 29 cm, 26 cm, 55 cm, 34 cm, 35 cm, 40 cm, 34 cm, 56 cm

(b) 99 kg, 135 kg, 182 kg, 49 kg, 189 kg, 196 kg, 78 kg, 192 kg, 182 kg

2. Find the mode from given frequency table

(a)

Marks obtained	5	10	15	20	25	30	35	40	45
No. of students	2	6	7	9	11	5	15	2	3

(b)

Wages (Rs.)	50	75	100	125	150	175	200	225
No. of workers	8	12	17	29	30	27	20	11

3. **Find the mode from the given frequency table**

(a)

Marks obtained.	20-25	25-30	30-35	35-40	40-45	45-50
No. of students	2	5	8	6	4	5

(b)

Wages (Rs.)	100-110	110- 120	120-130	130-140	140-150	150-160
No. of workers	5	6	4	7	5	4

(c)

Age (yrs)	20-25	25-30	30-35	35-40	40-45	45-50	50-55	55-60
No. of people	50	70	100	300	220	150	70	60

Answer

1. (a) 34 cm (b) 182 kg
2. (a) 35 (b) 150 kg
3. (a) 33 (b) 136 (c) 38.57

13.4 Quartiles

Activity 8

In class 9, we studied to find the first quartile and third quartile from the individual and discrete data. The data given below is the marks obtained in Mathematics by class 10 students of Janta Secondary School in the first terminal examination.

21, 23, 28, 14, 10, 18, 19, 29, 27, 25, 19, 17, 18, 20, 21, 17, 15, 16,

28, 23, 24, 17, 16, 19, 14, 24, 23, 27, 14, 15, 21, 24, 26, 24, 18

From the above data, find the value of the first quartile and the third quartile by making individual series and discrete series. Is the value of the first quartile and the third quartile obtained from different series also different? Discuss in the group of two.

Activity 9

How to find the first quartile and the third quartile from the data given in continuous series? Do the following activities in groups:

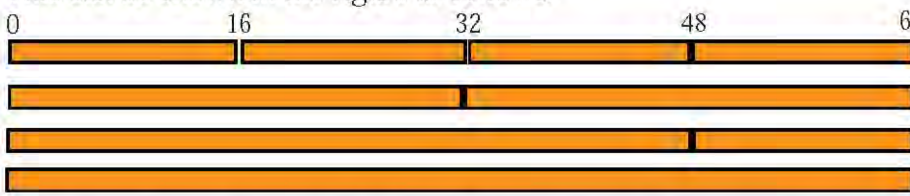
(a) Take any 4 sticks of length 64 cm.

(b) Divide first stick into 4 equal parts. Find the length of each piece. 16 divide by 64 into 4 equal parts. This is the first quartile.

(c) Let's divide the second stick into 2 equal parts and find the length of each piece.

(d) Divide the third stick into 4 equal parts such that 3 parts in one side and one part on another side.

This can be shown in the figure as follows.



As seen in the picture above, when divided into 4 equal pieces, 16 cm pieces are formed. When divided into 2 equal parts, 32 cm pieces are formed. It is also called median. Similarly, if you add 3 equal pieces out of 4 equal pieces, the total length is 48 cm. 16 is the first quartile (Q_1), 32 is the second quartile(Q_2) and 48 is the third quartile (Q_3).

For grouped series

(a) Find the less than cumulative frequency.

(b) The position of first quartile $Q_1 = \frac{N}{4}^{th}$ item and the position of the third quartile $Q_3 = (\frac{3N}{4})^{th}$ item

(c) Look at the cumulative frequency for (Q_1) in the class interval with cumulative frequency equal to or greater than $\frac{N}{4}$ and for (Q_3) in the class interval with cumulative frequency exactly greater than $\frac{3N}{4}$.

(d) After that, use the following formula:

$$Q_1 = L + \frac{\frac{N}{4} - cf}{f} \times h$$

Here,

L = Lower limit of the first quartile (Q_1) class

N = Total numbers of data

cf = Cumulative frequency of the class preceding the first quartile (Q_1) class

f = Frequency of the first quartile class

h = Length (width) of class interval

$$Q_3 = L + \frac{\frac{3N}{4} - cf}{f} \times h$$

Where, L = Lower limit of the third quartile class

N = Total numbers of data

cf = Cumulative frequency of the class preceding to the third quartile class

f = Frequency of the third quartile class

h = Length (width) of class interval

Example 1

Find the first quartile (Q_1) and third quartile (Q_3) from the following data.

Age (yrs)	20	25	28	30	32	35	42	46
No. of workers	2	8	12	10	14	7	5	1

Solution

Details of workers' age

Workers age X	No. of workers (f)	cf
20	2	2
25	8	10
28	12	22
30	10	32
32	14	46
35	7	53
42	5	58
46	1	59

The position of the first quartile $= \frac{N+1}{4}$ th term

$= \frac{59+1}{4}$ th term

$= \frac{60}{4} = 15$ th term

The corresponding value of the 15^{th} item is 28. So, the first quartile (Q_1) is 28.

Again, the position of the third quartile $= \frac{3(N+1)}{4}$ th term,

$= \frac{3(59+1)}{4}$ th term $= \frac{180}{4} = 45^{th}$ term

The corresponding value of the 15^{th} item is 32. So the third quartile (Q_3) is 32.

Example 2

The marks obtained in Mathematics by class 7 students in are given below. Find the first quartile (Q_1) and the third quartile (Q_3) from the data.

Marks obtained (X)	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of students (f)	2	8	15	14	10	8	3

Solution

Details of the mark obtained by students

Marks obtained (X)	No. of students (f)	Cumulative frequency (cf)
10-20	2	2
20-30	8	10
30-40	15	25
40-50	14	39
50-60	10	49
60-70	8	57
70-80	3	60
$\sum f = N = 60$		

The position of the first quartile (Q_1) = $\frac{N}{4}^{th}$ item = $\frac{60}{4}^{th}$ item = 15^{th} item

The class interval having the 15^{th} item is (30 - 40)

Now, lower limit of the first quartile class interval (L) = 30

Cumulative frequency of the class preceding the first quartile class (cf) = 10

Frequency of the first quartile class (f) =15

Length (width) of first quartile class interval (h) = 40 - 30 = 10

We know that, first quartile (Q_1) $= L + \dfrac{\frac{N}{4} - cf}{f} \times h$

$$= 30 + \frac{15 - 10}{15} \times 10$$

$$= 30 + \frac{50}{15}$$

$$= 30 + 3.34 = 33.34$$

The position of the third quartile = $\frac{3N}{4}^{th}$ item = $\frac{3 \times 60}{4}^{th}$ item = 45^{th} item

The class interval having the 45^{th} item is (50 - 60).

Now, lower limit of the third quartile class interval (L) = 50

Cumulative frequency of the class preceding the third quartile class (cf) = 39

Frequency of the third quartile class (f) =10

Length (width) of the third quartile class interval (h) = 60 - 50 = 10

We know that, third quartile $(Q_3) = L + \dfrac{\frac{3N}{4} - cf}{f} \times h$

$$= 50 + \frac{45 - 39}{10} \times 10$$

$$= 50 + \frac{60}{10}$$

$$= 50 + 6 = 56$$

Hence, the first quartile(Q_1) = 33.34 and the third quartile (Q_3) = 56

Example 3

In the following table workers' incomes are given. Find Q_1, Q_2 and Q_3 from the data.

Income (in thousands)	0-5	5-10	10-15	15-20	20-25	25-30	30-35
No. of workers	10	15	40	55	30	25	5

Solution

Details of workers' income

Income (in thousands) (X)	No. of workers (f)	Cumulative frequency (cf)
0-5	10	10
5-10	15	25
10-15	40	65
15-20	55	120
20-25	30	150
25-30	25	175
30-35	5	180

Total number of workers (N) = 180

The position of the first quartile = $\frac{N}{4}^{th}$ item = $\frac{180}{4}^{th}$ item = 45^{th} item

The class interval having the 45^{th} item is (10 – 15)

Now, lower limit of the first quartile class interval (L) = 10

Cumulative frequency of the class preceding the first quartile class (cf) = 25

Frequency of the first quartile class (f) =40

Length (width) of the first quartile class interval (h) = 15 - 10 = 5

We know that, first quartile $(Q_1) = L + \frac{\frac{N}{4} - cf}{f} \times h$

$$= 10 + \frac{45 - 25}{40} \times 5$$

$$= 10 + \frac{100}{40}$$

$$= 10 + 2.5 = 12.5$$

Again, the position of the second quartile or median = $\frac{N}{2}$ th item $= \frac{180}{2}$ th item $= 90$ th item

The class interval having 90^{th} item is (15 - 20)

Now, lower limit of the median class (L) = 15

Cumulative frequency of the class preceding the median class (cf) = 65

Frequency of the median class (f) = 55

Length (width) of median class interval (h) = 20 - 15 = 5

We know that, second quartile or median $(Q_2) = L + \frac{\frac{N}{2} - cf}{f} \times h$

$$= 15 + \frac{90 - 65}{55} \times 5$$

$$= 15 + \frac{25}{11}$$

$$= 15 + 2.27 = 17.27$$

Now, the position of the third quartile $= \frac{3N}{4}$th item $= \frac{3 \times 180}{4}$ th item = 135 th item

The class interval having the 135^{th} item is (20 - 25).

Now, lower limit of the third quartile class (L) = 20

Cumulative frequency of the class preceding the third quartile class (cf) = 120

Frequency of the third quartile class (f) =30

Length (width) of third quartile class interval (h) = 25 - 20 = 5

We know that, third quartile $(Q_3) = L + \dfrac{\frac{3N}{4} - cf}{f} \times h$

$$= 20 + \frac{135 - 120}{30} \times 5$$

$$= 20 + \frac{15}{6} = 20 + 2.5$$

$$= 22.5$$

Hence, the first quartile $(Q_1) = 12.5$, the second quartile or median $(Q_2) = 17.27$ and the third quartile $(Q_3) = 22.5$.

Example 4

Students' expenditure on tiffin in one week and the number of students are given in If the upper quartile of the data is 460, then find the value of p.

Expenditure (Rs.)	100-200	200-300	300-400	400-500	500-600
No. of students	15	18	P	20	17

Solution

Table to find quartile

Expenditure (Rs) (X)	*No. of students (f)*	*Cumulative frequency (cf)*
100-200	15	15
200-300	18	33
300-400	p	33 + p
400-500	20	53 + p
500-600	17	70 + p
	N = 70 + p	

Total number of students (N) = 70 + p

Since, third quartile $(Q_3) = 460$

The third quartile class interval is (400 - 500).

Now, lower limit of the third quartile class (L) = 400

Cumulative frequency of the class preceding the third quartile class $(cf) = 33 + p$

Frequency of the third quartile class $(f) = 20$

Length (width) of the third quartile class interval (h) = 500 - 400 = 100

We know that, third quartile $(Q_3) = L + \dfrac{\frac{3N}{4} - cf}{f} \times h$

$$\text{or, } 460 = 400 + \frac{\frac{3(70+p)}{4} - (33+p)}{20} \times 100$$

or, $460 - 400 = \frac{210 + 3p - 132 - 4p}{4 \times 20} \times 100$

or, $60 = \frac{78 - p}{4} \times 5$

or, $78 - p = \frac{60 \times 4}{5} = 48$

or, $78 - 48 = p$

or, $30 = p$

$\therefore p = 30$

Hence, the missing frequency (p) = 30

Exercise 13.4

1. **Find the value of Q_1 and Q_3 from the data given below:**

(a) 10, 12, 14, 11, 22, 15, 27, 14, 16, 13, 25

(b)

Marks obtained	42	48	49	53	56	59	60	65	68	70
No. of students	2	3	5	8	9	11	7	8	6	4

2. **Find the value of Q_1 and Q_3 from the data given below:**

(a)

Age (yrs)	2-4	4-6	6-8	8-10	10-12	12-14	14-16	16-18
No. of students	5	12	25	26	24	28	20	15

(b)

Marks obtained	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of students	2	3	6	12	13	11	7

(c)

Height (cm)	100-110	110-120	120-130	130-140	140-150	150-160	160-170
No. of students	3	4	9	15	20	14	7

(d)

Wages (Rs.)	100-150	150-200	200-250	250-300	300-350	350-400
No. of workers	6	11	21	34	25	22

(e)

Marks obtained	0-20	20-40	40-60	60-80	80-100	100-120	120-140
Frequency	8	12	15	14	12	9	10

(f)

Time (minutes)	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
Frequency	5	3	10	6	4	2

(g)

Time (minutes)	20-25	25-30	30-35	35-40	40-45	45-50
Frequency	2	5	8	6	4	5

3. (a) If $Q_1 = 8$, what is the value of k?

Age(yrs)	0-6	6-12	12-18	18-24	24-30	30-36
No. of pupils	9	6	5	k	7	9

(b) If $Q_1 = 31$, what is the missing frequency?

Class interval	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	4	5	?	8	7	6

(c) If $Q_1 = 51.75$, what is the value of q?

Weight (in kg)	40-44	44-48	48-52	52-56	56-60	60-64
Frequency	8	10	14	q	3	1

4. Find the value of Q_1 and Q_3 from the data given below:

(a)

Height (cm)	<125	<130	<135	<140	<145	<150	<155
No. of students	0	5	11	24	45	60	72

(b)

Weight (lbs)	110-119	120-129	130-139	140-149	150-159	160-169	170-179	180-189
Frequency	5	7	12	20	16	10	7	3

(c)

Expenditure (per day)	Less then100	100-200	200-300	300-400	400-500	More than 500
Frequency	22	34	52	20	19	13

(d)

Marks obtained	less than 20	less than 40	less than 60	less than 80	less than 100
No. of students	21	44	66	79	90

5. (a) The data given below represent the marks obtained by 30 students in an internal examination. Find the first and third quartiles by tabulating the data taking a class interval of 10.

42, 65, 78, 70, 62, 50, 72, 34, 30, 40, 58, 53, 30, 34, 51, 54, 42, 59, 20,40, 42, 60, 25, 35, 35, 28, 46, 60, 47, 52

(b) The number of eggs produced everyday in a chicken farm is given below. Find the first and third quartiles by tabulating the data taking a class interval of 20.

32, 87, 17, 51, 99, 79, 64, 39, 25, 95, 53, 49, 78, 32, 42, 48, 59, 86, 69, 57, 15, 27, 44, 66, 77, 92

Project work

Ask and write the total marks obtained by the 100 students of classes 9 and 10 in an internal examination out of 100 full marks.

(a) Construct the frequency distribution table with a suitable class interval of the given data.

(b) Prepare the more than and less than cumulative frequency table by using the given data.

(c) Prepare a report of all the work in sequential order and present it in the classroom.

Answer

1. (a) 12, 22 (b) 53, 65
2. (a) 7.74, 13.73 (b) 42.08, 64.09 (c) 131.33, 152.14 (d) 230.35, 334.5 (e) 40, 98.33 (f) 18.3, 37.5 (g) 30.31, 40.63
3. (a) 8 (b) 10 (c) 16
4. (a) 137.69, 148 (b) 136.16, 159.5 (c) 152.94, 360 (d) 20.45, 61.15
5. (a) 37.5, 59.28 (b) 38, 61.57

Lesson 14

Probability

14.0 Review

Make a suitable group and discuss the following questions, and find the answers.

(a) What is the probability of getting even or prime number when a die is thrown one time?

(b) What is the probability of getting an ace or face cards when a card is drawn from a well shuffled pack of cards?

(c) What is the probability of getting both heads when two coins are tossed together?

On the basis of the above questions

(a) Write the sample space of each experiment.

(b) Write each event.

(c) Find the probability of each event.

(d) Find what each event is.

Each group should work in group and present your task to the class.

14.1 Principles of Probabilities

(a) Mutually Exclusive

Activity 1

When a dice is tossed once, let's write the event where

(a) Even number or odd number

(b) Let's write the event where there is an even number or prime number.

Here, the set of numbers that appears on the top of the dice (S) = {1, 2, 3, 4, 5, 6}

Let, the sset of even numbers that appears on the top (A),

Set of odd numbers that appears on the top (B) and

Set of prime numbers that appears on the top (C)

A = {2, 4, 6}

B = {1, 3, 5}

C = {2, 3, 5,}

From the above example, we can see that event A and event B do not have their common element. So that event A and event B can not occur at the same time. Therefore, A and B are called mutually exclusive events.

Again, in events A and C, 2 is the common element. When we get 2 after rolling a die it might be even number or prime number. So that, there is a probability of gettign event A which is equal to gettign event C.

Similarly, what events happen in B and C? Discuss and write.

In an experiment, if the occurrence of any one event excludes the occurrence of the other event, then such events are called mutually exclusive events. In the same way, when one event occurs, another event may also occur, which is not a mutually exclusive event. In the above example events A and B are mutually exclusive events but events A and C are not mutually exclusive event.

(b) Addition Law of Probability

Activity 2

Write the sample space (S) when a die is rolled. The event of even number is (A), the event of odd number is (B), event of prime number is (C) and write its cardinality and probability. Based on this, answer the questions asked below and reach in conclusion.

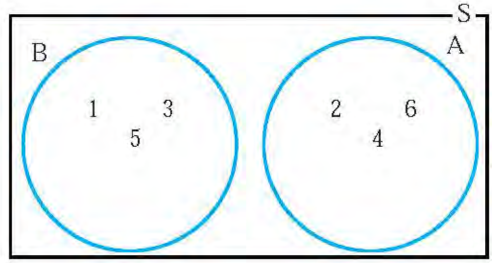
(a) What is the probability of getting even number or odd number?

(b) What is the probability of getting even number or prime number?

The possible outcomes when a die is thrown (S) = {1, 2, 3, 4, 5, 6}, n(S) = 6

Events	Cardinality	Probability
Getting even number (A) = {2, 4, 6}	$n(A) = 3$	$p(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$
Getting odd number (B) = {1, 3, 5}	$n(B) = 3$	$p(B) = \frac{n(B)}{n(S)} = \frac{3}{6} = \frac{1}{2}$
Getting prime number (C) = {2, 3, 5}	$n(C) = 3$	$p(C) = \frac{n(C)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

(a) Getting even number or odd number = {2, 4, 6} or {1, 3, 5}



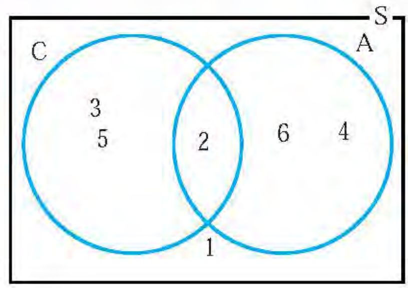
$(A \cup B) = \{1, 2, 3, 4, 5, 6\}$

$\therefore n(A \cup B) = 6$

$P(A \cup B) = \frac{n(A \cup B)}{n(S)} = \frac{6}{6} = 1$

Again, $P(A) + P(B) = \frac{1}{2} + \frac{1}{2} = 1$

$\therefore P(A \cup B) = P(A) + P(B)$



(b) Getting even number or prime number = {2, 4, 6} or {2, 3, 5}

$(A \cup C) = \{2, 3, 4, 5, 6\}$ $\therefore n(A \cup C) = 5$

$P(A \cup C) = \frac{n(A \cup C)}{n(S)} = \frac{5}{6}$

Here, $(A \cap C) = \{2\}$ $\therefore n(A \cap C) = 1$

$P(A \cap C) = \frac{n(A \cap C)}{n(S)} = \frac{1}{6}$

If A and B are mutually exclusive events, $P(A \cup B) = P(A) + P(B)$ and if A and C are not mutually exclusive event $P(A \cup C) = P(A) + P(C) - P(A \cap C)$. This is called addition law of probability.

Example 1

What is the probability of getting that a name beginning with W or a name beginning with T when one of the name of days is taken from a week.

Solution

Here, S = {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}; $n(S) = 7$

Let, A ={Days starting from W} = {Wednesday}; $n(A) = 1$ $\therefore P(A) = \frac{n(A)}{n(S)} = \frac{1}{7}$

B ={Days starting from T} = {Tuesday, Thursday}; $n(B) = 2$ $\therefore P(B) = \frac{n(B)}{n(S)} = \frac{2}{7}$

The probability of getting the names starting from W or T = $P(A \cup B)$ =?

A and B are mutually exclusive events so that,

$P(A \cup B) = P(A) + P(B) = \frac{1}{7} + \frac{2}{7} = \frac{3}{7}$

Hence, the probability of getting the names starting from W or T = $\frac{3}{7}$.

Example 2

A bag contains 4 black, 6 yellow and 5 red balls of same shape and size.

(a) If a ball is drawn randomly, find the probability of getting either black or red ball.

(b) Find the probability of getting either yellow or red ball when a ball is drawn randomly.

Solution

Here, total number of balls n(S) = (4 + 6 + 5) = 15

Number of black ball n(B) = 4, probability of getting black ball $P(B) = \frac{n(B)}{n(S)} = \frac{4}{15}$

Number of yellow ball n(Y) = 6, probability of getting yellow ball $P(Y) = \frac{n(Y)}{n(S)} = \frac{6}{15}$

Number of red ball n(R) = 5, probability of getting red ball $P(R) = \frac{n(R)}{n(S)} = \frac{5}{15}$

(a) Probability of getting either black or red ball $P(B \cup R) = ?$

Events B and R are mutually exclusive events, so by addition law of probability;

$P(B \cup R) = P(B) + P(R) = \frac{4}{15} + \frac{5}{15} = \frac{9}{15} = \frac{3}{5}$

Hence, the probability of getting either black or red ball is $= \frac{3}{5}$

(b) Probability of getting either yellow or red ball $P(Y \cup R) = ?$

Events Y and R are mutually exclusive events, so by addition law of probability;

$P(Y \cup R) = P(B) + P(R) = \frac{4}{15} + \frac{6}{15} = \frac{11}{15}$

Hence, the probability of getting either yellow or red ball is $P(Y \cup R) = \frac{11}{15}$.

Example 3

From the set of 20 cards numbered from 1 to 20, a card is drawn randomly. Find the probability of getting a number that is either divisible by 4 or by 3.

Solution

Here, total number of number cards n(S) = 20

Let A be the event of getting a number divisible by 4 and B be the event of getting a number divisible by 3.

Now, A = {4,8,12,16,20}

$n(A) = 5, \quad \therefore P(A) = \frac{n(A)}{n(S)} = \frac{5}{20}$

B = {3, 6, 9, 12, 15, 18} and $n(B) = 6, \therefore P(B) = \frac{n(B)}{n(S)} = \frac{6}{20}$

$A \cap B = \{12\}$; (12 is in both events A and B)

$n(A \cap B) = 1;$ $\qquad P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{20}$

$P(A \cup B) = ?$

Here, events A and B are not mutually exclusive events so by addition law of probability,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{5}{20} + \frac{6}{20} - \frac{1}{20}$$

$$= \frac{5+6-1}{20}$$

$$= \frac{10}{20}$$

$$= \frac{1}{2}$$

Hence, the probability of getting a number is either divisible by 4 or by 3 is $\frac{1}{2}$.

Example 4

From a well-shuffled pack of 52 playing cards, a card is drawn randomly. Find the probability of getting the king, queen or a jack.

Solution

Here, total number of cards $n(S) = 52$

Let K, Q and J be the events of getting the king, queen and jack respectively.

Nown, No. of king $n(K) = 4;$ $\qquad P(K) = \frac{n(K)}{n(S)} = \frac{4}{52} = \frac{1}{13}$

No. of queen $n(Q) = 4$; $P(Q) = \frac{n(Q)}{n(S)} = \frac{4}{52} = \frac{1}{13}$

No. of jack $n(J) = 4$; $= \frac{n(J)}{n(S)} = \frac{4}{52} = \frac{1}{13}$

Events K, Q and J are mutually exclusive events, so by addition law of probability;

$P(K \cup Q \cup J) = P(K) + P(Q) + P(J).$

$$= \frac{1}{13} + \frac{1}{13} + \frac{1}{13} = \frac{3}{13}$$

Hence, the probability of getting the king, a queen or jack is $\frac{3}{13}$.

Exercise 14.1

1. **Find out whether the given events are mutually exclusive or not.**

   (a) When a coin is tossed, A = getting t he head (H) and B = getting the tail (T).

   (b) When a dice is rolled, P = getting an even number and Q = getting an odd number die.

   (c) A card is drawn from well-shuffled pack of cards, F = getting a face card and A = getting a spade.

   (d) A card is drawn from well-shuffled pack of cards, T = getting 10 and A = getting an ace.

   (e) A bag contains 5 white, 8 green and 7 blue balls. A ball is drawn at random, G = getting green ball and B = getting blue ball

2. **Find the probability of the given events.**

   (a) Getting at least one head (H) when two coins are tossed

   (b) Getting prime number when a die is rolled

   (c) Getting a face card when a card is drawn from a well-shuffled pack of cards.

   (d) According to the English Months, a boy is born in a month with 30 days.

   (e) Getting a white ball when a ball is drawn from a bag containing 4 white, 7 green and 5 blue balls.

3. **What is the probability of the following events? Find.**

   (a) A bag contains 6 red, 5 yellow and 7 blue identical balls. If a ball is drawn randomly from the bag, it may be either, red or blue ball.

   (b) Three coins are tossed together, getting all three are head (H) or all three are tail.

   (c) When a die is rolled, getting prime number or 4.

   (d) When a card is drawn, from a well-shuffled pack of cards, 10 card or ace card (A).

   (e) When a card is drawn from a well-shuffled pack of cards, face card or spade cards.

4. **Find the probability of the following events.**

   (a) When a letter is drawn from MATHEMATICS, getting the letter M or T.

   (b) When a letter is drawn from STATISTICS, getting the letter S or T.

(c) When a letter is drawn from RHODODENDRON, getting the letter O or D.

(d) Out of 15 students of a class, 8 students opted English, 9 students Mathematics and 4 students opted both subjects. When a student is selected at random, what is the probability of getting Mathematics or English?

Answer

1. Show it to your teacher.

2. (a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{13}$ (d) $\frac{1}{3}$ (e) $\frac{1}{4}$

3. (a) $\frac{13}{18}$ (b) $\frac{1}{4}$ (c) $\frac{2}{3}$ (d) $\frac{2}{13}$ (e) $\frac{11}{26}$

4. (a) $\frac{4}{11}$ (b) $\frac{3}{5}$ (c) $\frac{1}{2}$ (d) $\frac{13}{15}$

14.2 Independent and Dependent Events

Activity 3

Compare the probabilities obtained from the following two cases

A bag contains 5 red, 7 green and 4 blue identical balls.

(i) First condition (with Replacement)

(a) What is the probability that the first ball is red?

(b) If the same ball (red ball) is placed in the same bag and the second ball is drawn, what is the probability that the second ball will be red?

(ii) Second condition (Without replacement)

(a) What is the probability that the first ball is red?

(b) If the same ball (red ball) is not replaced in the same bag and the second ball is drawn, what is the probability that the second ball will be red?

Comparing the probabilities obtained from the above two conditions

(i) First condition (With replacement)	(ii) Second condition (Without replacement)
Number of red balls n(R) = 5 Probability that the first ball is red $P_1(R)$ $= \frac{n(R)}{n(S)}$ $= \frac{5}{16}$ Now, there are 15 balls in the bag. The first drawn ball is replaced, so that there are again 16 balls in the bag. The probability that the second ball is red $P_2(R) = \frac{n(R)}{n(S)} = \frac{5}{16}$. It's the same as first.	Number of red balls n(R) = 5 Probability that the first ball is red $P_1(R)$ $P_1(R) = \frac{n(R)}{n(S)} = \frac{5}{16}$ Now, there are 15 balls in the bag. The first drawn ball is not replaced, so that there are again 15 balls in the bag. The probability that second ball is red $P_2(R) = \frac{n(R)}{n(S)} = \frac{4}{15}$. It is different than the first.
The occurrence of the first event does not affect the occurrence of the second event. So, these are independent events.	The occurrence of the first event affects the occurrence of the second event. So, these are dependent events

In two or more events, the occurrence or non-occurrences of any one event does not affect the occurrence or non-occurrences of any other events are called independent events. Similarly, in two or more events, the occurrence of any one event affects the occurrences of any other events are called dependent events.

Example 1

When a coin and a die are tossed at the same time, what is the event that head (H) comes up on the coin and 4 on the dice?

Solution

When a coin is tossed and a die is rolled simultaneously, we can get head (H) or tail (T) in the coin and we can get any numbers from 1 to 6 in the die. So, the event obtained in the dice does not affect the event we get in the coin. Hence, it is an independent event.

Example 2

Two cards are drawn from a well-shuffled deck of 52 playing cards one after another without replacement. What is the event that both cards are the king (K)?

Solution

Here, let K be the event of the king card in a sample space.

There are 4 king cards in a deck of 52 playing cards.

Therefore, n(K) = 4 and n(S) = 52

Probability of getting the king $P_1(K) = \frac{n(K)}{n(S)} = \frac{4}{52} = \frac{1}{13}$

If card 'K' obtained from the first case is not replace there are only 51 cards.

Now, in the second case, n(S) = 51 and n(K) = 3

Probability of getting the king second time $P_2(K) = \frac{n(K)}{n(S)} = \frac{3}{51} = \frac{1}{17}$. In this case, the probability of getting the king second time depends on the probability of getting the king in the first time. Hence, these are dependent events.

14.3 Multiplication Principle of Probability

Activity 4

Form pairs and solve the given problems.

When a coin is tossed and a cubical die is rolled,

(a) Write the possible sample space.

(b) What is the probability of getting the head (H) in a coin and 4 in a die?

(c) What types of events are these?

When a coin is tossed and a die is rolled simultaneously, we can get head (H) or tail (T) in a coin and we can get any numbers from 1 to 6 in a die. So, the events obtained in a die does not affect the event we get in a coin. Hence, these are independent events.

For a coin, sample space

$(S_1) = \{H, T\}$ $\therefore n(S_1) = 2$

For a cubical die, sample space $(S_2) = \{1, 2, 3, 4, 5, 6\}$ $\therefore n(S_2) = 6$

When a coin is tossed and a die is thrown, then the possible outcomes are(S) = { (H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)} $\therefore$ $n(S) = 12$.

Now, what is the probability of getting H in a coin and 4 in a die?

Let, A be an event of getting H in the coin, $\therefore$ n (A) = 1

Let, B be an event of getting 4 in the die. $\therefore$ n (B) = 1

H in the coin and 4 in the die $(A \cap B) = \{(H, 4)\}$; $n(A \cap B) = 1$

$P(A) = \dfrac{n(A)}{n(S_1)} = \dfrac{1}{2}$ र $P(B) = \dfrac{n(B)}{n(S_2)} = \dfrac{1}{6}$

Similarly, $P(A \cap B) = \dfrac{n(A \cap B)}{n(S)} = \dfrac{1}{12}$

Can you get a conclusions from the above solution?

$P(A) \times P(B) = \dfrac{1}{2} \times \dfrac{1}{6} = \dfrac{1}{12} = P(A \cap B)$

If two events A and B are independent events then, $P(A \text{ and } B) = P(A \cap B) = P(A) \times P(B)$.

Example 3

A coin is tossed and a spinner with three colors; green, blue and red is spun together. What is the probability of getting 'T' in the coin and the needle of the spinner stopping on green?

Solution

Here, sample space of the coin $(S_1) = \{H, T\}$

$\therefore n(S_1) = 2$

Probability of getting T on the coin $P(T) = \dfrac{n(T)}{n(S_1)}$
$= \dfrac{1}{2}$

Let G be the event of green. Similarly, the sample space of the spinner $(S_2) = \{$green, blue, red$\}$ $\therefore n(S_2) = 3$

Probability of the needle of the spinner stopping on green $P(G) = \dfrac{n(G)}{n(S_2)} = \dfrac{1}{3}$.

Probability of getting 'T' in the coin and the needle of the spinner stopping on green $P(T \cap G) = ?$

Since, getting 'T' in the coin and the needle of the spinner stopping on green is independent event, $P(T \cap G) = P(T) \times P(G) = \dfrac{1}{2} \times \dfrac{1}{3} = \dfrac{1}{6}$.

Alternative method

The possible outcomes of a coin and spinner S = {(H, Green), (H, Blue), (H, Red), (T, Green), (T, Blue), (T, Red)} $\therefore n(S) = 6$

The event of getting 'T' in the coin and the needle of the spinner stopping on green (A) = {(T, Green)} $\therefore n(S) = 1$ Probability of getting 'T' in the coin and the needle of the spinner stopping on green $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{1}{6}$.

Example 4

Two cards are drawn randomly in succession with a replacement from a well-shuffled pack of 52 cards. Find the probability of getting the first king card (K) and the second an ace card (A).

Solution

Here, there are 4 king cards in a deck of 52 cards. 'S' be the sample space.

Now, n(S) = 52, n(K) = 4

The probability of getting the first card king is $P(K) = \frac{n(K)}{n(S)} = \frac{4}{52} = \frac{1}{13}$.

If the first drawn card is replaced, there are 52 cards again. Number of ace cards also 4.

Now, $n(S) = 52$, $n(A) = 4$

The probability of gettig the second card ace is $P(A) = \frac{n(A)}{n(S)} = \frac{4}{52} = \frac{1}{13}$.

Probability of getting the first king card (K) and the second an ace card (A) = P (K ∩ A) =?

Since, K and A are independent events;

We know that, $P(K \cap A) = P(K) \times P(A) = \frac{4}{52} \times \frac{4}{52} = \frac{1}{169}$

Hence, the probability of getting the first king card (K) and the second an ace card (A) is = $\frac{1}{169}$.

Example 5

Two cards are drawn randomly from a well-shuffled deck of 52 cards in succession without replacement. Find the probability of getting the first king card (K) and the second an ace card (A).

Solution

Here, there are 4 king cards in a deck of 52 cards. Let, 'S' be the sample space.

Now, n(S) = 52, n(K) = 4

The probability of getting the first card king is $P(K) = \frac{n(K)}{n(S)} = \frac{4}{52} = \frac{1}{13}$.

If the second card is drawn without replacement of the first drawn card, so there are 52 - 1 = 51 cards remaining. But, number of ace card are still 4.

Now, $n(S_1) = 51$, $n(A) = 4$

The probability of getting the second card ace is $P(A) = \frac{n(A)}{n(S_1)} = \frac{4}{51}$

Probability of getting the first king card (K) and the second an ace card (A) = $P(K \cap A)$ = ?

Since, K and A are dependent events;

We know that, $P(K \cap A) = P(K) \times P(A) = \frac{4}{52} \times \frac{4}{51} = \frac{4}{663}$

Hence, the probability of getting the first king card (K) and the second an ace card (A) is $= \frac{4}{663}$.

Example 6

A bag contains 5 blue and 6 red marbles. Two marbles are drawn randomly one after another, what is the probability of getting the first one red and the second one blue?

(a) Second marble is drawn with replacing the first drawn marble

(b) Second marble is drawn without replacing the first drawn marble

Solution

Here, let, 'B' and 'R' be the events of red and blue marbles respectively. 'S' be the total number of balls in the bag.

Now, n(B) = 5, n(R) = 6 and n(S) = 5 + 6 = 11

Probability of getting a blue marble $P(B) = \frac{n(B)}{n(S)} = \frac{5}{11}$

Probability of getting a red marble $P(R) = \frac{n(R)}{n(S)} = \frac{6}{11}$

a) The second marble is drawn with a replacement of the first drawn marble

Probability of getting the first one red and the second one blue $P(B \cap R)$ = ?

Since, B and R are independent events;

We know that, $P(R \cap B) = P(R) \times P(B) = \frac{6}{11} \times \frac{5}{11} = \frac{30}{121}$

Hence, the probability of getting the first one red and the second one blue = $\frac{30}{121}$.

(b) Second marble is drawn without replacement of the first drawn marble

The probability of getting the first marble red $P(R) = \frac{6}{11}$

The second marble is drawn without replacement of the first drawn marble. So, there are 11 - 1 = 10 marbles remaining. But the number of blue marbles are 5.

Now, $n(S_1) = 10$, n(B) = 5

The probability of getting the second blue marble $P(B) = \frac{5}{10}$

Probability of getting the first red marble and second blue marble $P(R \cap B)$ =?

Since, the first drawn marble is not replaced, so R and B are dependent events;

We know that, $P(R \cap B) = P(R) \times P(B) = \frac{6}{11} \times \frac{5}{10} = \frac{3}{11}$

Exercise 14.2

1. When a coin is tossed and a dice is rolled simultaneously. What is the probability of getting tail (T) on coin and 3 on dice?

2. **A box contains 2 green, 3 red and 5 black balls of same shape and size. Two balls are drawn randomly and replaced. Then after, another ball is drawn. Find the probabilities of getting following balls.**

   (a) Both of them are of the same color.

   (b) Both of them are of the different color.

   (c) At least one ball is red or black

3. **A box contains 2 green, 3 red and 5 black balls of same shape and size. Two balls are drawn randomly and not replaced. Then after, another ball is drawn. Find the probabilities of getting following balls.**

   (a) Both of them are of the same color.

   (b) Both of them are of the different color.

   (c) At least one ball is red or black

4. A bag contains 7 red and 8 yellow balls of same shape and size. Two balls are drawn randomly one after another. Find the probabilities of getting both balls are red or yellow. (First drawn ball is not replaced in a bag)

5. **Two cards are drawn randomly from a well shuffled deck of 52 cards in succession without replacement.**

   (a) What is the probability of getting both are ace cards?

   (b) What is the probability of getting one is ace card or other is king card?

6. A bag contains one red, one green and one black marble of the same shape and size. Two marbles are drawn randomly and not replaced. Then after, another marble is drawn. Find all the probabilities.

Project work

Where is the use of probability in our daily life? Find. Prepare an article about its positive use and present it in the classroom.

Answers

1. $\frac{1}{12}$ 2. (a) $\frac{12}{25}$ (b) $\frac{31}{50}$ (c) $\frac{13}{20}$

3. (a) $\frac{14}{45}$ (b) $\frac{31}{45}$ (c) $\frac{57}{90}$ 4. $\frac{7}{15}$

5. (a) $\frac{1}{221}$ (b) $\frac{8}{663}$ 6. $\frac{1}{6}$

14.3 Tree Diagram

Activity 5

Make a list of the events that occur when a coin is tossed two times. For example; the results of tossing a coin twice can be shown as follows.

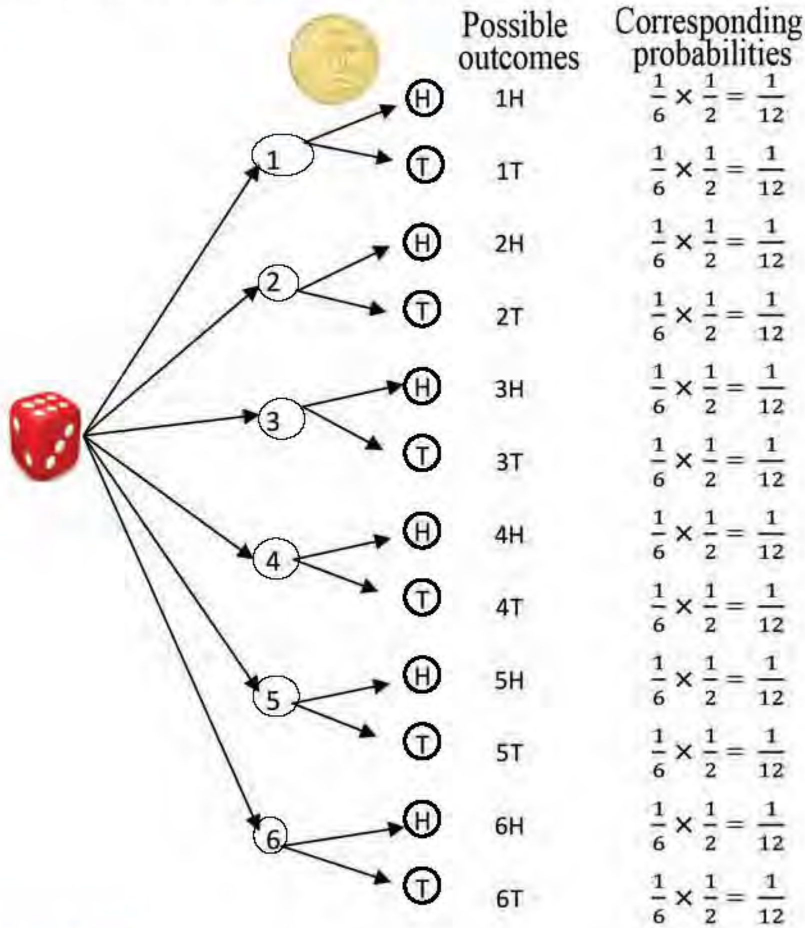
A picture like the above, which presents the results from a random experiment is called tree a diagram. Every branch of a tree diagram shows the probability of respective event. It gives all the possible outcomes and their probabilities. Like, from the above experiment sample space S = { HH ,HT, TH, TT}.

Example 1

Prepare a tree diagram showing the results and probabilities thay may come when a die is rolled and a coin is tossed simulation?

Solution

When a dice is rolled and a coin is tossed simultaneously the following tree diagram showin,g the events and their probabilities:



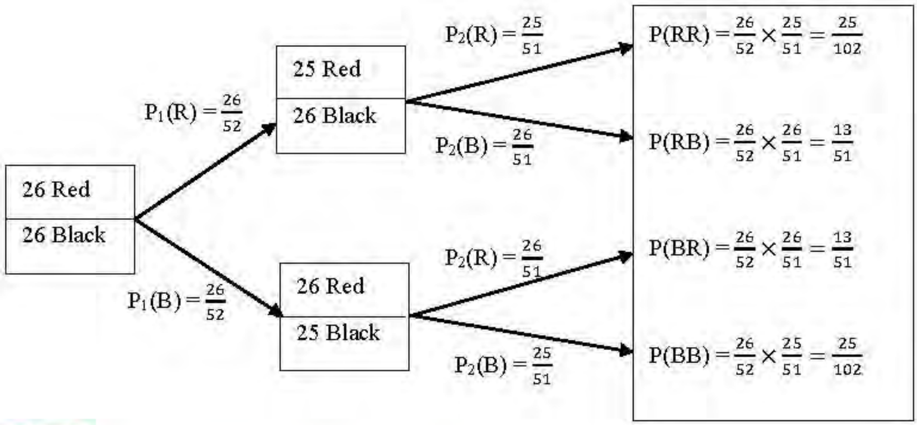
Example 2

Two cards are drawn randomly from a well-shuffled deck of 52 cards in succession without replacement. Draw a tree diagram to represent the probabilities of getting a red or black card.

Solution

Here,

Two cards are drawn randomly from a well-shuffled deck of 52 cards in succession without replacement. The following tree diagram shows the events and their probabilities:



Example 3

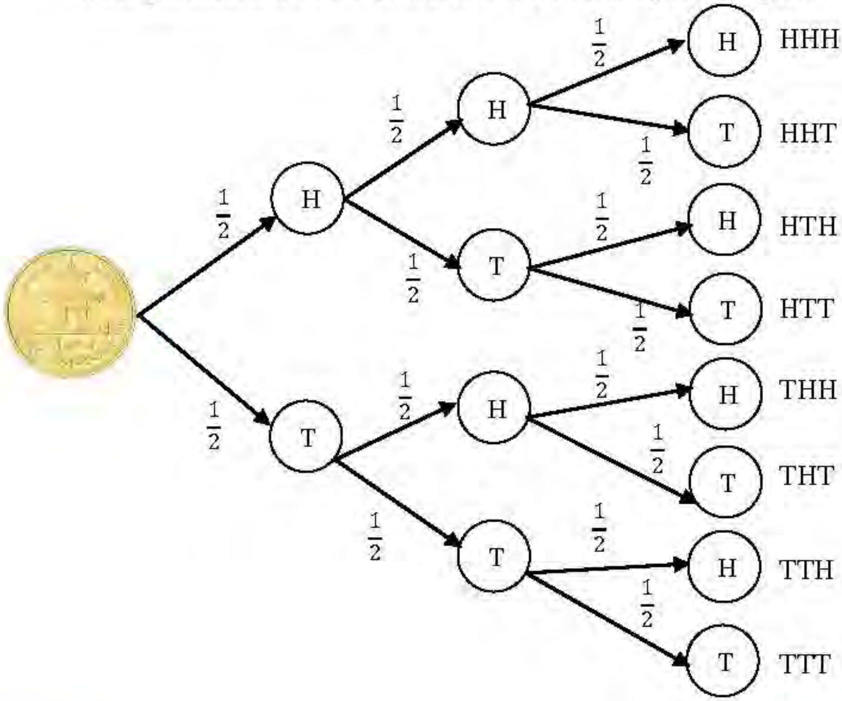
Three coins are tossed one after another.

(a) Draw a tree diagram by showing all the possible outcomes and their probabilities.

(b) Find the probabilities of getting at least two heads (H).

Solution

(a) When three coins are tossed one after another all the possible outcomes and their probabilities can be shown in a tree diagram as given below.



(b) The sample space (S) = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT} $\therefore n(S) = 8$

Let A be the event of getting at least 2 heads then, (A) = {HHH, HHT, HTH, THH}
$\therefore\ n(A) = 4$

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

Hence, the probabilities of getting at least two heads is $\frac{1}{2}$.

Example 4

A bag contains 12 blue, 15 green and 18 white identical balls. Two balls are drawn one after another without replacement. Show the probabilities of all the outcomes. By using a tree diagram, find the probabilities of the following.

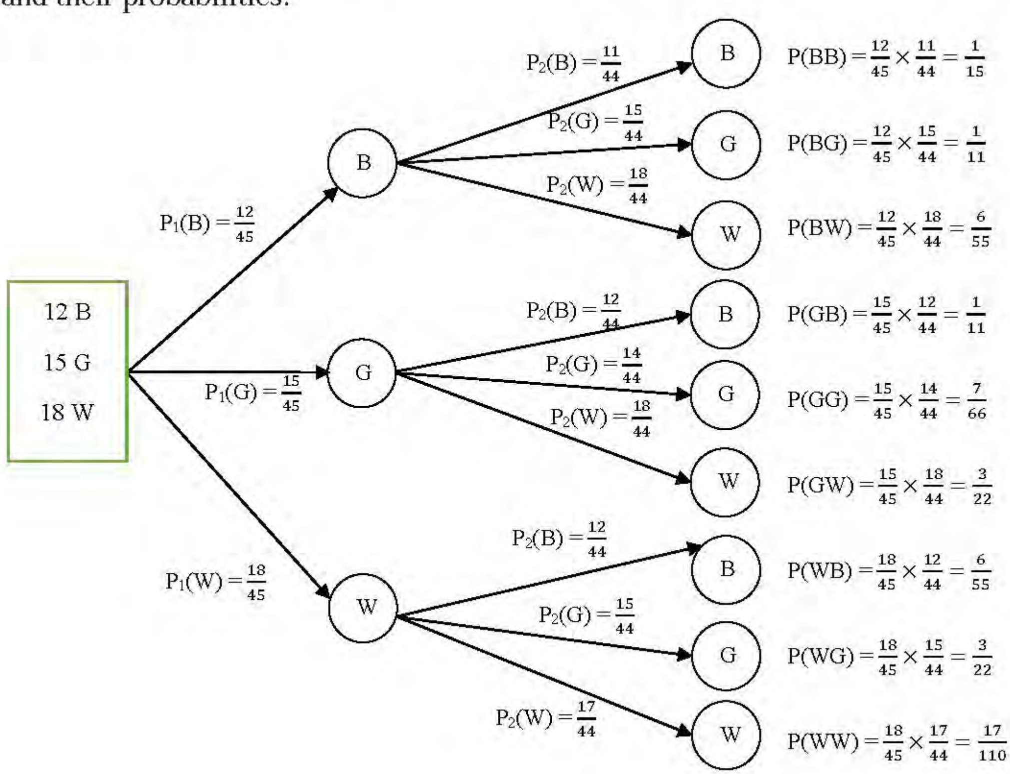
(a) Both balls are blue.

(b) First ball is white and the second ball is green.

(c) One ball is blue and the other ball is white.

Solution

A bag contains 12 blue, 15 green and 18 white identical balls. Two balls are drawn one after another without replacement. The following tree diagram shows the events and their probabilities:



Now, from the tree diagram,

(a) The probability of both balls are blue $P(BB) = \frac{1}{15}$

(b) The probability of the first ball is white and the second ball is green $P(WG) = \frac{3}{22}$

(c) The probability of one ball is blue and the other ball is white,

$P(BW) + P(WB) = \frac{6}{55} + \frac{6}{55} = \frac{12}{55}$

Exercise 14.3

1. **A coin is tossed three times. Draw a probability tree diagram to show all the possible outcomes. Find the probabilities of the following.**

   (a) All three are tails (T) (b) At least two heads (H)

   (c) Three tails (T)

2. **A spinner with three colors viz red, blue and brown is spun and a coin is tossed together. Draw a probability tree diagram to show all the possible outcomes. By using a probability tree diagram, find the probabilities of the following.**

   (a) The spinner can land on red and head (H) on coin.

   (b) The spinner can land on brown and tail (T) or head (H) on coin.

3. **A coin is tossed and a die is rolled one after another. Draw a probability tree diagram to show all the possible outcomes. By using a probability tree diagram, find the probabilities of the following.**

   (a) Head (H) on the coin and even number on the dice.

   (b) Tail (T) on the coin and square number on the dice.

4. Prepare a tree diagram to show the probability of getting a card (spade club, diamond and heart) from a well-shuffled pack of 52 cards and when a coin is tossed. Draw a probability tree diagram to show all the possible outcomes. By using a probability tree diagram, find the probabilities of the following.

   (a) Red on the card and head (H) on the coin.

   (b) Black on the card and tail (T) on the coin.

5. A bag contains 7 red and 5 green marbles. Three marbles are drawn one after another; (i) with replacement (ii) without replacement

   Draw a probability tree diagram to show all the possible outcomes.

Project work

Take a deck of 52 cards. Three cards are drawn one after another at random without replacement. Out of these three cards; draw probability diagram for the following.

(a) All three cards are spade.

(b) Only two cards are spade.

(c) Only one card is spade.

(d) All three cards are not spade.

Answer

1. $\frac{1}{8}$ (b) $\frac{1}{2}$ (c) $\frac{1}{8}$ 2. (b) $\frac{1}{6}$ (b) $\frac{1}{3}$

3. (a) $\frac{1}{4}$ (b) $\frac{1}{6}$ 4. (a) $\frac{1}{4}$ (b) $\frac{1}{4}$ 5. Show it to your teacher.

Mixed Exercise

1. The heights (in cm) of 50 people are given below.

Height (in cm)
125, 137, 155, 149, 122, 128, 133, 144, 115, 118, 142, 145, 151, 157, 159, 160, 165, 162, 156, 158, 155, 141, 147, 149, 148, 159, 154, 155, 166, 168, 169, 172, 174, 173, 176, 161, 164, 163, 149, 150, 154, 153, 152, 164, 158, 159, 162, 157, 156, 155

(a) Construct a frequency distribution table with the class interval 10 of the given data.

(b) Find the mean from the above frequency distribution table.

(c) Show the histogram of frequency distribution table prepared in (a).

(d) What is the median value of the above data?

2. The table given below represents the weight (in kg) of students of a school.

Weight (in kg)
18, 20, 13, 24, 35, 34, 56, 45,33, 23, 24, 56, 33, 22, 26, 35, 39,44, 42, 47, 46, 48, 55,51,44,40,47, 49,34, 31, 28, 29, 35, 39, 28, 48,51,50,47,23,19,27, 57, 42, 33, 23, 38,36, 45, 45,37,29, 27, 22, 28, 36, 35,57, 54, 40, 50, 30, 29

(a) Construct a frequency distribution table with the class interval 5 of the given data.

(b) Prepare a less than and more than cumulative frequency table.

(c) Find the difference of mode and median.

3. In a survey conducted in a community related to the job holder's age obtained the following data: :

Age (year)	0-15	15-30	30-45	45-60	60-75
No. of person	5	6	10	6	3

(a) In the above table, which age group has the maximum number of employees?

(b) What is the average age of the employees?

(c) Angel said that mean and median lie on the same class interval. Is it right?

4. The details of monthly electricity consumption by 40 households of Thaha Municipality are as follows:

Consumption unit	10-20	20-30	30-40	40-50	50-60	60- 70	70- 80
No. of household	5	6	8	9	7	4	1

(a) From the above table, prepare the less than and more than cumulative frequency table.

(b) On the basis of the answer of (a), draw the cumulative frequency curve.

(c) Do the mean and median lie on the same class interval?

5. The data given below represents the number of patients admitted in a hospital in a week.

Age (yrs)	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No. of patients	4	5	8	13	12	8	4	9	7

(a) From the above data, find the first and third quartiles.

(b) Which value of the given data divides the number of the patients into two equal parts?

(c) Niruta said that mode and median lie on the same class interval. Is she correct right?

6. The given data represents the monthly expenditure (in thousands) of the families of Lamapatan

Expenditure(000)	10-20	20-30	30-40	40-50	50-60
No. of family	3	2	6	5	4

(a) From the above data, write the class interval of mode.

(b) From the above data, what is the expenditure of the maximum family?

(c) From the above data, find the first and third quartiles.

7. The marks obtained in Mathematics by the students of class 10 in the second terminal examination are given below in the table.

Marks obtained	0 - 5	5 - 10	10 - 15	15 - 20	20 - 25	25 - 30
No. of students	2	4	5	3	2	4

(a) How many students of class 10 obtained the marks less than 20?

(b) Construct the cumulative frequency table on the basis of the given table.

(c) Find the values which divide the data given in the above table into 4 equal parts.

(d) What is the maximum number of students who obtained the marks less than the median, based on the median?

8. Three coins are tossed together then,

(a) Write the possible outcomes

(b) Find the probability of getting only one head.

(c) Find the probability of getting at least 2 tails (T).

(d) Find the probability of not getting head (H).

(e) Find the probability of getting 2 heads (H) or not to get H.

9. A card is drawn randomly from a well-shuffled deck of 52 cards then,

(a) What is the probability of getting an ace?

(b) Find the probability of getting a spade or diamond.

(c) Find the probability of getting a spade or ace.

(d) Find the probability of getting a club or face card.

10. A coin is tossed and a spinner with 4 colours (blue, green, red, purple) is spun together:

(a) Draw a probability tree diagram to show all the possible outcomes. Find the possible outcomes of this experiment separately and together.

(b) Find the probability of getting the head (H) in the coin and landing the needle at green or blue.

(c) Find the probability of getting head (H) in a coin and landing the needle at green.

11. Two cards are drawn randomly from a well-shuffled deck of 52 cards one after another, then,

(a) If the first drawn card is replaced and drawn another card, what is the probability of getting both cards are spade?

(b) If the first drawn card is not replaced and second card is drawn, what is the probability of getting the first card spade and the second card heart?

(c) Show the probability tree diagram for (a) and (b) separately.

12. **A bag contains 7 red and 8 yellow balls of the same shape and size:**

(a) If the balls are drawn one after another (without replacement), find the probability of getting both balls are red.

(b) If two balls are drawn one after another (with replacement), find the probability of getting both balls are red.

(c) Ramila said that both of the above conditions are independent. Is she correct? Write.

13. **A bag contains one red, one green and one black ball of the same shape and size. A ball is drawn randomly and without replacing the ball, another ball is drawn from the bag. Then,**

(a) Show the probability tree diagram for the events.

(b) Find the probability of the events.

14. **A coin is tossed and a die is thrown then,**

(a) Draw a probability tree diagram to show all the possible outcomes.

(b) By using the probability tree diagram, find the probabilities of the following.

(i) Head (H) in the coin and even number in the dice.

(ii) Tail (T) in the coin and square number in the dice.

(c) Are the events of (b) independent? Write with reason.

15. **A card is drawn randomly from a well-shuffled deck of 52 cards (spade, club, diamond and heart) and a coin is tossed.**

(a) Draw the probability tree diagram to show all the possible outcomes.

(b) On the basis of probability tree diagram, write the sample space.

(c) By using a probability tree diagram, find the probabilities of the following:

(i) Red in cards and H on the coin.

(ii) Black in cards and tail (T) in the coin.

(d) Bikash said that the events obtained from the cards and the coin is dependent. Is that statement, correct? Write.

16. **A bag contains 7 red, and 5 green marbles. 2 marbles are drawn randomly one after other: (a) With replacement (b) Without replacement. Draw a probability tree diagram to show all the possible outcomes in the following conditions.**

(i) On the basis of the tree diagram, what is the probability of getting both marbles red?

(ii) What is the probability of getting both marbles green?

(iii) Is the probability of getting first red and the second green equal to the probability of getting the first green and the second red? (A marble is drawn and not replaced before drawing second.)

17. From the number cards number from 1 to 20, a card is drawn at random,

(a) Make a list of number cards divisible by 3 and divisible by 5.

(b) What is the probability of getting a number card having a number divisible only by 3 or divisible only by 5?

(c) Kopila said that the events in question (a) are not mutually exclusive events. Is Kopila's statement correct? Write with reason.

(d) What is the probability of getting a number card having a number either divisible by 3 or by 5?

18. In a bag, there are 5 red and 3 blue balls. A ball is drawn randomly and replaced before ing the second ball then,

(a) Find the probability of getting both blue balls.

(b) What is the probability that none of them are blue?

(c) Show both the above probabilities in a tree diagram.

Answer

1 - 7 Show them to your teacher.

8. (a) {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

(b) $\frac{3}{8}$ (c) $\frac{1}{2}$ (d) $\frac{1}{6}$ (e) $\frac{1}{2}$

9. (a) $\frac{1}{13}$ (b) $\frac{1}{2}$ (c) $\frac{4}{13}$ (d) $\frac{11}{26}$

10. (b) $\frac{1}{4}$ (c) $\frac{1}{8}$ 11. (a) $\frac{1}{16}$ (b) $\frac{13}{204}$ (c) Show it to your teacher

12. (a) $\frac{1}{5}$ (b) $\frac{49}{225}$ (c) no

13. show them to your teacher

14. (b) (i) $\frac{1}{4}$ (ii) $\frac{1}{6}$ (iii) हुन्

15 - 16 show to your teacher.

17. (a) $M_3 = \{3, 6, 9, 12, 15, 18\}$, $M_5 = \{5, 10, 15, 20\}$ (b) $\frac{2}{5}$

(c) yes (d) $\frac{9}{20}$

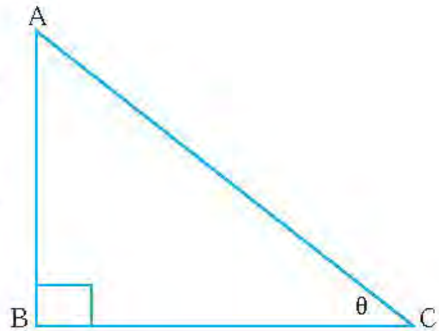
18. (a) $\frac{9}{64}$ (b) $\frac{25}{64}$

Lesson 15

Trigonometry

15.0 Review

A right angled triangle ABC is given. Based on this, discuss on the following questions.



(a) What is the area of the right angled triangle ABC?

(b) Find the trigonometric ratios sinθ, cosθ, and tanθ.

(c) State the relationship between the sides of a right angled triangle.? What is this relation called?

(d) If AB = 6 cm and AC = 10 cm, then find the trigonometric ratios.

(e) What are the values of sin θ, cos θ and tan θ. When θ = 30°, θ = 45°, θ = 60° and θ = 90°.

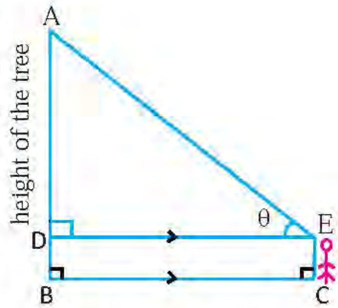
Values	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

15.1 Angle of Elevation and Angle of Depression

Activity 1

Ramnaresh is looking at the tree on the school ground and seems to be thinking about something. At the same time, his Mathematics teacher arrives.

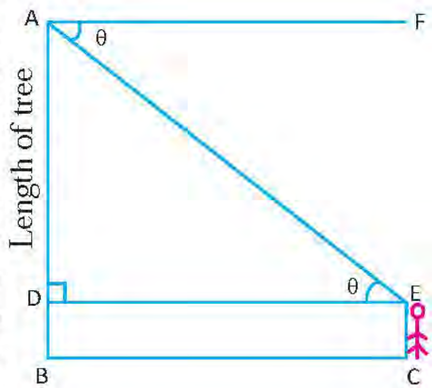
Teacher: What happened to you, Ramnaresh? Why are you staring at a tree continuously?



Ramnaresh: Sir Namskar. Sir, how tall is this tree? What is the distance between the tree and the place where I am standing? What is the height of the flagstaff of the school? Sir, I am thinking about how to find all these things.

Teacher: Wait Ramnaresh, you have asked how many questions. Let's go to the classroom, make a picture of a question, and discuss it with your friends. (Both go to the classroom.)

Teacher: Ramnaresh, the first question you asked was how tall the tree on the ground is, Right? Look at the picture on the whiteboard. Let x be the angle formed when we look at the top of the tree. In the figure, BD = CE. Now, if we can find the value of AD, we can find the height of the tree. A right-angled triangle ADE is formed. Let's try to remember how we derive the trigonometric ratios, in class 9.



Omkumari: What is the name of the angle, formed by the line of sight while looking at the top of the tree? How can we find it?

Teacher: Omkumari, first of all, we have to draw an imaginary line from our eye level parallel to the ground (DE in the picture). When we look at the top of the tree, an angle is formed by the line of our sight (EA in the figure) with the line parallel to the ground. This is called an angle of elevation. In the figure, $\angle AED$ is the angle of elevation.

Ramnaresh: Sir, even if you look from the top of the tree (point A in the picture) to point E, the angle is equal, isn't it? What is it called?

Teacher: Yes, you are right Ramnaresh. In this situation too, first we have to draw an imaginary line parallel to the ground from the top of the tree at the height of the top of the tree (AF in the figure). Yes, the angle made by our line of sight (AE in the figure) with the same line is the angle made by us from the top of the tree. This is called an angle of depression. In this figure, $\angle FAE$ is the angle of depression. Now, in which condition are the angle of elevation and the angle of depression formed in the following statements?

(a) Looking at the top of a house from the ground
(b) Looking at the top of a tree from the ground
(c) Looking at the top of a tower from the ground
(d) Looking at a vehicle from the roof of a house
(e) Observing the floor from the top of Dharahara
(f) Observing the ground from the top of a school building

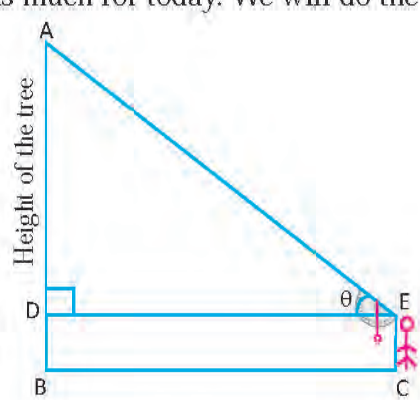
When an observer observes any object from a lower place to an upper place, the angle formed between the line of our sight and the line parallel to the ground is called the angle of elevation. In the figure, $\angle AED$ is called the angle of elevation. Similarly, when an observer observes any object from the upper place to the lower place, then the angle formed between the line of our sight and the horizon is called an angle of depression. In the figure, $\angle FAE$ is called the angle of depression. The instrument used to measure these angles is called a clinometer.

Ramnaresh: Sir, which is bigger, the angle of elevation formed when we look at the top of the tree or the angle of elevation formed when we look at the middle of the tree?

Teacher: Find out yourself. How the angle made by the line of sight changes when you look at the mid of the tree and top of the tree?

Ramnaresh: Looking at the top of the tree, angle made by the line of sight was greater than when looking at the mid-part of the tree. It seemed that the angle of elevation formed when looking at the top of the tree is larger than the angle of elevation formed when looking at the middle of the tree.

Teacher: Ramnaresh, what you said is correct. The angle of elevation formed when we look at the top of the tree is greater than the angle of elevation formed when we look at the middle of the tree. Similarly, the angle of depression formed when we look from the top of the tree is bigger when we look at a shorter distance than when we look from a farther distance. Let's do this much for today. We will do the rest the the question tomorrow.



Activity 2

(Next day in class)

Omkumari: Yesterday, we were talking about finding the height of trees, sir. How to find the height of the tree in the lawn?

Teacher: In the right triangle ADE, if we know the value of $\angle AED$ and the distance of the tree from us i.e. (ED), then we can find the height of the tree. Let's look at the top of the tree with a Clinometer. The angle of elevation of the top of the tree is found to be 45°. If the height of your eyes (CE) = 1.6 m, the angle of elevation $\angle AED = 45°$ and the distance from you to the bottom of the tree is BC = DE = 30 m then,

From the right angled triangle ADE

$\tan 45^\circ = \frac{AD}{30}$

or, $1 = \frac{AD}{30}$

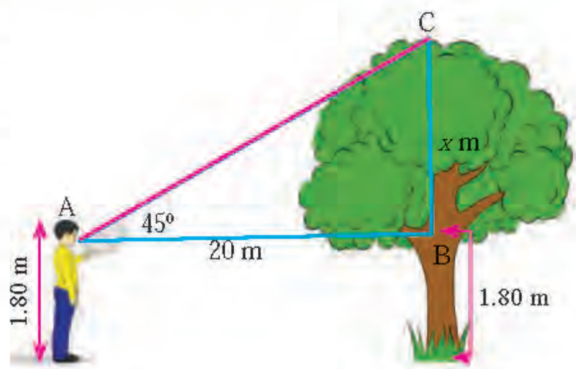
or, AD = 30 m

Hence, the total height of the tree = 30 + 1.60 = 31.60 m.

Example 1

A man of 1.80 meter high observes the angle of elevation at the top of tree and found it to be 45°. If the distance of the man and the tree is 20 meter, find the height of the tree.

Solution



When the man observes to the top of the tree, the angle between the line of sight and the line parallel to the ground is 45°.

In the figure, the height of the man is 1.80 meter and the total height of the tree = (x + 1.80) meter

In the right angled triangle ABC

We have,

$\tan 45^\circ = \frac{x}{20}$

or, $1 = \frac{x}{20}$

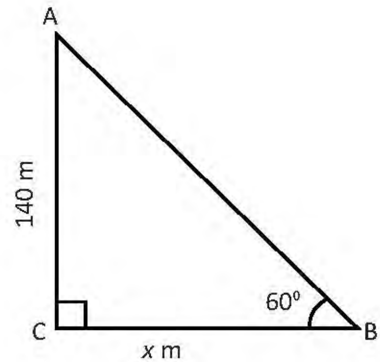
or, $x = 20$ m

Hence the height of tree = 20 + 1.80 = 21.80 meter.

Example 2

A man finds the angle of elevation of the top of a tower to be 60°. The height of tower is 140 m and the distance between man and tower is x m. Find the value of *x*.

Solution



Here,

Height of tower (AC) = 140 m

The distance between man and tower (BC) = x m

The angle of elevation $\angle ABC = 60°$

In right angled triangle ACB,

We have, $\tan 60° = \frac{AC}{BC}$

or, $\sqrt{3} = \frac{140}{x}$

or, $x\sqrt{3} = 140$

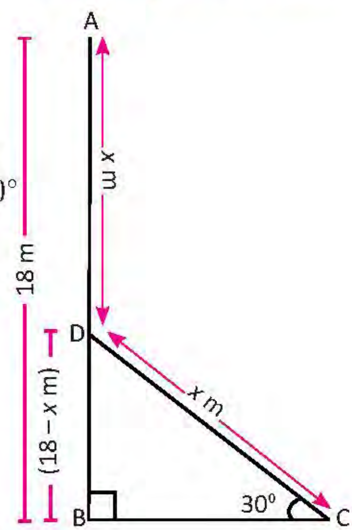
or, $x = \frac{140}{1.732}$ $\therefore$ BC = 80.83 m

Hence, the distance between man and tower is 80.83

Example 3

A tree 18 m high is broken by the wind so that its top touches the ground and makes an angle of 30° with the ground. Find the length of broken part of the tree.

Solution



Let, the height of tree (AB) = 18 m

The length of broken part of tree (AD) = CD = x m

The height of remaining part of tree (BD) = $(18 - x)$ m

The angle making by broken part of tree = $\angle DCB = 30°$

Now,

In right angled triangle CBD,

We have, $\sin 30° = \frac{BD}{CD}$

or, $\frac{1}{2} = \frac{18 - x}{x}$

or, $x = 36 - 2x$

or, $x + 2x = 36$

or, $3x = 36$

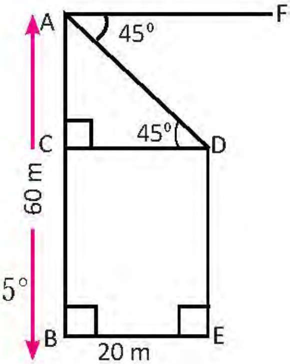
or, $x = 12$ m

Hence, the length of broken part of the tree is 12 m

Example 4

The distance between a tower and a house is one third of the height of the tower. If the height of the tower is 60 m and the angle of depression from the top of the tower to the house is 45°, find the height of the house.

Solution



Let, the height of the tower (AB) = 60 m

DE be the height of the house

BE be the distance between the tower and house.

The distance between the tower and house;

$BE = CD = 60 \times \frac{1}{3} = 20\ m$

We know that, DE = BC and by alternate angle = $\angle FAD = 45°$

Now, in the right angled triangle ACD,

We have, $\tan 45° = \frac{AC}{CD}$

or, $1 = \frac{AC}{20}$ or, AC = 20 m

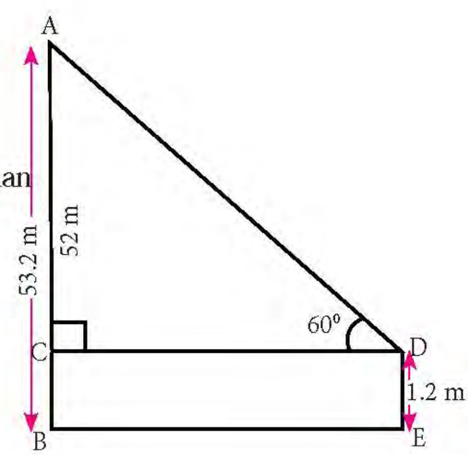
Again, BC = DE = AB - AC = 60 - 20 = 40 m

Hence, the height of the is 40 m.

Example 5

A man 1.2 m tall observes the angle of elevation at the top of a tower and finds to be 60°. If the height of the tower is 53.2 m, find the distance between the tower and the man.

Solution



Let, the height of the tower (AB) = 53.2 m

The height of the man (DE) = 1.2 m

BE be the distance between the tower and the man

The angle of elevation $\angle ADC = 60°$

We know that, DE = BC and BE = CD

($\because$ Being opposite sides of a rectangle)

Now, in the right angled triangle ACD,

We have, $\tan 60° = \frac{AC}{CD}$.

or, $\sqrt{3} = \frac{AB - BC}{CD}$

or, $\sqrt{3} = \frac{53.2 - 1.2}{CD}$

or, $\sqrt{3} = \frac{52}{CD}$

or, $\sqrt{3} \times CD = 52$

or, $CD = \frac{52}{\sqrt{3}}$

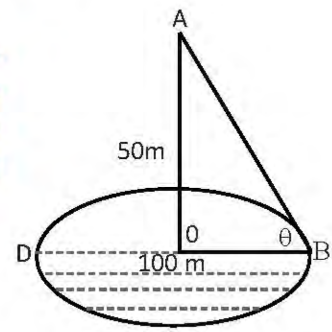
or, $CD = \frac{52}{1.732}$

or, CD = 30.02 m

Hence, the distance between the tower and the man is 30. 02 m.

Example 6

The diameter of a circular pond is 100 m. A pole is fixed at the centre of the pond and the height of the pole above the water surface is 50 m, what is the angle of elevation to the top of the pole observed from the edge of the pond? Find it.



Solution

Let, the diameter of the pond (BD) = d = 100 m

The radius of the pond (OB) = d/2 = 100/2 = 50 m

AO = 50 m be the height of the pole above the water surface.

The angle of elevation from B to A $\angle ABO = \theta$

Now, in right angled triangle AOB,

We have, $\tan\theta = \frac{OA}{OB} = \frac{50}{50} = 1$

or, $\tan\theta = \tan 45^\circ$

$\therefore \quad \theta = 45^\circ$

Hence, the angle of elevation to the top of pole observed from the edge of the pond is 45°.

Exercise 15

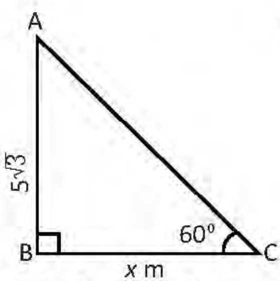
1. (a) Define the angle of elevation and depression with example.

   (b) In a right angled triangle, which trigonometric ratio is related to both perpendicular and base?

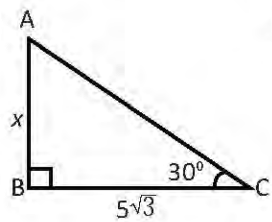
   (c) In a right angled triangle, which trigonometric ratio has the relation with perpendicular and hypotenuse?

2. **Find the value of *x* on the given right-angled triangles.**

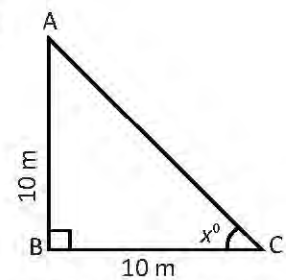
(a)



(b)



(c)



3. (a) The height of a tower is 60 m and the distance between a man and a tower is x m. A man finds the angle of elevation of the top of a tower to be $30°$. Find the value of x.

   (b) The height of tower is 12 m and the distance between man and tower is 12 m. The man finds the angle at elevation of the top of the tower to be $x°$. Find the value of x.

   (c) The height of a tower is x m and the distance between a man and the tower is 12 m. The man finds the angle of elevation at the top of the tower to be $45°$. Find the value of x.

4. (a) A tree of 14 m high is broken by the wind so that its top touches the ground (not separated from the main stem and makes an angle of $60°$ with the ground. Find the length of the broken part of the tree.

   (b) A tree is broken from the middle part by the wind so that it's top touches the ground (not separated from the main stem and makes an angle of $60°$ with the ground. If the length of the broken part of the tree is 7.5 m, find the height of the tree before it was broken.

   (c) A tree is broken at the middle part by the wind so that it's top touches the ground (not separated from the main stem and makes an angle of $30°$ with the ground. The length of the broken part of the tree is 30 m.

(i) Find the height of the tree before it was broken.

ii) How far does it meet the ground level from the base of the tree?

5. (a) A man of 1.7 m height observes the angle of elevation at the top of a tower and finds it to be 60°. If the distance between the tower and the man is 30 m, find the height of the tower.

(b) A man of height 2 m is flying a kite from the roof of a house of 33.6 m. If the length of the string of the kite is $90\sqrt{2}$ m and it makes an angle of $45^\circ$with the horizon, find the height of kite from the ground level.

(c) A 1.5 m tall man observes the angle of elevation at the top of a tree of 51.5 m height and finds it to be 45°, find the distance between the tree and the man.

6. (a) The distance between a tower and a man is 20 m. The height of the ree is 36.5 m. If the angle of elevation from the eye of the man to the top of the tower is 60°, find the height of the man.

(b) From the top of a house of 30 ft hight, the angle of depression at the top of a tree is 30°. If the distance between the house and the tree is $10\sqrt{3}$ m, find the height of the tree.

(c) The height of tower is 60 m and the distance between the tower and house is 35 m. If a man observes the angle of depression from the top of tower to the top of house is 45°, find the height of house.

7. (a) The diameter of a circular pond is 90 m. A pole is fixed at the centre of the pond and height of the pole above the water surface is 45 m, what is the angle of elevation of the top of pole observed from the edge of the pond? Find it.

(b) The diameter of a circular pond is 130 m. A pole is fixed at the centre of the pond. A person finds the angle of elevation of the top of the pole observed from the edge of the pond is 45°. What is the height of the pole above the water surface? Find it.

(c) At the centre of a circular pond, there is a pole of 11.62 m height above the surface of water. From a point on the edge of the pond, a man of 1.62 m height observed the angle of elevation at of the top of the pole and found it to be 30°. Find the diameter of the pond.

8. (a) On the occasion of a festival, Ramesh is flying a kite. The thread of the kite makes an angle of 30° with the horizon. If the length of the thread is 120 m and the Ramesh's height is 1.5 m, find the height of the kite from the ground.

(b) 1.5 m tall Ramsharan is flying a kite from the roof of a house of 9 m height. The string of the kite makes an angle of $30°$ with the horizon. If the height of the kite from the ground is 58 m, what is the length of the string. Find it.

(c) A man of height 2 m is flying a kite from the roof of a house of 32 m high. If the length of the string of the kite is $66\sqrt{2}$ m and makes an angle of $60°$ with the horizon, work out to calculate the height of the kite from the ground?

9. The length of the shadow of a pole of 20 m high at 2 pm is $20\sqrt{3}$ m. on the meantime. Find the length of the shadow of the tower with the height $25\sqrt{3}$ m?

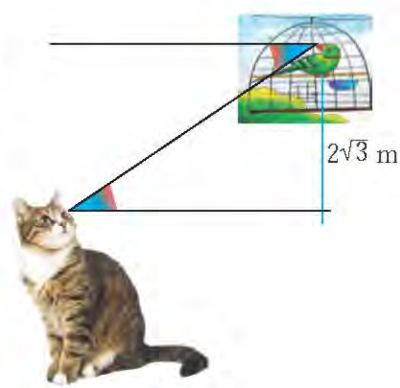
10. A tree 25 m high is just in the one corner of the ground of a school. A man of height 1.2 m is sitting on another corner of the ground. The distance between the man and the tree is 23.8 m.

(a) What is the angle of elevation at the top of the tree made by the man?

(b) Does the angle of elevation increase when the height of the tree is increases? Write with reason.

(c) Is the angle of elevation whether increases or decreases when the distance between the man and tree is decreases. Write with reason.

11. In the given figure two buildings 20 m and 32 m are shown. The distance between then is 12 m.



(a) What is the name of the the angle formed with the horizontal line when the parrot looks at the cat?

(b) Find the angle in degree with the horizontal line when the parrot looks at the cat?

(c) What is the distance between the pole and cat?

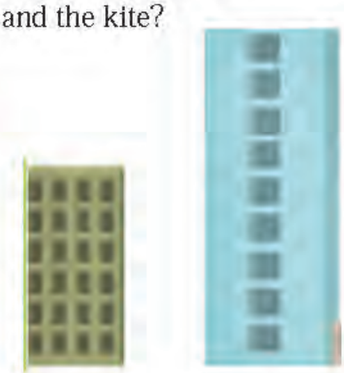
(d) Does the angle the parrot makes when looking at the cat decrease as the cat moves towards the pole? Write with reason.

12. The circumference of a circular pond is 176 m. A pole is fixed at the centre of the pond. From a point on the edge of the pond, a man of 1.6 m tall observed the angle of elevation of the top of the pole and found it be $45°$.

(a) Find the distance between the man and the pole.
(b) What is the height of the pole above the water surface? Find it.
(c) How much less should the height of the pole above the water surface such that it would have made the angle of elevation of 30°?

13. A man of height 1.2 m is flying a kite form the roof of a house 8.8 m high. If the length of the string of the kite is 180 m and makes an angle of 30° with the horizon.

(a) Represent the given relationship in a diagrammatical form.
(b) What is the height of the kite in meter from the ground? Find it.
(c) What is thecheck distance between the man and the kite? Find it.



14. In the figure, two buildings 20 m and 32 m high are shown. The distance between them is 12 m.

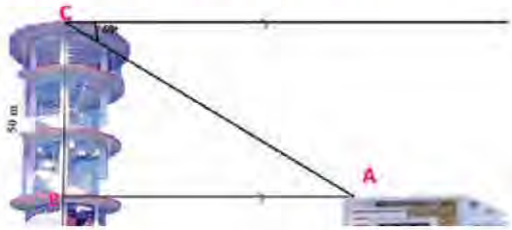
(a) What type of angle is formed when observed from the roof of tall building to the roof of the short building?
(b) Explain with reason the relation between the angle that forms when you observes the top of the tall building from the top of the short building and the top of the short building from the top of the big building.
(c) Calculate the angle in degree, when an observer observes from the roof of the small building to the roof of big building.
(d) If the ladder is put from the top of the small building to the top of the big building, what should be the length of the ladder? Calculate it.

15. The tower and the house are on the same ground level and distance between them is 25 m. The height of the house is 15 m.

(a) The angle of elevation from a point A of the roof of the house to the top of tower B is 45°, what is the height of the tower?
(b) From the figure, how the angle of elevation changes if the height of the tower increases. The angle of depression of a house 20 m to the east of the tower is 60° when observed from top of the tower of height 50 m.

16. **From the top of the 50 m hight tower an angle of 60° is formed when looking at the roof of a house of 20 m to the east of the tower.**

(a) What is the angle formed when observed from a point A on the roof of the house to the top of the view tower?



(b) What is the height of the part BC of the tower? Find it.

(c) What is the height of the house? Find it.

(d) What distance does she have to come down from the top of the tower to observe the terrace (top) of the house such that the angle of depression of 45°

Project work

Make a group of friends and find out the things in the highest and lowest areas at around your house. Find out practically the angles with respect to their height and distance or the height with respect to their angles and distance. Discuss the result obtained thus and find out the conclusion, and present it in the class.

Answer

2. (a) 5 m	(b) 5 m	(c) 45°	
3. (a) $60\sqrt{3}$ m	(b) 45°	(c) 12 m	
4. (a) 7.5 m	(b) 14 m	(c) 45 m, $15\sqrt{3}$ m	
5. (a) 53.6 m	(b) 125 m	(c) 50 m	
6. (a) 1.86 m	(b) 20 ft	(c) 25 m	
7. (a) 45°	(b) 65 m	(c) 11.55 m	
8. (a) 61.5 m	(b) 95 m	(c) 100 m	
9. 30°, 75 m	10. (a) 45°	11. (a) 30° (b) 6 m	
12. (a) 28 m	(b) 29 m	(c) 11.83 m	
13. (a) 100 m	(c) $90\sqrt{3}$ m	14. (a) and (b) Show ot to your teacher	
(a) 45°	(d) $12\sqrt{2}$ m	15. (a) 40 m	
16. (a) 60°	(b) 34.64 m	(c) 15.36 m	(d) 14.64 m